Chapter 30 SPECIAL RELATIVITY

1. $t = 3.1 \times 10^{-8}$ s

$t_0 = 2.6 \times 10^{-8}$ s

$$t = \frac{t_0}{\sqrt{1 - v^2/c^2}}$$

$$\therefore 1 - \frac{v^2}{c^2} = \frac{t_0^2}{t^2}$$

$$\therefore v = \sqrt{c^2\left(1 - \frac{t_0^2}{t^2}\right)} = c\sqrt{1 - \frac{(2.6 \times 10^{-8})^2}{(3.1 \times 10^{-8})^2}}$$

$$= \underline{0.54\ c}$$

2. $v = 2.85 \times 10^8$ m s^{-1}

$t = 2.50 \times 10^{-8}$ s

Proper time, t_0, for an elementary particle is called its rest life.

$$t_0 = t\sqrt{\left(1 - \frac{v^2}{c^2}\right)} = 2.5 \times 10^{-8}\sqrt{\left(1 - \frac{(2.85 \times 10^8)^2}{(3.0 \times 10^8)^2}\right)}$$

$$= 2.50 \times 10^{-8} \times 0.31$$

$$= \underline{7.9 \times 10^{-9}\ s}$$

3. (a) $$v = \frac{1.8 \times 10^7\ m\,s^{-1}}{3.0 \times 10^8\ m\,s^{-1}} = \underline{0.06\ c}$$

(b) $0.95c = 0.95 \times 3 \times 10^8\ m s^{-1} = \underline{2.85 \times 10^8\ m s^{-1}}$

(c) $30\ ly = \underset{years}{30} \times \underset{days}{365} \times \underset{hours}{24} \times \underset{sec}{3600} \times \underset{c\ in\ km/s}{3 \times 10^5}$

$= \underline{2.8 \times 10^{14}\ km}$

(d) $$3 \times 10^{15}\ km = \frac{3 \times 10^{15} \times 10^3\ m}{365 \times 24 \times 3600 \times 3 \times 10^8\ m\ yr^{-1}}$$

$= \underline{317\ ly}$

4. $L_0 = 40.0$ m

$v = 630$ m s^{-1}

$L = ?$

$$L = L_0\sqrt{1 - v^2/c^2} = 40\sqrt{1 - \frac{(630)^2}{(3 \times 10^8)^2}}$$

$= \underline{40\ m}$

5. $L = 20$ ly

$L_0 = 85$ ly

$v = ?$

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}$$

$$\therefore \sqrt{1 - \frac{v^2}{c^2}} = \frac{L}{L_0}$$

$$1 - \frac{v^2}{c^2} = \frac{L^2}{L_0^2}$$

$$\therefore v = \sqrt{c^2\left(1 - \frac{L^2}{L_0^2}\right)} = c\sqrt{1 - \frac{20^2}{85^2}}$$

$$= \underline{0.97\ c}$$

6 (a) $v = 0.80c$

$L_0 = 4.35$ ly $\left\{\text{Note: 4.35 ly is a distance, measured from Earth}\right.$

$$t = \frac{L_0}{v} = \frac{4.35}{0.8} = \underline{5.44\ y}$$

(b) $$t_0 = t\sqrt{1 - \frac{v^2}{c^2}} = 5.44 \times 0.6$$

$= \underline{3.26\ y}$

(c) $$L = L_0\sqrt{1 - \frac{v^2}{c^2}} = 4.35 \times 0.6$$

$= \underline{2.61\ ly}$

7 (a) Parallel tracks

$\overrightarrow{v_{AE} = 0.75c}$ $\quad \overrightarrow{v_{BE} = 0.50c}$ $\quad$ E

$$v_{AB} = \frac{v_{AE} - v_{BE}}{1 - \frac{v_{AE} v_{BE}}{c^2}}$$

$$= \frac{0.75c - 0.50c}{1 - \frac{0.75c \times 0.50c}{c^2}} = \frac{0.25c}{1 - \frac{0.375c^2}{c^2}}$$

$$= \frac{0.25c}{1 - 0.375} = \underline{0.40c}$$

(b) Antiparallel tracks

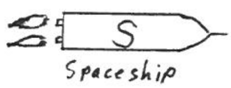
$\overrightarrow{v_{AE} = +0.75c}$ $\quad$ E $\quad \overleftarrow{v_{BE} = -0.50c}$

$$v_{AB} = \frac{v_{AE} - v_{BE}}{1 - \frac{v_{AE} v_{BE}}{c^2}} = \frac{0.75c - -0.50c}{1 - \frac{0.75c \times -0.50c}{c^2}}$$

$$= \frac{1.25c}{1 + 0.75 \times 0.5} = \frac{1.25c}{1.35}$$

$$= \underline{0.91c}$$

8.



$\overleftarrow{A}$ $v_{AS} = 0.90c$ $\quad \overleftarrow{B}$ $v_{BS} = 0.70c$

$$v_{AB} = \frac{v_{AS} - v_{BS}}{1 - \frac{v_{AS} \times v_{BS}}{c^2}} = \frac{0.90c - 0.70c}{1 - \frac{0.90c \times 0.70c}{c^2}}$$

$$= \frac{0.20c}{1 - \frac{0.63c^2}{c^2}} = \frac{0.20c}{1 - 0.63}$$

$$= \underline{0.54\ C}$$

9. A (Li) $\xrightarrow{\quad}$ $v_{ALab} = 0.70c$ $\quad$ (β) $\xrightarrow{v_{\beta A} = 0.80c}$ $\quad v_{\beta Lab} = ?$

$$v_{A\beta} = \frac{v_{ALab} - v_{\beta Lab}}{1 - \frac{v_{ALab} v_{\beta Lab}}{c^2}}$$

$$\therefore -0.80c = \frac{0.70c - v_{\beta Lab}}{1 - \frac{0.70\cancel{c} v_{\beta Lab}}{c^{\cancel{2}}}}$$

$$\therefore -0.80c = \frac{0.70c - v_{\beta Lab}}{1 - \frac{0.70\ v_{\beta Lab}}{c}}$$

$$\therefore -0.80c\left(1 - \frac{0.70\ v_{\beta Lab}}{c}\right) = 0.70c - v_{\beta Lab}$$

$$\therefore -0.80c + \frac{0.56\cancel{c}\ v_{\beta Lab}}{\cancel{c}} = 0.70c - v_{\beta Lab}$$

$$\therefore 0.56\ v_{\beta Lab} = 0.70c + 0.80c - v_{\beta Lab}$$

$$\therefore 1.56\ v_{\beta Lab} = 1.50c$$

$$\therefore v_{\beta Lab} = \underline{0.96c}$$

10. $$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} = \frac{2.4 \times 10^{-28}\ kg}{\sqrt{1 - \frac{(0.87c)^2}{c^2}}}$$

$$= \frac{2.4 \times 10^{-28}\ kg}{\sqrt{1 - \frac{0.76c^2}{c^2}}} = \frac{2.4 \times 10^{-28}}{0.49}$$

$$= \underline{4.9 \times 10^{-28}\ kg}$$

11. $$v_{esc} = 40\ 000\ km\ hr^{-1} = 1.1 \times 10^4\ m s^{-1}$$
$$m_0 = 3.8 \times 10^5\ kg = 380\ 000\ kg$$
$$\Delta m_0 = \ ?$$

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} = \frac{3.8 \times 10^5\ kg}{\sqrt{1 - \frac{(1.1 \times 10^4)^2}{(3 \times 10^8)^2}}}$$

$$= \frac{3.8 \times 10^5\ kg}{\sqrt{1 - \frac{(0.00011 \times 10^8)^2}{(3 \times 10^8)^2}}} = \frac{3.8 \times 10^5\ kg}{\sqrt{1 - 1.3 \times 10^{-9}}}$$

$$= \frac{3.8 \times 10^5\ kg}{\sqrt{0.999999998}} = \frac{3.8 \times 10^5\ kg}{0.999999999}$$

$$= 380\ 000.000\ 3$$

$$\therefore \Delta m = \underline{0.000\ 3\ kg}$$

12. $$mc^2 = m_0c^2 + E_k$$
$$\therefore E_k = mc^2 - m_0c^2$$

$$= \frac{m_0}{\sqrt{1 - v^2/c^2}}c^2 - m_0c^2$$

$$= \frac{1.673 \times 10^{-27}\ kg}{\sqrt{1 - \frac{(9.2 \times 10^5)^2}{(3 \times 10^8)^2}}}c^2 - 1.673 \times 10^{-27}c^2$$

$$= \frac{1.673 \times 10^{-27}\ kg}{\sqrt{1 - 9.4 \times 10^{-6}}}c^2 - 1.673 \times 10^{-27}c^2$$

$$= c^2\left(\frac{1.673 \times 10^{-27}\ kg}{0.999995297} - 1.673 \times 10^{-27}\right)$$

$$= c^2 \times 7.85 \times 10^{-33}$$

$$= 9 \times 10^{16} \times 7.85 \times 10^{-33} = \underline{7.1 \times 10^{-16}\ J}$$

13. $$E_k = mc^2 - m_0c^2$$
$$m = 5 \times m_0$$

$$\therefore E_k = 5m_0c^2 - m_0c^2 = c^2(5m_0 - m_0)$$
$$= c^2 \times 4m_0$$
$$= (3 \times 10^8)^2 \times 4 \times 9.11 \times 10^{-31}\ kg$$
$$= \underline{3.3 \times 10^{-13}\ J}$$

14. The ball will land on the car, directly beneath where it was thrown.

15. Einstein's 2nd postulate states that light propagates through empty space with a speed c which is independent of the speed of the source or the observer.

Therefore, the light from the star would pass the space ship at c, the speed of light.

16. $$t_0 = 2.6 \times 10^{-8}\ s$$
$$t = 5.2 \times 10^{-8}\ s$$
$$v = \ ?$$

$$t = \frac{t_0}{\sqrt{1 - v^2/c^2}}$$

$$\therefore t\sqrt{1 - v^2/c^2} = t_0$$

$$\therefore \sqrt{1 - v^2/c^2} = \frac{t_0}{t}$$

$$\therefore 1 - \frac{v^2}{c^2} = \frac{t_0^2}{t^2}$$

$$\therefore v = \sqrt{c^2 - \frac{c^2t_0^2}{t^2}} = c\sqrt{1 - \frac{t_0^2}{t^2}}$$

$$= c\sqrt{1 - \frac{(2.6 \times 10^{-8})^2}{(5.2 \times 10^{-8})^2}} = c\sqrt{1 - 0.25}$$

$$= \underline{0.87\ c}$$

17. (a) $2.6 \times 10^8\ m s^{-1} = \frac{2.6 \times 10^8}{3 \times 10^8}\ c = \underline{0.867\ c}$

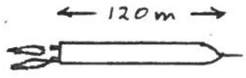
(b) $$t = \frac{t_0}{\sqrt{1 - v^2/c^2}} = \frac{t_0}{\sqrt{1 - \frac{(0.867c)^2}{c^2}}}$$

$$\therefore 2.2 \times 10^{-7}\ s = \frac{t_0}{\sqrt{1 - 0.75}} = \frac{t_0}{\sqrt{0.25}}$$

$$\therefore t_0 = \underline{1.1 \times 10^{-7}\ s}$$

18. Relativistic length is calculated by multiplying the rest length by $\sqrt{1 - v^2/c^2}$, while relativistic time and relativistic mass are calculated by dividing the rest value by $\sqrt{1 - v^2/c^2}$. When v is relatively small the term is almost zero, so the relativistic value equals the rest value. (Even v of $0.1c$ {ie 30 000 km s^{-1}} only changes rest mass, length and time by 0.5%.)

19. $$L = 120\ m$$
$$v = 0.75\ c$$



$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}$$

$$\therefore L_0 = \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{120}{\sqrt{1 - \frac{(0.75c)^2}{c^2}}}$$

$$= \frac{120}{\sqrt{0.44}} = \underline{181\ m}$$

20. $$L = 40\ ly \qquad v = \ ?$$
$$L_0 = 80\ ly$$
$$\frac{L}{L_0} = \sqrt{1 - \frac{v^2}{c^2}}$$

$$\therefore 1 - \frac{v^2}{c^2} = \frac{L^2}{L_0^2}$$

$$\therefore v = \sqrt{c^2 - \frac{c^2L^2}{L_0^2}} = c\sqrt{1 - \frac{L^2}{L_0^2}}$$

$$= c\sqrt{1 - \frac{40^2}{80^2}} = c\sqrt{1 - 0.25}$$

$$= c\sqrt{0.75}$$

$$= \underline{0.87\ c}$$

21 $$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$
$$= 24\sqrt{1-\frac{(2.4\times10^8)^2}{(3\times10^8)^2}} = 24\sqrt{1-0.64}$$
$$= 24\times0.6 = 14.4 \text{ ly}$$

22. (a) Parallel Tracks

$\beta \xrightarrow{+0.65c}$

$\alpha \xrightarrow[+0.85c]{}$ (E)

$$v_{\alpha\beta} = \frac{v_\alpha - v_\beta}{1-\frac{v_\alpha v_\beta}{c^2}}$$
$$= \frac{0.85c-0.65c}{1-\frac{0.85c\times0.65c}{c^2}} = \frac{0.20c}{1-0.5525}$$
$$= 0.45c$$

(b) Antiparallel Tracks

$\xrightarrow[+0.85c]{\alpha}$ (E) $\xleftarrow[-0.65c]{\beta}$

$$v_{\alpha\beta} = \frac{v_\alpha - v_\beta}{1-\frac{v_\alpha v_\beta}{c^2}} = \frac{0.85c - -0.65c}{1-\frac{0.85c\times-0.65c}{c^2}}$$
$$= \frac{1.5c}{1+0.5525} = 0.966c$$

23 

$$v_{AB} = \frac{v_A - v_B}{1-\frac{v_A v_B}{c^2}} = \frac{c - -c}{1-\frac{c\times -c}{c^2}}$$
$$= \frac{2c}{1+1} = \frac{2c}{2} = c$$

24. $$m = 2m_0$$
$$\therefore 2m_0 = \frac{m_0}{\sqrt{1-v^2/c^2}}$$
$$\therefore 2m_0\sqrt{1-v^2/c^2} = m_0$$
$$\therefore 2\sqrt{1-v^2/c^2} = 1$$
$$1-\frac{v^2}{c^2} = \left(\tfrac{1}{2}\right)^2 = \frac{1}{4}$$
$$\therefore v = \sqrt{c^2\times0.75} = c\sqrt{0.75}$$
$$v = 0.87c$$

25. (a) $$m = 800m_0$$
$\therefore 800m_0\sqrt{1-\frac{v^2}{c^2}} = m_0$ (see prob. 24 for detailed working)
$$\therefore v = \sqrt{c^2\left(1-\left(\tfrac{1}{800}\right)^2\right)} = c\sqrt{0.999\,998\,437}$$
$$v = 0.999\,999\,218c$$

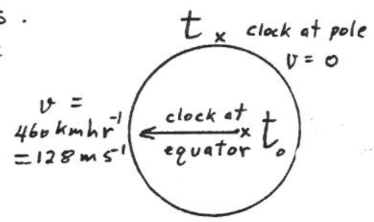
(b) $$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$
$$= 1500\sqrt{1-\frac{(0.999\,999\,218)^2}{c^2}}$$
$$= 1500\times1.25\times10^{-3}$$
$$\approx 1.9 \text{ m}$$

26. (a) $$E = mc^2 = 10^{-9}\text{ kg}\,(3\times10^8\text{ m s}^{-1})^2$$
$$= 9\times10^7 \text{ J}$$
(b) $$W = mgh$$
$$\therefore 9\times10^7 = m\times10\times10$$
$$\therefore m = \frac{9\times10^7}{10^2} = 9\times10^5\text{ kg}$$

27. You, travelling at 0.5c, would see no change in your own mass, height and waistline.
To an observer on earth, your height would remain the same but your width (and hence your waistline) would decrease and your mass would increase.

28. The following are dependent on their speed:
(a) thickness: If you are looking side on at the page which moves in the direction of its flatside
(b) mass: increases as v increases (c) volume decrease since L decreases (g) colour changes in direct.^{n} of motion since L decreases, λ decreases
The following are independent of their speed:
(d) no. of atoms in the paper does not change
(e) chemical composition is unchanged (f) c is a constant.

29. Technically, yes.
The clock at the equator goes slower since it is moving:



$$t = \frac{t_0}{\sqrt{1-v^2/c^2}}$$
$$\therefore \frac{t_0}{t} = \sqrt{1-v^2/c^2} = \sqrt{1-\frac{(128)^2}{(3\times10^8)^2}}$$
$$= \sqrt{1-0.182044444\times10^{-12}}$$
$$\approx 0.999\,999\,999\,999\,9$$
This means the equator gains 1 second roughly every 999 999 999 999 9 seconds the clock at pole shows. This is 1 second every 3.2 million years!

30. (S') $\xrightarrow{0.999\,987c}$ s' moves with electron
electron

$\longleftarrow$ 3.00 m $\longrightarrow$

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$
$$= 3.00\sqrt{1-\frac{(0.999\,987c)^2}{c^2}}$$
$$= 3\times5.1\times10^{-3}$$
$$\approx 1.5\text{ cm}$$

31. (a) $L = L_0\sqrt{1 - v^2/c^2}$

$\therefore L_0 = \dfrac{L}{\sqrt{1 - v^2/c^2}}$

$= \dfrac{5.80}{\sqrt{1 - \frac{(0.76c)^2}{c^2}}}$

$= \dfrac{5.80}{\sqrt{1 - 0.5776}}$

$= \underline{8.92\ m}$ (no height change)

(b) $t = \dfrac{t_0}{\sqrt{1 - v^2/c^2}}$

$\therefore t_0 = t\sqrt{1 - \frac{v^2}{c^2}} = 20.0\sqrt{1 - 0.5776}$

$= \underline{13.0\ s}$

32. $L_0 = 4\ ly = 4(c \times 365 \times 24 \times 60 \times 60)$

$L = 3(v \times 365 \times 24 \times 60 \times 60)$

$L = L_0\sqrt{1 - \frac{v^2}{c^2}}$

$\therefore 3v \times 365 \times 24 \times 60 \times 60 = 4c \times 365 \times 24 \times 60 \times 60 \sqrt{1 - \frac{v^2}{c^2}}$

$\dfrac{3v}{4c} = \sqrt{1 - \frac{v^2}{c^2}}$

$\therefore \dfrac{9v^2}{16c^2} = 1 - \dfrac{v^2}{c^2}$

$\dfrac{9v^2}{16c^2} + \dfrac{v^2}{c^2} = 1$

$\therefore \dfrac{9v^2 + 16v^2}{16c^2} = 1$

$\therefore 25v^2 = 16c^2$

$\therefore v = \frac{4}{5}c = \underline{0.8c}$

33 (a) $t_0 = 7.00\ y$

$v = 0.80\ c$

$L = ?$

Time for cat to die $(t) = \dfrac{t_0}{\sqrt{1 - v^2/c^2}} = \dfrac{7.00}{\sqrt{1 - \frac{(0.8c)^2}{c^2}}}$

$\therefore t = 11.67$ years

Dist. from earth $= 11.67 \times 0.8\ c$

$= \underline{9.3\ ly}$

(b)
Total time for the message to reach earth = Time for cat to die + Time for signal to reach earth

= 11.67 years + 9.3 years

$\doteq$ <u>21</u> years

34 (a) $m = \dfrac{m_0}{\sqrt{1 - v^2/c^2}} = \dfrac{m_0}{\sqrt{1 - \frac{(0.25c)^2}{c^2}}}$

$\therefore m = 1.033\ m_0$ ($m_0 = 20\ 000$ kg)

$E_k = mc^2 - m_0c^2$

$= c^2(1.033\ m_0 - m_0)$

$= c^2 \times 0.033 \times 20\ 000\ kg$

$= \underline{5.9 \times 10^{19}\ J}$

(b) $E_k = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 20\ 000 \times \left(\frac{1}{4} \times 3 \times 10^8\right)^2$

$= 5.6 \times 10^{19}\ J$

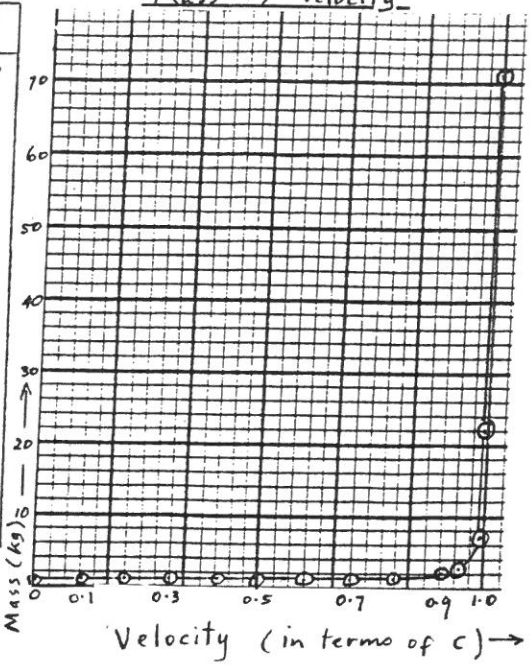
% difference $= \dfrac{5.9 \times 10^{19} - 5.6 \times 10^{19}}{5.9 \times 10^{19}} \times 100$

$\approx \underline{5\ \%}$

35.

vel (in c)	mass (kg)
0.0	1.000
0.1	1.005
0.2	1.021
0.3	1.048
0.4	1.091
0.5	1.155
0.6	1.250
0.7	1.400
0.8	1.667
0.9	2.294
0.95	3.203
0.99	7.089
0.999	22.366
0.9999	70.712

Mass Vs Velocity



36. For the Holden driver, he sees the sports car as:

$L_{\text{sports car}} = L_0\sqrt{1 - \frac{v^2}{c^2}} = 6.00\sqrt{1 - \frac{(0.18c)^2}{c^2}}$

$= 6.00\sqrt{1 - 0.0324} = \underline{5.90\ m}$

$L_{\text{Holden}} = L_0\sqrt{1 - \frac{v^2}{c^2}}$

$\therefore L_0 = \dfrac{L}{\sqrt{1 - v^2/c^2}} = \dfrac{6.15}{\sqrt{1 - \frac{(0.18c)^2}{c^2}}}$

$= \underline{6.25\ m}$

37. (a) $t = \dfrac{L_0}{v} = \dfrac{4.225}{0.8} = \underline{5.28\ years}$

(b) $t_0 = t\sqrt{1 - \frac{v^2}{c^2}} = 5.28\sqrt{1 - \frac{(0.80c)^2}{c^2}}$

$= 5.28\sqrt{1 - 0.64} = \underline{3.17\ years}$

(c) $L = L_0\sqrt{1 - \frac{v^2}{c^2}} = 4.225\sqrt{1 - \frac{(0.80c)^2}{c^2}}$

$= 4.225 \times 0.6 = \underline{2.54\ ly}$

chapter 30

Question 38

(a) $E_k = E_{Tot} - E_0$

$= mc^2 - m_0c^2$

$= c^2(20\times10^{-31} - 9.11\times10^{-31})$

$= 9\times10^{16} \times 10.89\times10^{-31}$

$= 98\times10^{-15}$

$= 9.8\times10^{-14}$ J

$= 9.8\times10^{-14} \times 6.24\times10^{18}$ eV

$= 61.15\times10^{4}$ eV

$\approx 0.61\times10^{6}$ eV

$= 0.62$ MeV

(b) $m = \dfrac{m_0}{\sqrt{1 - v^2/c^2}}$

$\therefore \sqrt{1-\frac{v^2}{c^2}} = \dfrac{m_0}{m}$

$1 - \frac{v^2}{c^2} = \dfrac{(9.11\times10^{-31})^2}{(20\times10^{-31})^2} = 0.207$

$\therefore 1-0.207 = \dfrac{v^2}{c^2}$

$v = \sqrt{c^2\times0.7925}$

$\therefore v = 0.9c$

Question 39 Chapter 30

Proton

$$m c^2 = 3 \times m_o c^2 \Rightarrow m = 3 \times m_o$$

$$E_{Total} \qquad E_o$$

(a) $$E_o = \frac{1.673 \times 10^{-27} \times (3 \times 10^8)^2}{1.6 \times 10^{-19}}$$

$$= 9.414 \times 10^8 \text{ eV}$$

$$\simeq 9.4 \times 10^8 \text{ eV} = 940 \text{ MeV}$$

(b) $$E_k = m c^2 - m_o c^2$$

$$= 3 m_o c^2 - m_o c^2$$

$$= 2 m_o c^2 = 2 \times 940 \text{ MeV}$$

$$= 1880 \text{ MeV}$$

(c) $$m = \frac{m_o}{\sqrt{1 - v^2/c^2}}$$

But $m = 3 m_o$

$$\therefore 3 m_o = \frac{m_o}{\sqrt{1 - v^2/c^2}}$$

$$\sqrt{1 - \frac{v^2}{c^2}} = \frac{\cancel{m_o}}{3 \cancel{m_o}}$$

$$1 - \frac{v^2}{c^2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

$$\frac{v^2}{c^2} = 1 - \frac{1}{9} = \frac{8}{9}$$

$$v^2 = \frac{8}{9} c^2$$

$$\therefore v = \sqrt{\frac{8}{9}} \times c$$

$$\therefore v = 0.94 c$$