CHAPTER 03

Vectors and Graphing

3.1 VECTORS AND SCALARS

To completely specify some physical quantity it is not sufficient to just state its magnitude. In the previous chapter you saw that to describe the motion of an object you needed to state its speed and the direction it was heading. In other words, its velocity. Displacement and acceleration needed a direction too. They were all called **vector** quantities. The word 'vector' comes from the Latin *vectus* meaning 'to carry' — a word implying direction. In biology a vector is an organism that carries disease from one place to another; for example, a mosquito is the vector for malaria. In physics it means a quantity that needs both magnitude and direction to specify it fully. This chapter continues the discussion about the nature of vectors as used in physics. Later in the chapter there is a discussion on graphing and how graphs can be used to solve problems.

Some of the questions that puzzle students about vectors and graphs are:

- Can two vectors having different magnitudes be combined to give a zero result? Can three?
- Can a vector have zero magnitude if one of its components is not zero?
- If time has magnitude and a direction (past → present → future), is it a vector?
- How can a statistician look at an unemployment graph and say the unemployment rate is increasing whereas another statistician can say the rate is decreasing?

Table 3.1 Some scalar and vector quantities

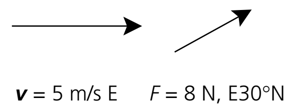
SCALAR	VECTOR
Length	displacement
Speed	velocity
Time	acceleration
Volume	force
Mass	weight
Energy	momentum
Frequency	torque
Pressure	moment
Power	electric current
Temperature	electric field
Charge	magnetic flux density

Photo 3.1
XYZ Plotter. A computer controls the X, Y and Z coordinates of the cutting head in a tool-maker's workshop. A steel mould is cut and used to make plastic parts by injection moulding.

Representation of vectors

A vector quantity can be represented by an arrow called a vector. The length of the arrow represents the magnitude of the vector quantity, and the direction of the arrow shows the direction of the vector quantity. For example, Figure 3.1 shows two vectors representing the vector quantities velocity and force:

Figure 3.1



NOVEL CHALLENGE

A ranger at Mt Mungo National Park published an booklet entitled Twenty Family Walks. In the introduction he wrote, 'The walks are short, ranging from a kilometre and a half to five kilometres; the average is two and a half kilometres.'

A What is the total length of all twenty walks?

B What is the greatest possible number of walks more than 4 km long?

C If there are three walks of 5 km each, what is the greatest possible number of walks shorter than 5 km but longer than $2\frac{1}{2}$ km?

Note: **wind directions** are confusing. A wind direction is where the wind is coming *from*. For instance, a south-easterly breeze comes *from* the south-east (S45°E or E45°S) but is *heading* north-east. Be careful to draw your diagrams carefully when wind directions are mentioned.

Scalar quantities require no statement about direction. For example, time = 3.5 s, mass = 25.5 g and current = 2.0 A are scalar quantity measurements — no direction has to be specified. The word 'scalar' comes from the Latin *scalaris* meaning 'pertaining to a ladder'. This refers to the stepwise change in the size of something without any reference to direction. *Note:* in maths class you may be taught how to work with 'unit vectors' using the symbols **i**, **j** and **k**. You could still use this system in physics if you like but it won't be discussed further in this book.

SR Activity 3.1 Finding your way home

Photocopy a map of your local area (e.g. a street directory). Draw in the route you normally take to school and estimate the distance travelled. Draw a straight line from your home to the school and determine your displacement or distance 'as the crow flies'. Include the direction.

VECTOR MANIPULATION 3.2

There are many cases in the world around us where more than one vector quantity is involved. When this is the case, we need rules to perform some form of arithmetic. We apply normal rules to scalar quantities — rules for addition, subtraction, multiplication and division. In the world of vector arithmetic, these rules must also take into account direction of the vector quantities. If you go 3 m N and then 4 m E your displacement is not 7 m. You'll see why below.

In this book we will represent a vector by printing it in ***bold italics***. For example, vector *A* will be represented as ***A*** and vector *v* as ***v***. Some books and teachers may prefer to underline the vector with a tilde (~), e.g. $\underset{\sim}{A}$, $\underset{\sim}{a}$, $\underset{\sim}{v}$ instead of using bold.

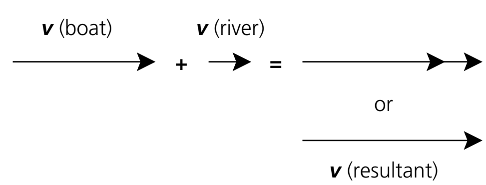
Vector addition

Two or more vector quantities can be combined to produce a single **resultant** vector.

Case 1

Consider rowing a boat at 5 m s^{-1} E in water that is also moving E at 1 m s^{-1}. Your actual velocity is 6 m s^{-1} E and is found by placing the two vector arrows head-to-tail. The resultant is a line drawn from the tail of the first arrow to the head of the second arrow (Figure 3.2).

Figure 3.2



Note: when *adding* vectors they should be placed *head-to-tail* and the resultant will always start at the tail of the first arrow and end at the head of the second arrow.

Case 2

Consider the same boat being rowed *against* the current. In this case the velocity of the river is 1 m s^{-1} W and is in the direction opposite to that of the boat and hence will slow the boat down:

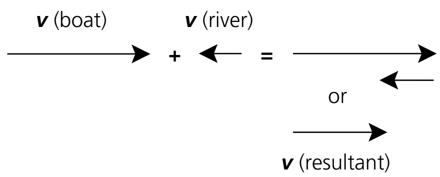


Figure 3.3

The resultant velocity is 4 m s^{-1} E. Note that when two vectors in the same line are added, the resultant has a direction the same as the larger vector.

Case 3: Vectors not in a line

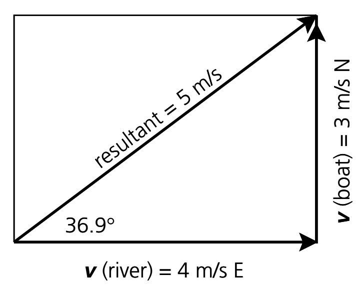
Imagine you are rowing north at 3 m s^{-1} across a river but the river current is flowing east at 4 m s^{-1}. You would be dragged off-course by the current and your resultant velocity would be 5 m s^{-1} (Figure 3.4). Note again that the two vectors are added head-to-tail. The resultant is a line drawn from the tail of the first arrow to the head of the second arrow. This resultant can also be drawn as the diagonal of the parallelogram constructed by using the two given vectors as sides. Either method is acceptable.

The solution to this problem is in two parts — a magnitude component (5 m s^{-1}) and a direction component (E36.8°N). This is achieved in the following manner:

1. *Magnitude* Because the starting vectors for the boat and river are at right angles (N and E), Pythagoras's theorem can be used. Resultant = $\sqrt{4^2 + 3^2}$. If a scale diagram was used, the resultant could be measured with a ruler.
2. *Direction* Because the two vectors and the resultant form a right-angled triangle, trigonometry can be used: i.e.

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{3}{4} = 0.75\text{, hence } \theta = 36.9°$$

Figure 3.4



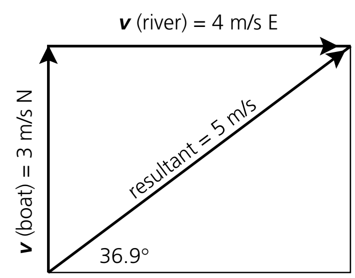
Note that the order of addition is not important. Figure 3.4 could also be drawn as shown in Figure 3.5. The resultant is still the same.

Refresher The trigonometric ratios for the right-angled triangle shown in Figure 3.6 are given below.

$$\sin\theta = \frac{\text{opposite side length}}{\text{hypotenuse}}; \cos\theta = \frac{\text{adjacent side length}}{\text{hypotenuse}}; \tan\theta = \frac{\text{opposite side length}}{\text{adjacent}}$$

Hint: when the calculator displays 0.75, usually you need to press 'shift' then either sin, cos or tan to convert this to the angle. *Note:* Latin *sinus* = 'curve'.

Figure 3.5



Questions

1 Calculate the values of θ in the following right-angled triangles (do not write in this book):

Table 3.2

	ADJACENT	OPPOSITE	HYPOTENUSE	RATIO	θ
(a)	10	7		tan θ =	
(b)	8		13	cos θ =	
(c)		9	20	sin θ =	
(d)	200	50		tan θ =	

Figure 3.6