CHAPTER 05

Projectile, Circular and Periodic Motion

5.1 MOTION IN TWO DIMENSIONS

The previous chapters have considered motion mainly in a straight line. This is called **recti-linear** motion (Latin *rectus* = 'straight' and *linea* = 'line'). This chapter will be looking at motion in two dimensions, that is, **curvilinear** motion.

Projectiles from cannons, a shotput, throwing a cricket ball, motorcyclists jumping rows of cars; and ballet dancers all involve curvilinear motion.

But there are facts and fallacies about such motion:

- Before Galileo, universities taught that when a cannon ball ran out of 'impetus' it would stop in its path and fall vertically to Earth. That's not true, is it?
- Soldiers in war have often reported that enemy bullets fired from miles away fell vertically in to their trenches. How can that be true?
- In the Olympic Hammer Throw, the hammer continues in a circular path for a fraction of a second after it is let go. True or false?
- Bombs and bullets fired at 45° have the greatest range. Well, cricket balls do; so should bullets.
- A pendulum will swing forever in a vacuum because air resistance is nil. True or false?

To make sense of these ideas, it helps if you have first-hand knowledge of some curvilinear motions.

NEI Activity 5.1 THINGS THAT DON'T GO IN STRAIGHT LINES

1 Watch a microwave oven in operation.

(a) Does the carousel rotate clockwise or anticlockwise? Does everyone else in the class get the same result?

(b) Measure the 'period' of the carousel. This is the time for one complete revolution. Time the carousel for five turns to get better accuracy. Is 12 seconds about the class average?

2 If you have a **CD player** and still have the manual, look up the rotation speed of the disk. Is it constant or is there a range of speeds?

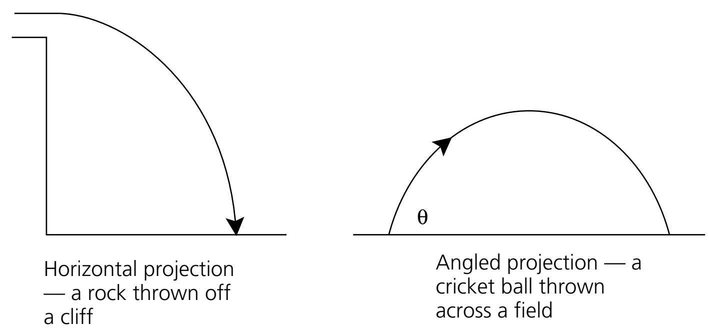
3 **Billiard** players talk about putting 'English' on the ball. What does that mean?

4 The **javelin** design was changed in 1998 so that it couldn't be thrown as far. Consult the *Guinness Book of Records* to find out how this was achieved and by approximately how much its range was reduced.

PROJECTILES 5.2

Good examples of projectiles are 1. a rock thrown straight out from the top of a cliff; 2. a cricket ball thrown across a field. (See Figure 5.1.) The word **projectile** comes from the Latin *jacere* meaning 'to throw' and *pro* meaning 'forward'. Projectile motion can be separated into two components — a **vertical** (up and down) motion and a **horizontal** motion. The vertical motion is the same as discussed in Chapter 2 — the ball is under the influence of gravity and accelerates at $-10\ \text{m s}^{-2}$ directed downward (the negative direction). In the horizontal direction, there are no net forces acting on the object so the velocity is constant. In all cases we are assuming air resistance is negligible. If you are to ever take air resistance into account in a problem you will be specifically told to do so. The path of a moving object is called its **trajectory** (Latin *trajectus* = 'crossing' or 'passage').

Figure 5.1



Note: in all examples that follow, the positive direction is upward and the negative direction is downward. You may choose a different convention in your problem-solving. It's up to you and your teacher.

Horizontal projection

This is the example of the rock thrown off the cliff. In this case the value of $\boldsymbol{v}_h$ equals the initial horizontal velocity ($\boldsymbol{u}_h$), which remains constant. The vertical velocity starts at zero ($\boldsymbol{u}_v = 0$) but increases as time passes.

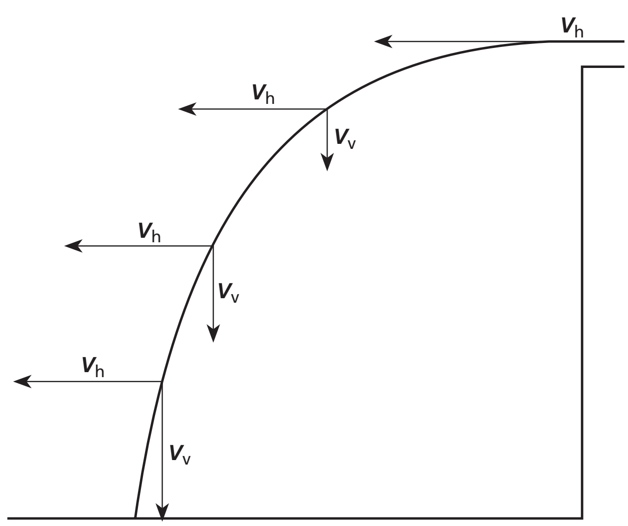


Figure 5.2
The horizontal velocity remains constant while the vertical velocity increases.

Example

A golf ball is thrown horizontally off a cliff at a velocity of $20\ \text{m s}^{-1}$ and takes 4 s to reach the ground below. Calculate **(a)** the height of the cliff; **(b)** how far the ball will land from the base of the cliff; **(c)** the impact velocity of the ball.

Solution

(a) In the vertical direction:

$$\boldsymbol{u}_v = 0 \text{ m s}^{-1},\ \boldsymbol{a} = -10 \text{ m s}^{-2},\ t = 4 \text{ s},\ \boldsymbol{s}_v = ?$$

$$\boldsymbol{s}_v = \boldsymbol{u}_v t + \tfrac{1}{2}\boldsymbol{a}t^2$$

$$= 0 + \tfrac{1}{2} \times -10 \times 4^2$$

$$= -80 \text{ m}$$

(b) In the horizontal direction:

$$\boldsymbol{u}_h = 20 \text{ m s}^{-1},\ \boldsymbol{a} = 0 \text{ m s}^{-2},\ t = 4 \text{ s},\ \boldsymbol{s}_h = ?$$

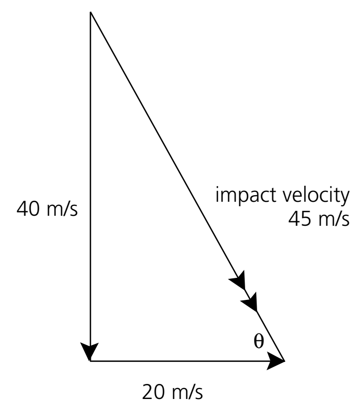
$$\boldsymbol{s}_h = \boldsymbol{u}_h t + \tfrac{1}{2}\boldsymbol{a}t^2$$

$$= 20 \times 4 + 0$$

$$= 80 \text{ m}$$

(c) Impact velocity is the sum of horizontal velocity, which remains constant at 20 m s^{-1}, and the final vertical velocity. This is a vector summation.
The vertical velocity on impact, $\boldsymbol{v}_v = \boldsymbol{u}_v + \boldsymbol{a}t = 0 + -10 \times 4 = -40$ m s^{-1}.

Figure 5.3



Using Pythagoras's theorem:

$$\boldsymbol{v}^2 = 40^2 + 20^2 = 1600 + 400$$

$$\boldsymbol{v} = \sqrt{2000} = 45 \text{ m s}^{-1}$$

The angle of impact, θ, can be found from $\tan\theta = \frac{40}{20} = 2.0$.

Hence θ = 63°.

Questions

1 A motorcycle is driven off a cliff at a horizontal velocity of 25 m s^{-1} and takes 5 s to reach the ground below. Calculate **(a)** the height of the cliff; **(b)** the distance out from the base of the cliff that the motorcycle lands; **(c)** the impact velocity.

2 A rock is thrown horizontally at 8 m s^{-1} off a 100 m high cliff. Calculate **(a)** how long it takes to hit the ground; **(b)** its impact velocity; **(c)** how far out from the cliff it lands.