CHAPTER 30

Special and General Relativity

30.1 WHY 'SPECIAL', WHY 'GENERAL'?

Einstein's name is always attached to the **theory of relativity**, yet the work of many famous scientists before him underpins his theory. He questioned the accepted theories of time and motion of earlier nineteenth-century physics and came up with a special theory of his own. People today still ask some of the questions that bothered Einstein:

- Can you travel faster than light?
- Can you travel into the past or into the future?
- If I ran at the speed of light with a mirror in my hand, could I see my own reflection?
- When two rockets are moving relative to each other, can you tell which one is really moving?
- If a torch was moving, wouldn't its light travel faster than if the torch was at rest?
- In *Star Trek*, 'warp speeds' faster than light are equal to $2^n c$, where c is the speed of light and n is the 'warp number'. Can this be true?

In this chapter, we will investigate Einstein's Special Theory of Relativity and later the General Theory.

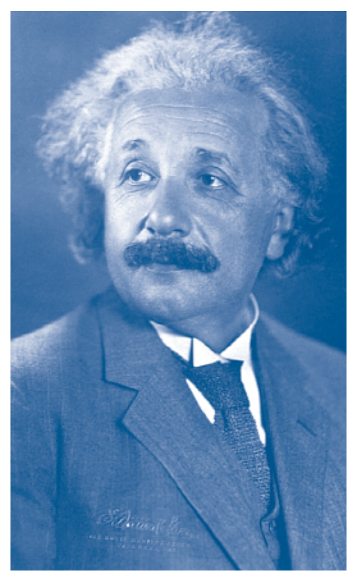
In the beginning

For more than two centuries after its inception, the Newtonian view of the world ruled supreme, to the point that scientists developed an almost blind faith in this theory. And for good reason: there were very few problems that could not be accounted for using this approach. Nevertheless, by the end of the nineteenth century new experimental data began to accumulate that were difficult to explain using Newtonian theory. New theories soon replaced the old ones. In 1884 Lord Kelvin said that there were 'nineteenth-century clouds' hanging over the physics of the time, referring to certain problems that had resisted explanation using the Newtonian approach. Among the problems of the time were the following:

- Light appeared to be a wave, but the medium for its propagation (the 'ether') was undetectable.
- The equations describing electricity and magnetism were inconsistent with Newton's description of space and time.
- The orbit of Mercury didn't quite match the Newtonian calculations.
- Materials at very low temperatures did not behave according to the predictions of Newtonian physics.
- Newtonian physics predicted that a hot object (a black-body radiator) at a stable constant temperature would emit an infinite range of energies — not so!

During the first quarter of the twentieth century, Albert Einstein created revolutionary theories that explained these phenomena. They also completely changed the way we understand nature. To deal with the first two problems he developed the **special theory of relativity** (in 1905). The third item required the introduction of his **general theory of relativity** (1915).

Photo 30.1
Albert Einstein.



The last two items can be understood only through the introduction of a completely new mechanics: quantum mechanics. This chapter deals with special and general relativity. The previous chapter introduced quantum mechanics.

Special Relativity is a deceptively simple theory and has only two assumptions or 'postulates'. They are presented here so you know what is coming, but without any explanation:

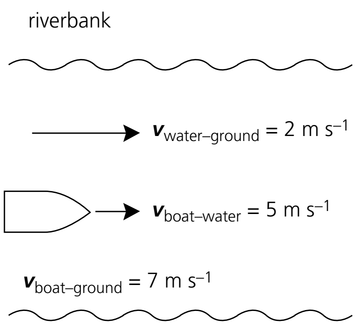
- The laws of physics are the same in all uniformly moving reference frames. No preferred frame exists.
- The speed of light in free space has the same value, c, in all uniformly moving reference frames.

Hmm! That doesn't seem too complicated. In fact, most physicists agree that the second postulate is redundant as it is a logical consequence of the first.

General Relativity does away with the restriction of 'uniform motion' which tends to make it more complicated — philosophically, physically and mathematically. In fact, it took Einstein 10 years, with many false starts and wrong turns, from introducing special relativity to the complication of his general theory in its final form. Along the way, the general theory became a whole new way of understanding gravity.

FRAMES OF REFERENCE 30.2

Figure 30.1
The speed of the boat is affected by the speed of the current.



In your earlier work on mechanics, you generally used the ground or Earth as your frame of reference. For example, when a car is going at 60 km h^{-1} along a road, this is with reference to the ground. But when a boat travels down a river, we can state its motion relative to the ground or relative to the water (Figure 30.1). The choice is arbitrary. This notion of reference frames had been discussed at length by Galileo and Newton and we need to begin there.

Inertial frames of reference

This chapter deals with **inertial reference frames** — that is, frames in which Newton's first law (the law of inertia) is valid. If an object experiences no net force due to other bodies, the object either remains at rest or remains in motion with constant velocity (in a straight line). Accelerating frames of reference, rotating or otherwise, are non-inertial frames, and we will not be concerned with them here. The Earth is not quite an inertial frame because it rotates. But it is close enough that for most purposes we can consider it an inertial frame. We could also carry out inertia experiments aboard a ship that is travelling at constant speed. It, too, is an inertial frame.

For Newton, there was a 'master' or absolute inertial frame: a frame stationary relative to absolute space. And any reference frame that is moving at a uniform velocity in a straight line relative to this master inertial frame, he said, will also be an inertial frame. Any reference frame that is accelerating with respect to absolute space, such as the car's frame when the light turns green and the driver accelerates, will not be inertial.

Now imagine that you are riding in the car at, say, 100 km/h down a straight highway and fluffy dice are hanging motionless from the rear view mirror. The principle of inertia is true for you. A second observer is standing beside the highway, watching the car go by. For her the dice are moving in uniform motion in a straight line. So the second observer is also in an inertial frame.

In this case, a good question is: 'Who is moving?' The answer is that you are moving relative to the observer beside the highway, but the observer beside the highway is moving relative to you. So you are both moving relative to each other. Both your inertial frame and her inertial frame are equally 'valid'. This realisation is often called 'Galilean relativity'.

An old favourite to illustrate this further is a cannonball dropped from the mast of a boat sailing along past an observer on the shore (Figure 30.2). For a sailor on the ship the cannonball appears to fall straight down (Figure 30.2**(a)**). From the point of view of an observer on shore, the ball falls with a uniform acceleration downwards while moving with constant speed in the horizontal direction — that is, it follows a parabolic path relative to the shore just like a rock thrown horizontally off a cliff (Figure 30.2**(b)**). However, for both observers the cannonball lands at the base of the mast, and the laws of inertia are the same in both reference frames although the paths are different. We can say:

> A reference frame that moves with constant velocity with respect to an inertial reference frame is itself also an inertial reference frame.

However, in frames moving relative to each other, the velocity of an object will appear different.

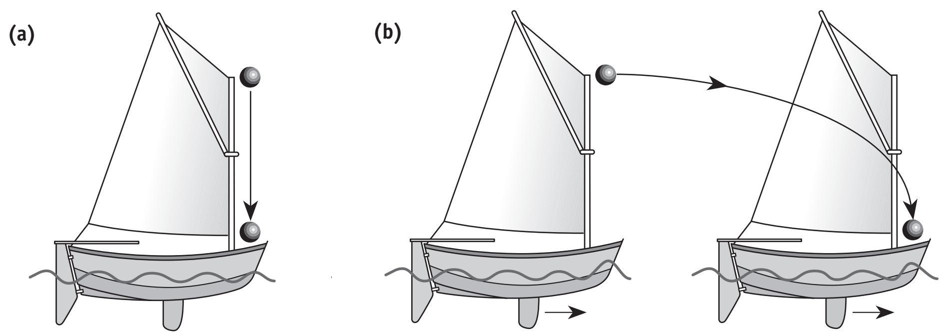


Figure 30.2
A falling cannonball travels different paths depending on your frame of reference: **(a)** from aboard the boat; **(b)** from the shore as the boat travels past you.

SR Activity 30.1 A 'GEDANKEN' (THOUGHT) EXPERIMENT

Before you read any further, you should sort out these questions (well, except for **(f)**):

(a) What would the path in Figure 30.2**(b)** look like if gravity was **(i)** less than that on Earth, **(ii)** more than that on Earth, **(iii)** zero?

(b) How would the path in Figure 30.2**(b)** differ if the cannonball was half the original mass?

(c) If the mast was 20 m high, and the boat sailed at 20 m s^{-1} relative to the shoreline, how many seconds would the cannonball take to hit the deck in Figure 30.2**(a)** and in Figure 30.2**(b)**?

(d) How far would the boat have travelled to the right in this time?

(e) Relative to the shore, what is the displacement and average velocity of the cannonball in its journey shown in Figure 30.2**(b)**?

(f) A very difficult one! How far would the cannonball have travelled in Figure 30.2**(b)** relative to the shore line? You will need to work out the 'arc length' of the parabola. How's your calculus?

Not all things change when viewed in different reference frames. For example, the number of atoms in an object doesn't change. If you time your pulse rate on Earth as 72 per minute, then you'll time it as 72 per minute aboard a moving bus. But if you are sitting down on the bus as it travels along a road at 5 m s^{-1}, you could say your speed is zero relative to the bus ($\boldsymbol{v}_{\text{person–bus}}$ = 0 m s^{-1}), and the speed of the bus relative to the Earth ($\boldsymbol{v}_{\text{bus–Earth}}$) = 5 m s^{-1}. Your