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An approach to Poiseuille's law in an undergraduate laboratory experiment

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Abstract

The continuous growth of computer and sensor technology allows many researchers to develop simple modifications and/or refinements to standard educational experiments, making them more attractive and comprehensible to students and thus increasing their educational impact. In the framework of this approach, the present study proposes an alternative experimental setup, which allows the confirmation of Hagen–Poiseuille's law, governing the flow of real fluids through tubes, a law with numerous important applications in both technology and medicine. In the proposed educational procedure, experimental measurements of fluid outflow are performed with the use of a motion sensor and a suitable computer program, allowing the determination of both the hydrostatic pressure and the flow rate. The dependence of the flow rate on parameters such as viscosity of the fluid, length and radius of the tube and the pressure difference between the ends of the tube are also studied, providing a laboratory activity which is useful and attractive for first year students, especially those of technologically oriented departments.

(Some figures in this article are in colour only in the electronic version)

Introduction

Flow of real fluids has long been a subject of considerable interest and intense study since it is dealt with in numerous areas of both science and technology. The role of the subject in various university and college curricula is equally important; this is reflected by the demand for educational studies aiming at helping students to understand the laws regulating flow. The knowledge of these laws is very important in various fields, especially in hemorheology and hemodynamics, both fields of physiology, in physical and chemical investigations and in the development of industrial engineering projects [1–4].

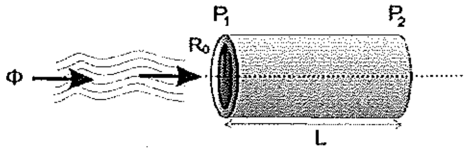

Figure 1. Voluminal laminar flow Φ of uniform viscous liquid through a cylindrical tube with the constant circular cross-section. P_1 is the input pressure of the fluid at that point; P_2 is the output pressure or the pressure of the fluid at the next point of the fluid; R_o is the tube radius; L is the length and Φ is the flow rate, equal to the fluid volume per unit time.

Gotthilf Heinrich Ludwig Hagen and Jean Louis Marie Poiseuille formulated Hagen–Poiseuille's law, a physical law governing the voluminal laminar stationary flow of an incompressible uniform viscous liquid through a cylindrical tube with a constant circular cross-section.

The most important feature of this law is the sensitive dependence of the flow rate on the channel width, or the pipe radius. For instance, for a pipe with a fixed pressure gradient, a 20% reduction in the pipe radius leads to a 60% reduction in the flow rate.

This clearly has important implications, for instance in physiology, small amounts of plaque accumulation in arteries can lead to very large reductions in the rate of blood flow [5].

In general, changes regarding the pressure difference between the ends of the tube ($\Delta P = P_2 - P_1$), the tube radius R_0 and length L (see figure 1), and the flowing fluid, as represented by its viscosity η, can influence the volume flow rate [2].

According to Poiseuille–Hagen's law, the flow rate Φ is expressed as in equation (1)

$$\Phi = \frac{\pi \cdot R_o^4 \cdot \Delta P}{8\eta L}, \tag{1}$$

(the derivation of the above equation can be found in textbooks and is obviously out of the scope of the present paper [6–8]).

The pressure difference between the free fluid surface of the container and the level of the container where the pipe is placed is proportional to the fluid height, $h_t(t)$ (figure 2). Consequently, the flow rate can be rewritten as in equation (2),

$$\Phi = -\frac{\pi R_o^4}{8\eta}\frac{\rho g}{L} h_t(t) \tag{2}$$

where Φ is the flow rate, equal to the fluid volume per unit time flowing through the pipe, $h_t(t)$ presents the height at each time moment, ρ and η are the density and the viscosity of the fluid, respectively (figures 1 and 2).

According to the law of continuity, the flow rate Φ of the pipe is equal to the flow rate of the container, which is by definition the volume rate $\mathrm{d}V/\mathrm{d}t$, which in turn can be expressed as the product of the container's surface A with the free fluid surface velocity ($\mathrm{d}h_t/\mathrm{d}t$), as shown in equation (3).

$$\Phi = \frac{\mathrm{d}V}{\mathrm{d}t} = A \cdot \frac{\mathrm{d}h_t(t)}{\mathrm{d}t}. \tag{3}$$

If we set $\frac{\pi R_o^4}{8\eta}\frac{\rho g}{L} = \lambda$, equation (2) can be written in the following compact form

$$\Phi = -\lambda \cdot h_t(t). \tag{4}$$

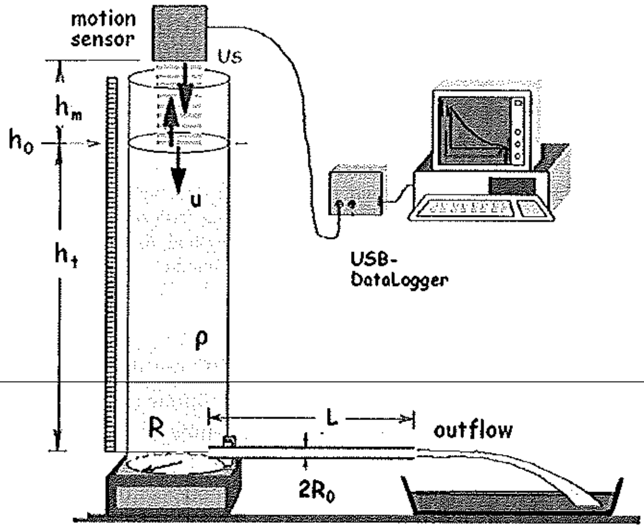

Figure 2. A schematical drawing of the experimental setup used for the measurements, where US are the ultrasonic waves, h_m, h_0 and h_t are the measured, the initial and the present height respectively, u is the velocity of the free fluid surface, ρ is the fluid density, R and R_0 are the radius of the container and of each pipe, respectively and L is their length.

From equations (3) and (4), one is led to equation (5), in which the height is expressed as an exponential function of time

$$-\lambda \cdot h_t(t) = A \cdot \frac{dh_t(t)}{dt} \xRightarrow{\int} h_t(t) = h_o(t) \exp\left[-\frac{\lambda}{A} \cdot t\right], \tag{5}$$

where h_o is the initial height of the fluid in the container (figures 1 and 2).

Materials and methods

The experiments presented have been designed so that students are able to verify directly the laws which govern the flow of fluids and how critically the flow rate depends on parameters such as pressure, length and diameter of the tubes. The experimental setup allows variation of all the quantities (ΔP, R_o, L and η) on which fluid flow through tubes depends.

The proposed setup

A cylindrical container filled with the fluid that one studies, a set of tubes of different radii (0.2–2.5 mm) and lengths (5–70 cm), liquids of various viscosity and a digital sensor of motion with the appropriate PC interface and software, e.g. PS-2103 with the PS-2100 and DataStudio of Pasco. A schematical drawing of the experimental setup is shown in figure 2.

This particular setup is based on an older version of the experiment, in which the flow rate was measured continuously and indirectly with the aid of a force sensor by weighing the liquid flowing out of the container [6].

The measurement method

In our experiment, we have used tubes of various lengths and radii. They were common laboratory pipettes of different volumes.

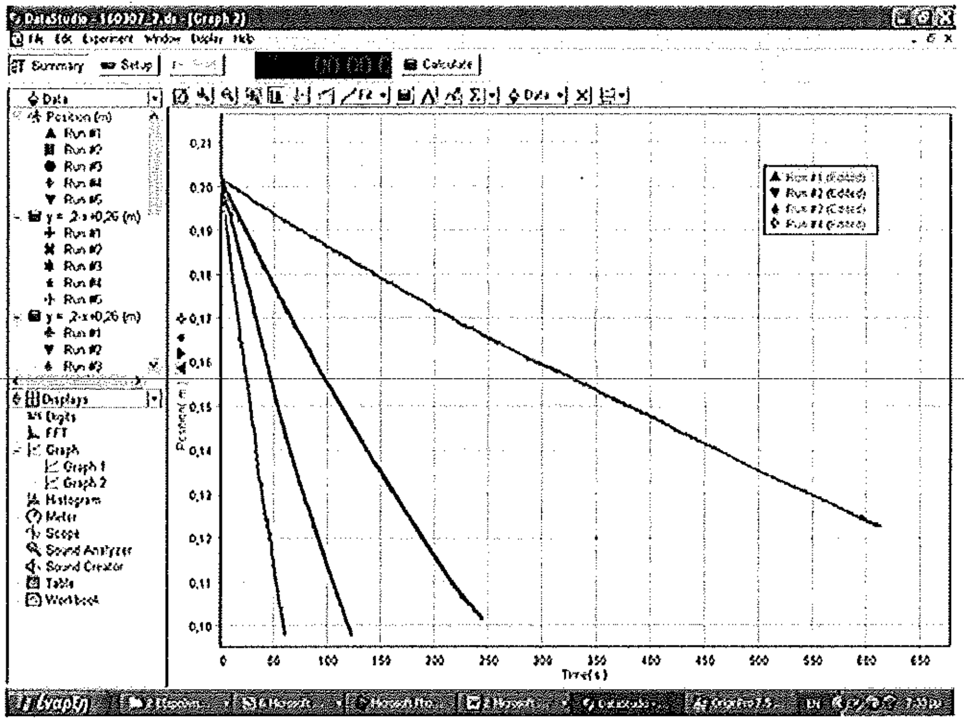

Figure 3. The print screen of the DataStudio with curves of the height versus time as measured by varying the tubes radius.

The fluid (de-ionized water, glycerine or their mixture) was placed in the cylindrical container until a certain initial height h_o (figure 2). As soon as the fluid is allowed to flow out of the tube, the motion sensor at the top starts to monitor the change of the fluid height (the free level fall) into the container, via the computer program. After a definite period of time, the measurement is stopped and the results appear both in the form of a raw data table (for further processing) and as a curve describing an exponential decay, as seen in figure 3. In this figure, the decrease in the height h_t is shown versus the time of the experiment.

An appropriate fitting to the experimental data, in relation to equation (1), is provided by the data processing, in order to obtain the right function, which describes the physical phenomenon. The measured data $h_t(t) = f(t)$ can be converted into the following function $\Phi = f(\Delta P)$, according to Poiseuille–Hagen's law, where Φ is the deduced flow rate and ΔP is the pressure difference. The dependence of the flow rate on parameters such as viscosity of the fluid, length and radius of the pipe and the pressure difference are objects of our experimental work, as shown in figures 4–7.

Results and discussion

The exponential behaviour of the fluid height (equation 5) is adequately confirmed by our experimental data (figure 3), measured with the help of the motion sensor placed on the top of the container. After an appropriate fitting, the parameter λ/A can be derived and the flow rate of the pipe Φ can easily be estimated, using equation 4.

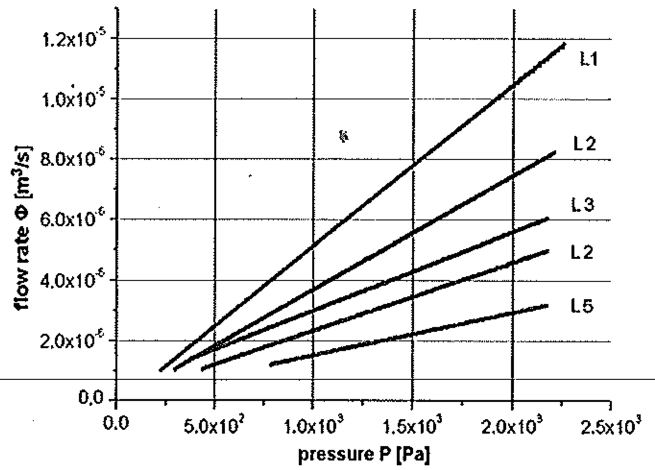

Figure 4. Flow rate Φ as a function of the pressure P for tubes of $R = 0.88$ mm and different lengths, L.

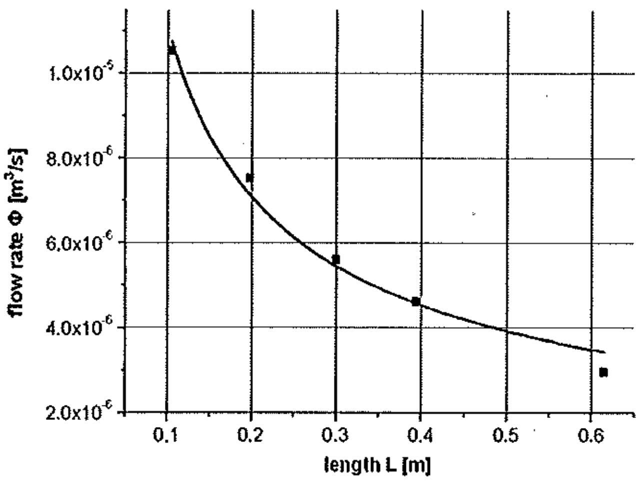

Figure 5. Flow rate Φ as a function of the length of tubes of $R = 0.88$ mm.

The results are grouped into the three following activities.

(a) *Different pressures*: results of the flow rate Φ of the liquid as a function of pressure difference ΔP can easily and straightforwardly be derived since the decrease in the height level of the liquid is a continuous process. From the measured data, hydrostatics pressure of the liquid (head) can be extracted.
As can be seen from the results presented in figure 4, for different lengths (L1–L5, 20–65 cm) of the tubes under operation, the flow rate (Φ) is a linear function of the pressure (ΔP), in accordance with Hagen–Poiseuille's law.

(b) *Different tube lengths*: in figure 5, flow rate versus different tube lengths is shown. The fitting procedure almost confirmed the mathematical linear relation of $\Phi \approx k \cdot (1/L)$, where k is a constant factor [1, 2, 7, 8].

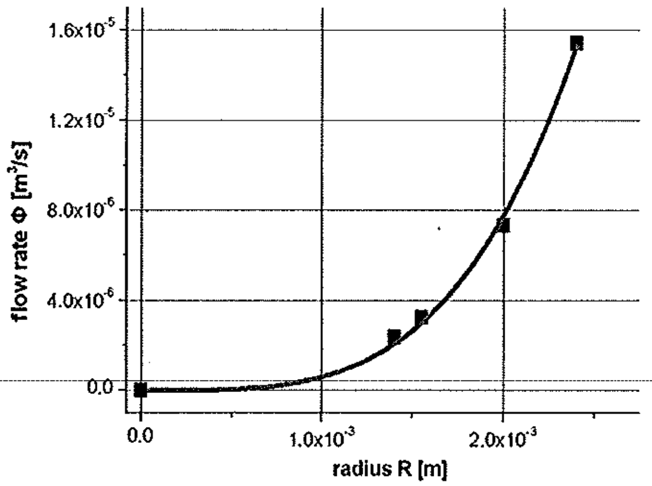

Figure 6. Plot of the flow rate, Φ, as a function of the radius, for tubes of equal length (0.30 m) and for the same solution.

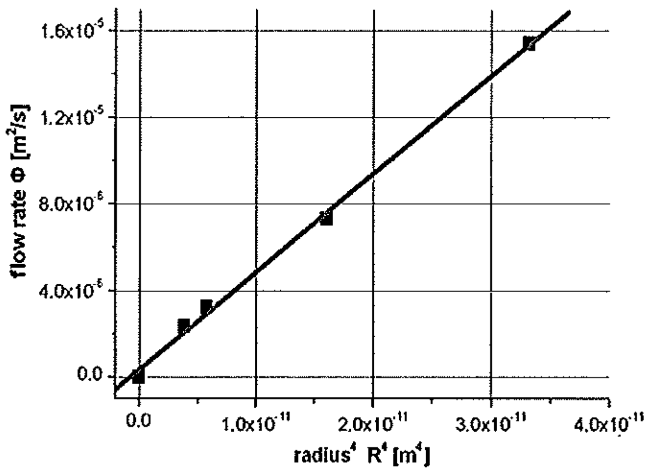

Figure 7. Plot of the flow rate, Φ, as a function of the fourth power of radius, for tubes of equal length (0.30 m) and for the same solution.

(c) *Different tube radii*: figures 6 and 7 present the flow rate as a function of the tube radius (length, pressure and viscosity are kept constant) [1, 2, 7, 8]. The exponential dependence on the radius tube is obvious, as the law of Poiseuille indicates.

The mathematical relation of the dependence of the flow rate on the fourth power of the radius of the tube could be deduced.

(d) *Different viscosity*: in order to study the influence of the viscosity on the flow, we measured liquids with different viscosities, with the rest of the variables kept constant. Such fluids could be obtained, e.g., with different concentrations of glycerine and water, presenting viscosities between about 0.8 and 34 mPa s. Ten solutions of glycerine in water were prepared, with different viscosities, repeating the experiment with each of them using a tube (20.0 ± 0.1) cm in length and with a radius of (2.50 ± 0.01) mm. Experimental values of Φ were obtained and fitted by means of a function of the type $y = ax^b$. We reached the conclusion that Φ is inversely proportional to the viscosity.

Conclusions

With the proposed exercise, the student is able to infer Poiseuille's law, which determines the flow rate of a fluid through a tube by means of an experimental design that clearly illustrates the different steps of the scientific method.

The proposed experimental setup is relatively simple, inexpensive, easy to construct and offers very satisfactory results. This law is behind many physiological phenomena of the circulatory system, either normal or pathological, such as the local regulation of blood flow, the narrowing of a vessel, the generation of critical stenosis, etc. Accurate studies of the relationship between pressure and flow rate can be performed with the proposed experimental setup, even though its construction is quite simple and its cost remains reasonable.

This experiment provides a pleasant and essential educational activity in the field of experimental and analytical processes for undergraduate students.

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