A simple and surprising experiment is performed by physical science students

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The simple act of rolling a ball down an inclined plane onto a tabletop can be analysed theoretically by students. When they test their predictions experimentally the agreement is satisfying.

Student-led research projects are typically the highlight of my liberal-arts physical science course at the University of Wisconsin Oshkosh. However, helping my students find appropriate research problems can be challenging. Recently, my physical science students and I stumbled across a research problem that was particularly interesting.

Suppose a ball is released from the perimeter of a vertical circle and rolls down a ramp toward the *centre* of the circle (see figure 1). At the centre of the circle, the ball strikes a level tabletop and bounces away. The ramp may be rotated about the circle's centre. For what angle will the time of travel across the tabletop be the *least*?

The solution to the problem is given for ideal conditions below. However, before reading on, make an intuitive guess at the answer. Would your students agree with you? Brace yourself for a surprise!

At end of the guide, the ball's horizontal component of velocity is

$$v_x = v\cos\theta \qquad (1)$$

where v is the magnitude of velocity, or speed, and θ is the angle of the incline with respect to the tabletop. As the ball bounces across the top of the table, the horizontal component of velocity is unchanged (neglecting energy losses at each bounce). Neither the upward force of the table nor the force of gravity contributes to the horizontal motion of the ball. Therefore, the value of v_x at the bottom of the ramp determines the time of travel across the table. In particular, the *maximum* value of v_x corresponds to the *minimum* time of travel.

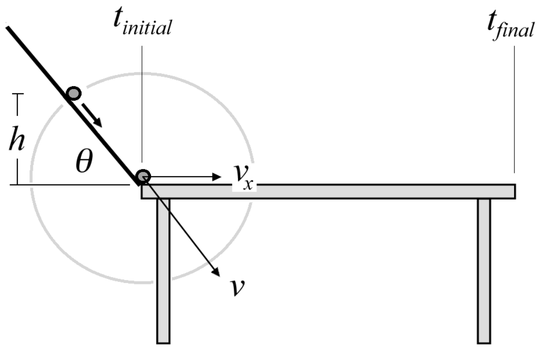

Figure 1. The experimental set-up. For what angle will the time of travel across the tabletop be the least?

The relationship between the initial height of the ball (h) and the speed of the ball (v) at the end of the ramp may be determined through energy conservation:

$$mgh = \tfrac{1}{2}mv^2 \qquad (2)$$

which reduces to

$$v = \sqrt{2gh} \qquad (3)$$

where g is the acceleration due to gravity. Since the initial height of the ball varies as $\sin\theta$, the speed of the ball at the bottom of the incline varies as the square root of $\sin\theta$, i.e. *more* rapidly than

$\sin\theta$! As a result, the horizontal component of velocity varies as

$$v_x \propto \left(\sqrt{\sin\theta}\right)\cos\theta. \quad (4)$$

The quantity v_x is *not* symmetric about 45° as you might expect! To determine the maximum value of v_x, we set the first derivative of v_x with respect to θ to zero, and solve for θ. This relation reduces to

$$\sin\theta = \sqrt{1/3}. \quad (5)$$

Hence, the maximum value of v_x, and therefore the minimum time of travel, occurs at $\theta \approx 35.3°$! But will a real experiment show this?

Initially, my students predicted an angle of least time of about 90°, reasoning that the speed of the ball at the end of the guide will be greatest at this angle. However, students quickly realized their error. As one student put it, 'At 90°, the ball just goes up and down and doesn't get anywhere!' After more extensive discussion, the group settled on a prediction of 45°, which is also incorrect, but closer to correct answer.

To test their prediction, students rolled a golf ball down a 2.0 m aluminium track (the ones used for rolling carts) and across a hard tile floor. The track was mounted on adjustable support beams to make pivoting the track about a point near the floor easy. Angles between the track and floor were determined using a protractor.

Students measured time with a digital stopwatch. One student stood near the bottom of the guide and clapped at the instant the ball reached the floor. The student operating the stopwatch started timing at the clap and stopped timing when the ball crossed a pre-defined plane on the floor several metres away.

Students took measurements at each of 18 angles between 10° and 55°. Their results are shown in figure 2. The error bars indicate the range of values obtained over ten measurements. The theoretical fit (performed by the instructor) takes into account student reaction time. Good agreement is observed between data and theory. In particular, the minimum time of travel occurs at $\theta \approx 35°$ (not 45°!)

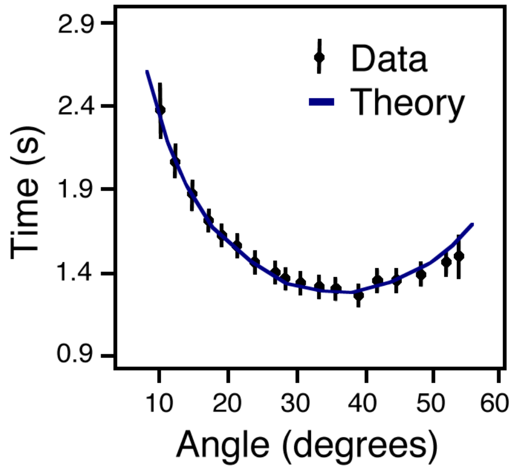

Figure 2. Comparison of theory and data. The error bars indicate the range of results over ten trials. The angle for the minimum time of travel is approximately 35°.

Lacking a deep mathematical background, students were asked to obtain the correct theoretical result from *Interactive Physics*† using the actual physical parameters of their experiment. We found that the dynamic velocity vectors of the simulation are particularly educational. To the great satisfaction of the students, the model calculation agreed with their experimental results.

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† *Interactive Physics*™ is a physics modelling and simulation software program developed by Knowledge Revolution (San Mateo, CA).