Short coil experiment – magnetic field strength and axial distance

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There are several experiments that can be done on the magnetic field about a short coil and this one is about the variation in *B* with axial distance, *x*. A short coil is one in which L << R, unlike a solenoid where L >> R.

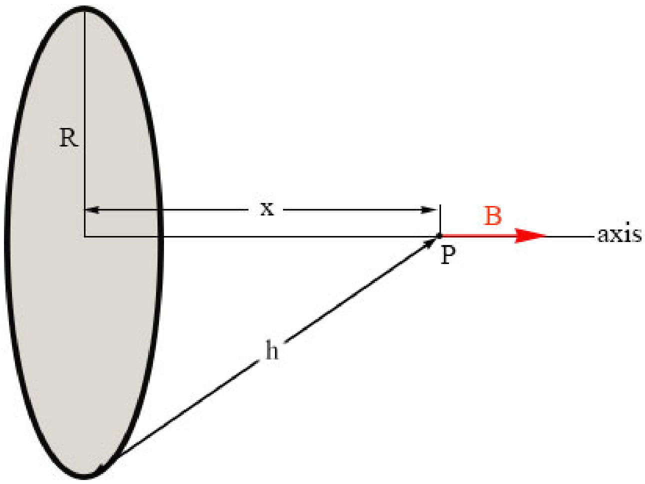


FIGURE 1. The axial distance and field

For a short coil of *N* turns, a radius *R,* and a current *I*, the magnetic field strength in the axial direction perpendicular to the plane of the loop is modelled by:

$$B = N\frac{\mu_0 I R^2}{2\left(R^2 + x^2\right)^{3/2}}$$

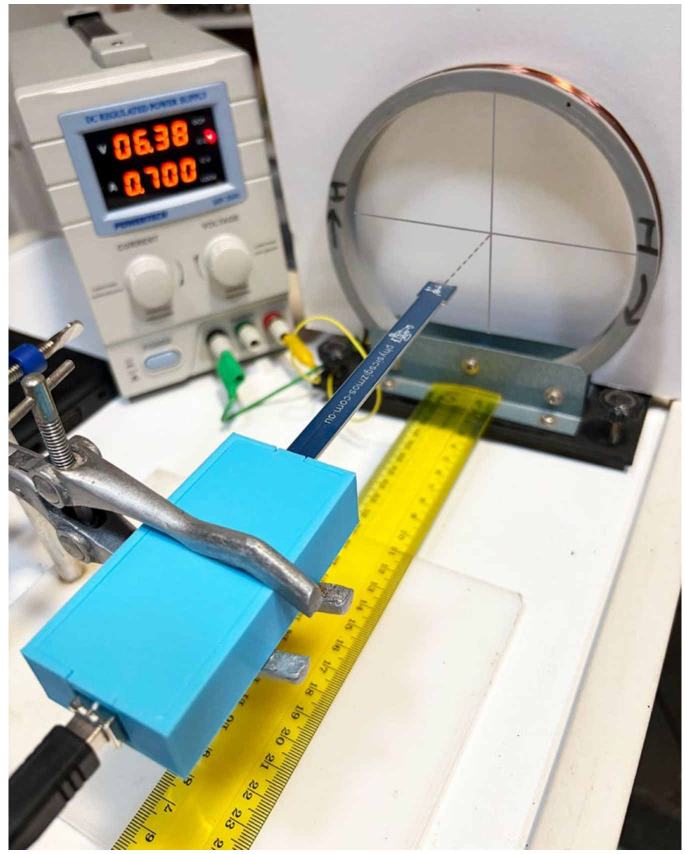


FIGURE 2. The magnetometer set to read the field in the y-direction which is along along the axis of the coil (y-direction). The coil has 200 turns with a radius of 8.0 cm and the current of 0.700 A.

RESULTS

x(cm)	B ex (uT)	B th (uT)	$1/h^3$
1.0	923	1075	1953
2.0	811	1004	1908
3.0	681	903	1783
4.0	598	787	1603
5.0	502	671	1398
6.0	417	563	1191
7.0	345	469	1000
8.0	284	389	832
9.0	233	323	691
10.0	196	268	573
11.0	161	224	476
12.0	133	188	397
13.0	120	158	333
14.0	104	134	281
15.0	89	115	239
16.0	78	98	204
17.0	68	85	175
18.0	60	74	151
19.0	54	64	131
20.0	48	56	114

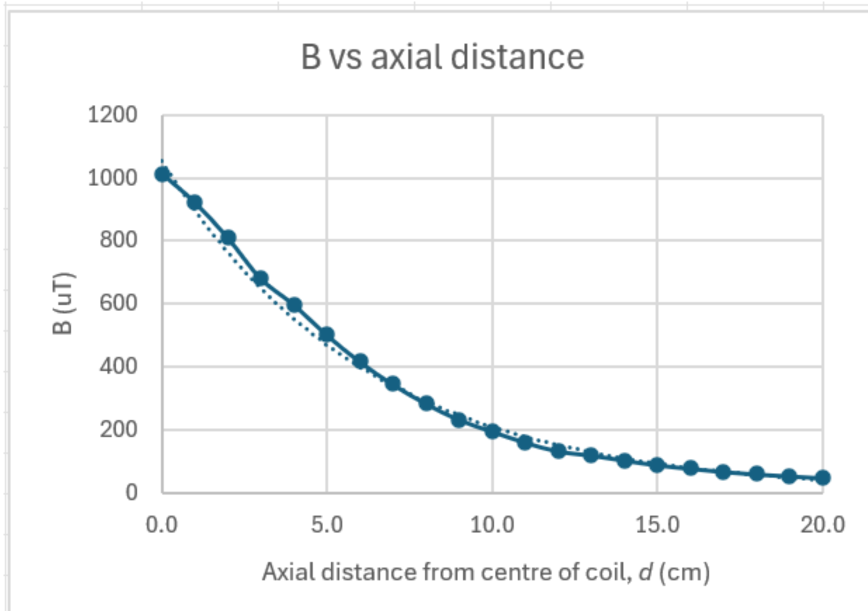


FIGURE 3. Graph of raw data

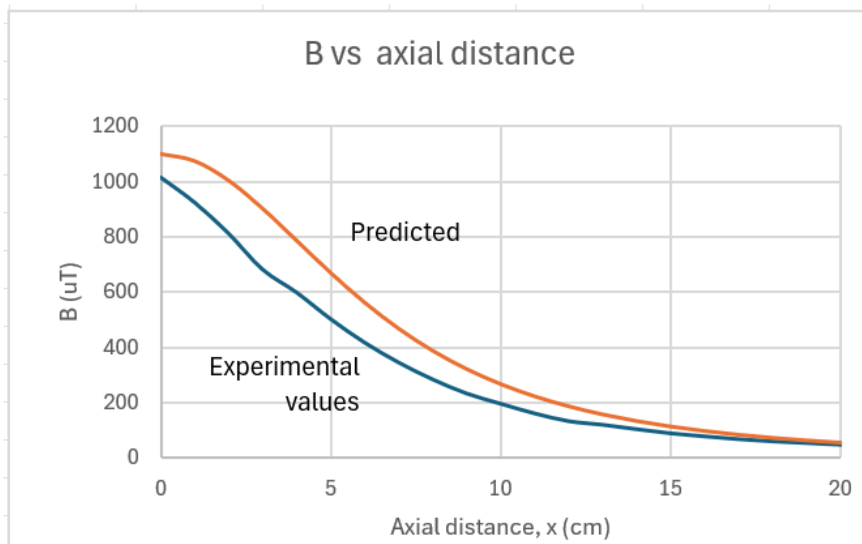


FIGURE 4. Experimental and predicted values compared.

Graph of experimental values and values predicted by the model. The shapes are generally the same, but the predicted values are higher than the observed values. On the left the 'B ex' stands for the experimental or observed values, and 'B th' are the theoretical values predicted by the model (equation).

The column labelled as $1/h^3$ is the value of $\frac{1}{(R^2+x^2)^{3/2}}$ in the accepted formula. It is just the inverse of the cube of the hypotenuse. It will be used to linearise the data and to show that the magnetic field strength is proportional to the inverse cube of the distance from the sensor to the wire.

ANALYSIS

The equation for the axial magnetic field strength as a function of distance, *x*, for a short coil is:

$$B = N\frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$$

This can be rearranged to separate the x-term (the independent variable):

$$B = \tfrac{1}{2} N \mu_0 I R^2 \times \frac{1}{(R^2 + x^2)^{3/2}}$$

$$\tfrac{1}{2} N \mu_0 I R^2 \times \frac{1}{h^3}$$

$$y = \quad m \quad \times \quad x$$

To linearise the graph we can plot *B* (y-axis) and $\frac{1}{h^3}$ (x-axis). The gradient will be:

Bh^3 which will be equal to $\tfrac{1}{2} N \mu_0 I R^2$

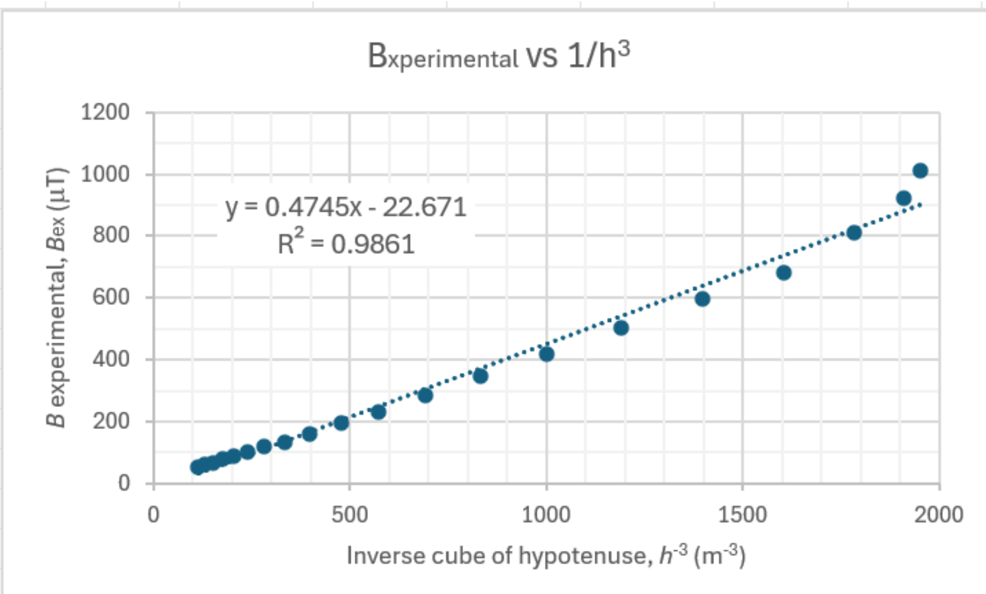


FIGURE 5. Linearised graph.

$$\begin{aligned} Bh^3 &= \tfrac{1}{2} N \mu_0 I R^2 \\ 0.4745 \times 10^{-6} &= \tfrac{1}{2} N \mu_0 I R^2 \\ \mu_0 &= \frac{2 \times 0.4745 \times 10^{-6}}{NIR^2} \\ &= \frac{0.9490 \times 10^{-6}}{200 \times 0.700 \times 0.080^2} \\ &= 1.06 \times 10^{-6}\,\mathrm{T\,A\,m^{-1}} \end{aligned}$$

In practical terms we can say that we've found that:

$$B = N\frac{0.95\mu_0 IR^2}{2h^3}$$

$$B \propto \frac{1}{h^3}$$

- when N, I, R are kept constant.

ERROR ANALYSIS

The accepted value for μ_0 is 1.26×10^{-6} T A m^{-1}.

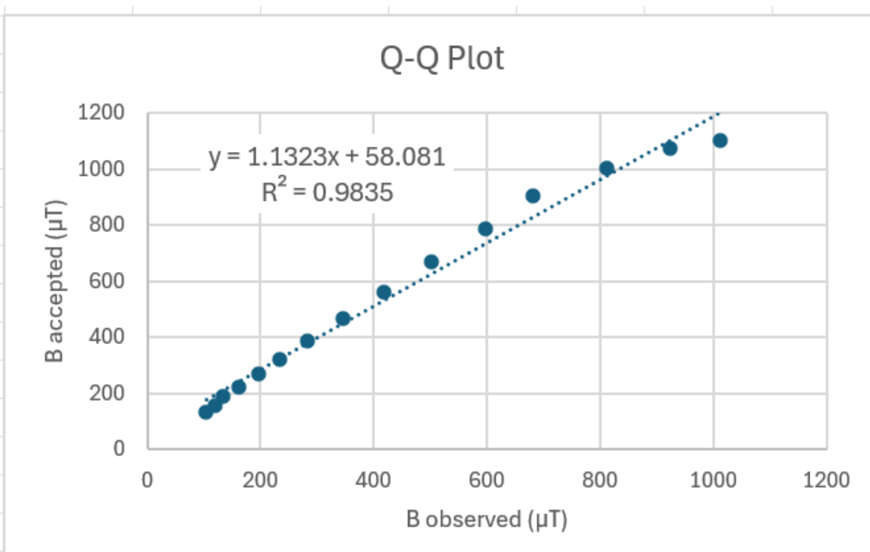
$$\begin{aligned} E\% &= \left|\frac{x_{\mathrm{O}} - x_{\mathrm{A}}}{x_{\mathrm{A}}}\right| \times 100 \\ &= \left|\frac{1.06\times10^{-6} - 1.26\times10^{-6}}{1.26\times10^{-6}}\right| \times 100 \\ &= 16\% \end{aligned}$$

OTHER CHECKS OF ACCURACY

The percent error in the value of the magnetic permeability constant μ_0 for this experiment 16% according to the analysis of the graph. We can also compare the experimental values of B with the values obtained from the model (equation). There are two simple ways of comparing observed (experimental) and theoretical (accepted, predicted) results:

METHOD 1: The Q-Q Plot

This is a graph of the predicted values against the observed values. If the gradient of the line is 1.0 it means the observed value can be predicted from the model's values. It means your data are a good fit with the model.



The gradient is 1.13 which means there is a 13% difference in the observed values compared to the model. We could say that the observed values can be successfully predicted from the model 87% of the time. The R^2 value is 0.98 which indicates that the equation y = 1.1323x + 58 is a good fit for the data.

METHOD 2: The coefficient of variation, CV

The residuals for each data point are calculated (absolute value of the difference) and a mean and standard deviation calculated for the whole data set of residuals. You then calculate the ratio of SD/average. If it is <1 it means the observed data is similar to the predicted data. For my experiment I found for the residuals that the average = 60.1 and the SD is 68.7 giving a CV = 1.1. This is greater than 1.0 so we can say the observed and predicted data are not that similar.

EQUIPMENT

Magnetic field sensor. I used Ian Thompson's magnetic field sensor ($75) that worked like a charm for all of these electromagnetism pracs. See: https://sciencewithmat.com.au/products/magnetic-field-sensor-set-of-6-by-physics-gizmos

DC Regulated power supply. I used one from Jaycar ($260 but not sure of school price) but they're also available from https://sciencewithmat.com.au/products/0-to-30vdc-0-to-5a-regulated-power-supply ($219).

Short coil. I use a short coil from a current balance, but normally would use a 100 mm PVC DWV cap from Bunnings ($3.50) as the coil former and 0.5 mm enamel copper wire from Jaycar ($15 for 57 m spool) for the windings.

My thanks to Ian Thompson for his help with this prac and report.