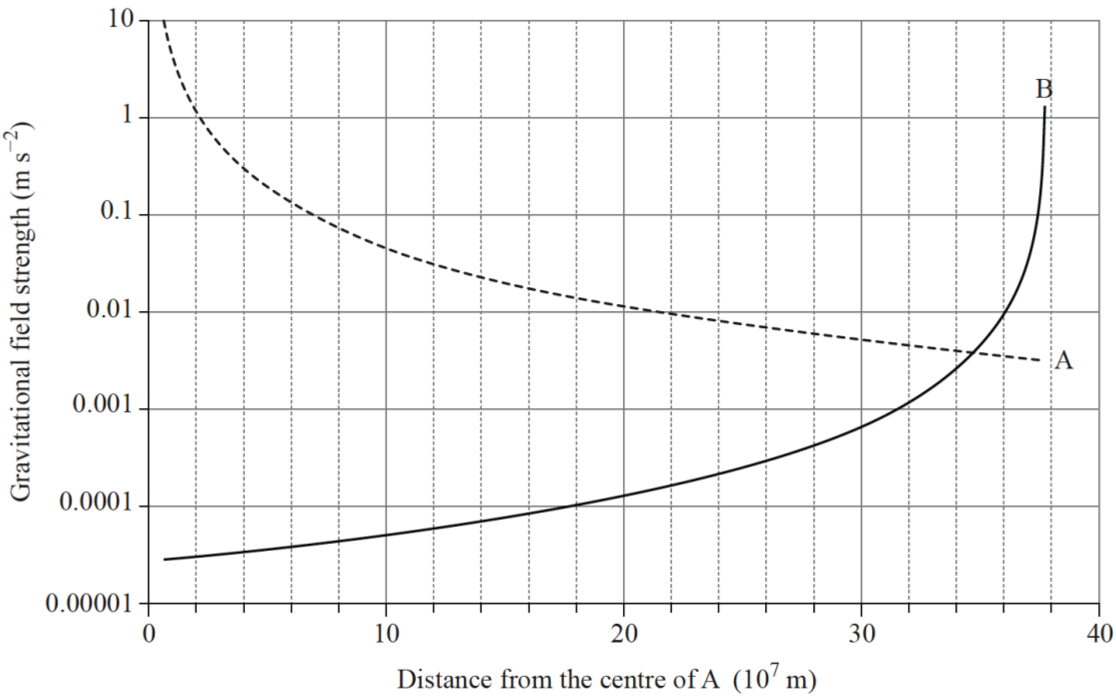
2022 Physics EA Paper 1 Question 27 Solutions

- from Richard Walding

This question is taken from the 2022 General Sequence EA Physics paper. One solution was provided by QCAA in the EA Marking Guide but there are many approaches that can be taken, and these are acknowledged as being appropriate in the EAMG. There is no unique answer to this question and different approaches will give different answers - all of which are correct. This ambiguity arises because the two graphs in the question are not compatible. Here goes:

QUESTION 27 (5 marks)

Object A is five times the mass of object B. The graph shows the contribution of each object towards the strength of the net gravitational field between them.



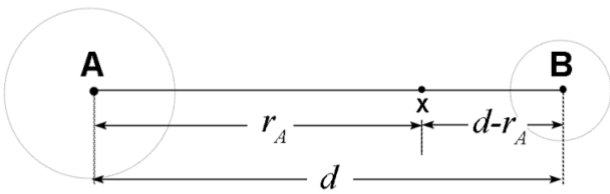
Determine the total distance between the centre of the two objects. Show your working.

SOLUTIONS

The two graphs are incompatible so there is no unique solution. Here are several approaches which would all be awarded full marks (5/5).

SOLUTION 1 (similar to the QCAA *EA marking guide*) 5/5 marks

Let the distance between the objects be d. Point X is on a line between the objects and a mathematical relationship between the gravitational field strengths of the objects can be stated. The distance from the centre of object A to point X is r_A, and from the centre of object B to X is r_B, which equals $d - r_A$.



In this solution, let X be the point where the gravitational field strengths are equal and opposite. This is where the two lines intersect:

$$g_A = \frac{Gm_A}{(r_A)^2} = \frac{G \times 5m_B}{(r_A)^2}$$

$$g_B = \frac{Gm_B}{(r_B)^2}$$

- g_A has a value between 0.001 and 0.01 m s^{-2} but cannot be stated precisely because of the logarithmic scale.
- $r_A = 35 \times 10^7$ m or 35 units (allow 34.5 to 35),
- $r_B = d - 35 \times 10^7$ m or ($d - 35$) units

$$g_A = g_B$$

$$\frac{Gm_A}{(r_A)^2} = \frac{Gm_B}{(r_B)^2}$$

$$\frac{G \times 5m_B}{(r_A)^2} = \frac{Gm_B}{(d - r_A)^2}$$

$$\frac{5}{(r_A)^2} = \frac{1}{(d - r_A)^2}$$

$$\frac{\sqrt{5}}{r_A} = \frac{1}{d - r_A}$$

$$\frac{r_A}{\sqrt{5}} = d - r_A$$

$$d = \frac{r_A}{\sqrt{5}} + r_A$$

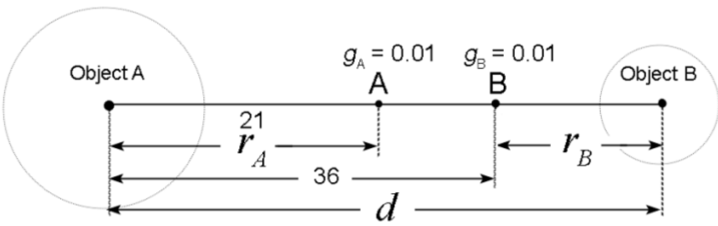
$$= \frac{35}{\sqrt{5}} + 35$$

$$= 50.7\ units$$

$$= 51 \times 10^7 m\ (2\ sf)$$

SOLUTION 2 (5/5 marks)

Let A and B be separate points between the objects where the gravitational field strengths, g_A due to object A, and g_B due to object B, are equal to 0.01 m s^{-2} but in opposite directions.



$r_A = 21$ units, $r_B = (d - 36)$ units, $g = 0.01$ m s^{-2}

$$g_A = g_B$$

$$\frac{Gm_A}{(r_A)^2} = \frac{Gm_B}{(r_B)^2}$$

$$\frac{G \times 5m_B}{(21)^2} = \frac{Gm_B}{(d - 36)^2}$$

$$\frac{5}{(21)^2} = \frac{1}{(d - 36)^2}$$

$$\frac{\sqrt{5}}{21} = \frac{1}{d - 36}$$

$$\frac{21}{\sqrt{5}} = d - 36$$

$$d = \frac{21}{\sqrt{5}} + 36$$

$$= 45\ units$$

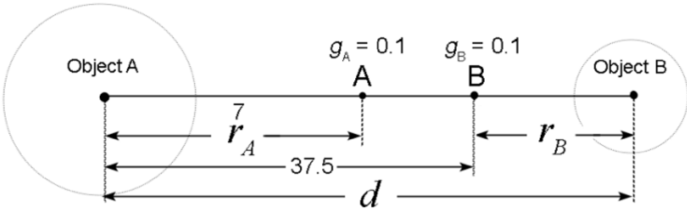
$$= 45 \times 10^7 m\ (2\ sf)$$

SOLUTION 3 (5/5 marks)

This is similar to Solution 2, but the selected gravitational field strength is 0.1 m s^{-2}. Let A and B be separate points between the objects where the gravitational field strengths, g_A due to object A, and g_B due to object B, are equal to 0.1 m s^{-2} but in opposite directions.

Let Point A be where g_A = 0.1 m s^{-2}, that is: r_A = 7 units,

Let Point B be where g_B = 0.1 m s^{-2}, that is: r_B = d – 37.5 units



$$g_A = g_B$$

$$\frac{Gm_A}{(r_A)^2} = \frac{Gm_B}{(r_B)^2}$$

$$\frac{G \times 5m_B}{(7)^2} = \frac{Gm_B}{(d - 37.5)^2}$$

$$\frac{5}{(7)^2} = \frac{1}{(d - 37.5)^2}$$

$$\frac{\sqrt{5}}{7} = \frac{1}{d - 37.5}$$

$$\frac{7}{\sqrt{5}} = d - 37.5$$

$$d = \frac{7}{\sqrt{5}} + 37.5$$

$$= 41\ units$$

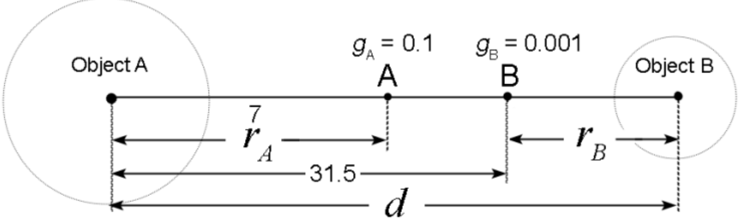
$$= 41 \times 10^7 m\ (2\ sf)$$

SOLUTION 4 (5/5 marks)

This is similar to Solution 2, but the selected gravitational field strengths are different, but their relationship is known.

Let Point A be where g_A = 0.1 m s^{-2}, that is: r_A = 7 units,

Let Point B be where g_B = 0.001 m s^{-2}, that is: r_B = d – 31.5 units



$$\frac{g_A}{g_B} = \frac{0.1}{0.001} = 100$$

$$g_A = 100\ g_B$$

$$\frac{Gm_A}{(r_A)^2} = \frac{100\ Gm_B}{(r_B)^2}$$

$$\frac{G \times 5m_B}{(7)^2} = \frac{100\ Gm_B}{(d-31.5)^2}$$

$$\frac{1}{(7)^2} = \frac{20}{(d-31.5)^2}$$

$$\frac{1}{7} = \frac{\sqrt{20}}{d-31.5}$$

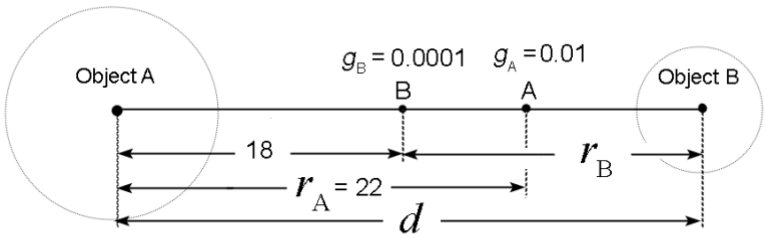
$$7\sqrt{20} = d - 31.5$$

$$d = 7\sqrt{20} + 31.5$$

$$= 63\ units$$

$$= 63 \times 10^7 m\ (2\ sf)$$

SOLUTION 5 (5/5 marks)
This is similar to Solution 4, where the selected gravitational field strengths are different, but their relationship is known.
Let Point A be where $g_A = 0.01$ m s^{-2}, that is: $r_A = 22$ units,
Let Point B be where $g_B = 0.0001$ m s^{-2}, that is 18 units from object A: $r_B = d - 18$ units



$$\frac{g_A}{g_B} = \frac{0.01}{0.0001} = 100$$

$$g_A = 100\ g_B$$

$$\frac{Gm_A}{(r_A)^2} = \frac{100\ Gm_B}{(r_B)^2}$$

$$\frac{G \times 5m_B}{(22)^2} = \frac{100\ Gm_B}{(d-18)^2}$$

$$\frac{1}{(22)^2} = \frac{20}{(d-18)^2}$$

$$\frac{1}{22} = \frac{\sqrt{20}}{d-18}$$

$$22\sqrt{20} = d - 18$$

$$d = 22\sqrt{20} + 18$$

$$= 107\ units$$

$$= 110 \times 10^7 m\ (2\ sf)$$

SOLUTION 6 (5/5 marks)

The following solution does not have perfect alignment with the EAMG but is sufficiently detailed to get full marks. There are three major concepts that form the solution.

• The graph shows that as the curve for B approaches the asymptote the gravitational field strength moves towards infinity. **[1 mark]** • The asymptote is at approximately 37.5 × 10^7 m. **[1 mark]**
• The equation describing this situation is Newton's law of universal gravitation, $g = \frac{GM}{r^2}$ which shows that as the radial distance from B, r_B, approaches zero the gravitational field strength, g, approaches infinity. **[1 mark]** • The value of r_B, is zero at the centre of object B if it was a point object. **[1 mark]**
• Thus the distance from A to the centre of B is 37.5 × 10^7 m. **[1 mark]**

Note: Firstly, this is not true for a real (astronomical) object and depends on its size and composition, but this is outside the scope of the Units 3 and 4 in the syllabus. Secondly, it uses only the data from one of the curves and more information is provided in the graph which is not used in the solution.
Nevertheless, it is a plausible answer.

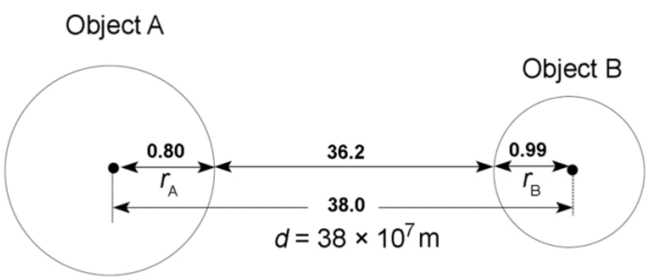
SOLUTION 7 (5/5 marks)

The following solution also uses the asymptote approach but considers both graphs. It is provided by Clinton Jackson, Brisbane Adventist College. It does not have perfect alignment with the EAMG but is more than sufficiently detailed to get full marks.

- The equation describing this situation is Newton's law of universal gravitation, $g = \frac{GM}{r^2}$.
  [EAMG: *recognises the scenario relates to gravitational field strength* (1 mark)]

- From the equation, the value of gravitational field strength asymptotically approaches infinity as $r \to 0$.
  [EAMG: *identifies distance from A where net gravitational field strength is zero* (1 mark)]

- On the graph there are two asymptotic regions, one corresponding to the approach to object A and the other corresponding to the approach to object B.



  - Curve B does not have any values d < 0.8 × 10^7 m, which likely corresponds to the surface of A, suggesting a radius of A, r_A, of 8 × 10^6 m.

- In a similar way the values for curve A do not have any values of $d > 37 \times 10^7$ m, which like curve B is asymptotic, and likely corresponds to the surface of B.
- As such, the distance between the two surfaces is $37 \times 10^7 - 0.8 \times 10^7 = 36.2 \times 10^7$, and the distance to the surface of B from the centre of A is 37×10^7 m.

  [EAMG: *constructs an equation that can be solved for the distance between the objects* (1 mark)]

- A value for the radius of B can be found as follows:
  - For curve A, when $d_A = 21 \times 10^7$ m, $g = 0.01$m s^{-1}. This gives $M_A = 10.9295 \times 10^{24}$ kg.
  - Thus $M_B = 2.1859 \times 10^{24}$ kg, from which r_B can be found as follows:

    On curve B, $g = 1.5$ m s^{-2} (roughly the surface) and using $g = \frac{GM}{r^2}$. gives $r_B = 9.9 \times 10^6$ m.

    [EAMG: *provides appropriate mathematical reasoning* (1 mark)]

- Adding this to the distance from the centre of A to the surface of B, gives a total distance between centres of 38×10^7 m.

  [EAMG: determines the distance between the centre of the two objects (1 mark)]

My thanks to Takashi Yamamoto from Highfields State Secondary College for his paper on Q27 presented on the Discussion List in February 2023, and for checking these solutions.