UNIT 01

Measurement & Physical Quantities

CHAPTER 01

Measurement and Physical Quantities

1.1 MEASUREMENT

Physics, like the other sciences, is all about explaining the natural world. Measurement is at its very heart. Ever since humans have been thinking about their place in the universe, they have been making measurements. Have you ever wondered about any of these:

- What would have been the first sort of measurement made by humans?
- When you use the unit of length, *foot*, whose foot was the standard?
- What is the shortest length of time that can exist? Is there no limit?
- Time passes but why can't it go backwards?
- Just how heavy is the universe? How did they weigh it?
- Is cream more dense than milk and, anyway, who invented *density*?

Questions like these have always intrigued people. As you study physics some of them will become clearer. But hopefully you will ask your own questions and make your own measurements, for this is what the study of physics is all about.

Activity 1.1 ESTIMATING

EI

1. Estimate the length of this page to the nearest millimetre. Now measure it. Were you over or under?
2. Now that you've had practice, estimate the length of this line:

   ____

   Were you any more accurate?
3. How far is it from the floor to the ceiling? Write down your estimate and then find the actual value.
4. Can you estimate 30 seconds? Look at your watch, cover it and uncover it when you think 30 seconds is up. Repeat it until you are accurate to within 1 second. How did you count off the seconds? How did others in the class count off the seconds?
5. How good are you at estimating mass? Estimate the mass of this book in grams without lifting it and then again after lifting it. Did you lift it up and down to estimate mass? Why?
6. Feel the thickness of one page of this book. How many pages do you estimate this book has? Check.

NOVEL CHALLENGE

Here are a few 'Fermi' questions (named after US physicist Enrico Fermi, who used to drive his students nuts with them).

A How quickly does hair grow?
B How many piano tuners are there in your capital city?
C How many ping-pong balls can you fit in a suitcase?
D How quickly does grass grow?

NOVEL CHALLENGE

The four compass directions North, East, South, West are derived from old foreign words. Can you match up the original meanings with the compass directions:

A Indoeuropean *wes* = Sun goes 'down'.
B Italian *nerto* = 'to the left' as one faces the Sun.
C German *suntha* = region in which the 'Sun' appears in the Northern Hemisphere.
D Indoeuropean *aus* = Sun 'rises'.

NOVEL CHALLENGE

If you were transported in a time machine to an unknown date in Australian history, how could you work out the date? See our Web page for some suggestions.

Estimating measurements is important. You can see whether answers are reasonable or nonsense if you have a feeling for some of the common units of measurement in physics. The three quantities you've measured in the activity are the most basic measurements in physics: **length, time** and **mass**. But your estimates probably differed from others in your class and that's why standards were developed. The importance of measurement grew as human societies became more complex.

The first measurement the earliest humans are believed to have used was the 'day'. Hence, the 'day' became the first unit of measurement, well before any concept of length or mass. Which unit do you think came next? Perhaps the 'month' — from one new moon or full moon to the next; and then perhaps the 'year' when people noticed that the Sun rose again in the same constellation of stars after many new moons.

Neanderthal burial sites from 50 000 years ago suggest that people were conscious of the past, the present and the future — something that most other animals are believed to be unaware of.

As humans have progressed, so too has their need for new units of measurement. The need for a unit comes before a unit is invented. Only recently have units like the *barn* been invented. The size of a nucleus as seen by a high speed atomic particle is as big as the side of a barn, hence the name. One barn equals 10^{-28} m^2. There was no need for this unit until Einstein produced the 'theory of relativity' and physicists applied it to atomic structure.

PHYSICAL QUANTITIES 1.2

There are a number of things in the world we want to measure. As well as the three mentioned above (length, time and mass), there are others, such as temperature, electric current and weight. These measurable features are called **physical quantities**. There are also some non-physical quantities, for example intelligence, beauty and personality, that are difficult-to-measure. Attempts have been made to devise measurements for quantities such as these but have always ended up in disagreement and, in many cases, failure.

The **international system** of units called **SI** (from the French name for the system, *Système International d'Unités*), is now commonly used around the world. It is sometimes called the **metric system** (from the Greek *metron* to 'measure').

The seven fundamental (or base) units of this system are shown in Table 1.1.

Table 1.1 SI UNITS

PHYSICAL QUANTITY	SYMBOL OF QUANTITY	NAME OF UNIT	SYMBOL FOR UNIT
Length	l	metre	m
Mass	m	kilogram	kg
Time	t	second	s
Electric current	I	ampere	A
Temperature	T	kelvin	K
Amount of substance	n	mole	mol
Luminous intensity		candela	cd

To get multiples of the base units, prefixes are added. Table 1.2 lists some of these prefixes that will be used throughout your physics course. You should remember from nano to mega. Check with your teacher if you need any others.

Table 1.2 PREFIXES

PREFIX	SYMBOL	MEANING	VALUE	FACTOR
Pico	p	one million-millionth	0.000 000 000 001	10^{-12}
Nano	n	one thousand-millionth	0.000 000 001	10^{-9}
Micro	µ	one millionth	0.000 001	10^{-6}
Milli	m	one thousandth	0.001	10^{-3}
Centi	c	one hundredth	0.01	10^{-2}
Deci	d	one tenth	0.1	10^{-1}
Kilo	k	one thousand	1 000	10^{3}
Mega	M	one million	1 000 000	10^{6}
Giga	G	one thousand million	1 000 000 000	10^{9}
Tera	T	one million million	1 000 000 000 000	10^{12}

Example of using a prefix with a unit: 1 millimetre = 10^{-3} metre = 0.001 metre.

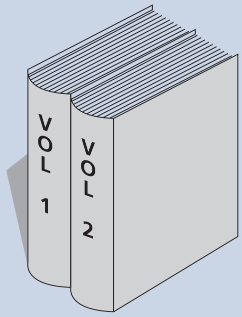
Rarely used prefixes are:

- 10^{-15} femto (f) — radius of a proton is 1 fm
- 10^{-21} zepto (z) — charge on the electron is 160 zC
- 10^{-24} yocto (y) — mass of the hydrogen atom is 1.66 yg
- 10^{-27} xenno (x) — magnetic moment of a proton is 14 xJ T^{-1}
- 10^{21} zetta (Z) — distance to Andromeda galaxy is 20 Zm
- 10^{24} yotta (Y) — mass of the Earth is 5977 Yg.

Others you'd never use are vendeko (v) 10^{-33} and vendeka (V) 10^{33}. Can you think of any practical use of these prefixes? Mathematicians also use the term *googol* to represent 10^{100} and *googolplex* for 10 raised to the power of a googol: $10^{10^{100}}$. The biggest number in the world (apart from infinity) is Grahams' number. If all the material in the world was turned into paper there still wouldn't be enough paper to write it down. Now that's big!

NOVEL CHALLENGE

You have two 100-page volumes of a dictionary on your shelf. A worm eats its way from Volume 1 page 1 through to Volume 2 page 100.
How many pages does it eat through?



Standards

Standards have to be agreed upon for units to be useful throughout the world. For instance, the temperatures in different countries couldn't be compared until a universal temperature scale was devised. The following shows how some of these units have developed.

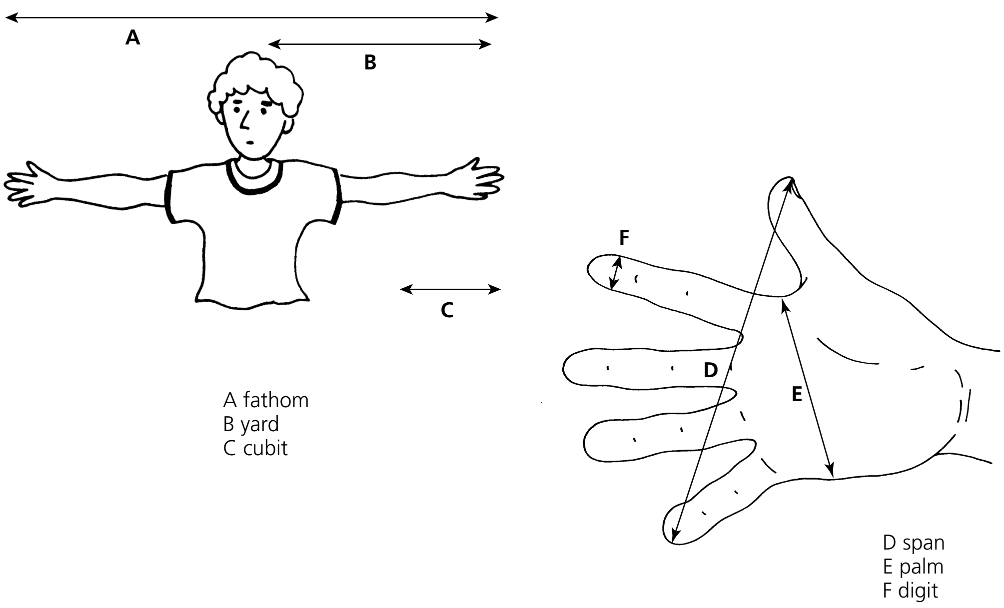
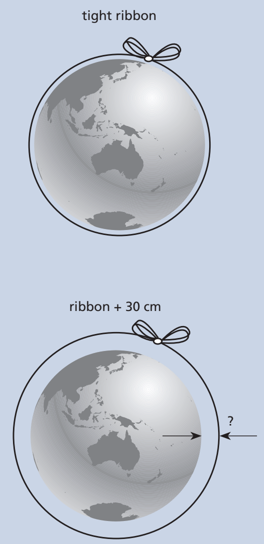


Figure 1.1
Body measurements.

NOVEL CHALLENGE

Consider the Earth to have a circumference of 40 000 km and a ribbon to be put tightly around it. If you cut the ribbon and inserted a 30 cm piece, how far would the ribbon be from the earth if it was evenly spaced?



Length

As with most early units, people used the most convenient measures — themselves. The length of a foot or a stride was a convenient measure. So was the span of a hand or the thickness of a thumb. But as civilisations grew, these ways of measuring became inadequate. How could a foot be used as a measure when one person's foot was so much longer than another's? Hands and thumbs were different too. In ancient times a measure that was used in one country was often later adopted by others through trade or invasion. Roman measures spread throughout Europe, Asia, England and Africa as the Romans conquered and occupied these lands but gradually, through mistakes in copying and figuring, the standards became so confused that most of them dropped out of use. By the sixteenth century most people in Europe had returned to the old body measurements and we still use some of these today.

The shortest unit of length was the *digit*, the width of a finger, or three-quarters of an inch. An inch is the width of a thumb; a hand is four inches and the span is nine inches. To try to standardise these units, Edward II of England ruled that one inch 'shall be equal to three grains of barley, dry and round, placed end to end lengthwise'.

The *foot* was about 11.5 inches in Greece, 12 inches in England and other English speaking countries, and 11 to 14 inches anywhere else. The earliest attempt to standardise the foot was in 2100 BC when it was decreed that a foot was the length of the foot of the statue of the ruler of Gudea of Lagash in Babylonia. It was 10.41 inches long and divided into 16 parts.

The *pace* was another common measure. It was about 5 feet — the length of two complete steps. Roman soldiers paced off the miles as they marched. A thousand paces made up a mile, just a little less than the modern mile, which is 5280 feet. Now we measure a pace as a single step — about 2.5 feet.

Lastly, the *yard*. The yard was defined in two ways: in northern Europe it was the length of an Anglo-Saxon's belt whereas in the south it was a double cubit. A cubit is 18 inches — the distance from the elbow to the wrist. Henry I, at the beginning of the twelfth century, fixed the yard as the distance from his nose to the thumb of his outstretched arm.

The metric system (SI) was invented by the French in 1790, following the French Revolution. It was a part of a plan for a new beginning, a whole new social and economic life in France, without any ties to the past. The metre, the basic unit of length, was supposed to be one ten-millionth part of the distance from the North Pole to the Equator. But in the eighteenth century instruments were not as accurate as they are today, so there was a measurement error. By the time the error was realised, the metre was so well established at 39.37 inches that it was left at that.

A platinum–iridium bar exactly this distance long was made and this became the standard for the metre. In 1960 the standard metre was redefined to be the length equal to 1 650 763.73 wavelengths in a vacuum of the red-orange light emitted by the krypton-86 atom. Since 1983, however, the metre has been redefined as the length of the path travelled by light in a vacuum during a time interval of 1/299 792 458 of a second.

NOVEL CHALLENGE

Quick now — is a physics lesson longer or shorter than a microcentury?

Table 1.3 SOME LENGTHS

LENGTH	METRES	LENGTH	METRES
To furthest quasar	10^{26}	Thickness of a page	10^{-4}
To nearest star	10^{16}	Radius of H atom	10^{-10}
To Pluto	10^{13}	Radius of a proton	10^{-15}
Radius of Earth	10^{7}		

Time

The first way of measuring time was to keep a record of the repetition of natural events. From sunrise to sunrise was the most fundamental of periods as it was so easy to measure and hence the day became the first unit of time. We do not know how long ago people started using the idea of days but it would certainly have been tens of thousands of years ago.

People realised that the Sun and the Moon were the best timekeepers of all. They called the time taken for the Earth to make one orbit of the Sun a year. We now know it to be 365 days, 5 hours, 48 minutes and 45.7 seconds long. The extra hours, minutes and seconds are collected together every 4 years to make the additional day we have in a leap year. To keep the timing more accurate, the start of a century is classified as a *non*-leap year.

The length of a day is fixed by the Earth's rotation on its axis. But at different times of the year the length of the day varies from place to place so we have to take this into account. What we end up with is the mean solar day but its duration is now standardised in terms of the hour, minute and second.

It was the ancient Babylonians who divided their measurements into sixty parts and we have kept their divisions for the hour and minute. A minute is 60 seconds and an hour is 60 minutes. The fundamental unit, the second, is now defined as the time for 9 192 631 770 vibrations of light (of a specified frequency) emitted by a caesium-133 atom. In principle, two caesium clocks would have to run for 6000 years before they differed by 1 second. In practice, atomic clocks do better than that. The latest Hewlett Packard 5071A caesium clock achieves an accuracy of 1 second in 1.6 million years and only costs about $90 000. Some experimental clocks are within 1 second in 30 billion years. Every physics lab should have one.

But a fundamental question about time has always bothered physicists. What does the passage of time mean? What is the difference between the past and the future apart from the passage of time? Nobel prize-winning physicist Richard Feynman said, 'We physicists work with time every day but don't ask me what it is. It's just too difficult to think about'.

Table 1.4 SOME TIME INTERVALS

TIME INTERVAL	SECONDS
Lifetime of a proton	10^{39}
Age of the universe	10^{18}
Age of the pyramid at Cheops	10^{11}
Human life expectancy	10^{9}
Length of a day	10^{5}
Physics lesson	10^{4}
Lifetime of a muon	10^{-6}
Shortest time interval known*	10^{-43}

* This is known as Planck time — the earliest time after the 'Big Bang' at which the laws of physics as we know them can be applied.

Mass

Measurements of mass and weight came a long time after measurements of length and time. An early way of thinking about weight was the amount a person could carry. At first, people compared weight by balancing small objects, one in each hand, and estimating whether one was heavier than another. About 7000 years ago, the Egyptians devised a crude scale — a stick hanging by a cord tied around its middle acting as a balance. By 3000 BC small stone weights were used as a comparison, but as trade developed, different weights were used for different objects. Honey, medicine and metal all had different units of weight, many of which have persisted into modern times. For example, the avoirdupois system of weights includes ounces, pounds and the ton. Grain was measured in bushels; liquids were measured in pints and gallons or in the case of oil, in barrels. But no mention in early history has been made of the quantity known as mass.

Mass and weight are different quantities but people use them as if they mean the same thing. In Chapter 4 you will see the difference. Mass is a measure of an object's resistance to motion when being pushed or pulled. A 1 kilogram mass will be just as hard to push around no matter where in the universe it is. Weight, on the other hand, is a measure of the force of gravity acting on an object and will vary depending on how strong gravity is in that place. But weight has always been the quantity people have associated with heaviness; after all,

NOVEL CHALLENGE

In the first paragraph of Charles Dickens' *The Pickwick Papers* he states that he was at the bottom of a deep well and could see the stars in the daytime. Aristotle made the same claim in *On the Generation of Animals* in 350 BC.

Is this possible? Propose points for and against this idea. See the Web page for an answer.

PHYSICS UPDATE

The Time Service Department, US Naval Observatory, Washington, DC provides time signals for use throughout the USA and other parts of the world. You can access their clock on the Internet at http://tycho.usno.navy.mil/what.html and even set your computer's clock against their master signal.

NOVEL CHALLENGE

The world is broken up into many time zones based on the longitude of the various regions. Queensland is 1/2 hour ahead of South Australia, for instance.

But what time zone is the South Pole? No emailing Casey Station to find out.

PHYSICS FACT

The word 'hour' comes from the Greek word meaning 'season'. The length of daylight depends on the season. The word 'day' comes from the Saxon word 'to burn', referring to the hot days of summer.

NOVEL CHALLENGE

Under the system of measurement adopted during the reign of Queen Elizabeth I:

1 mouthful	= 1 cubic inch
1 handful	= 2 mouthfuls
1 jack	= 2 handfuls
1 gill	= 2 jacks
1 cup	= 2 gills
1 pint	= 2 cups
1 quart	= 2 pints

If 1 cubic inch = 14.7 mL, how many cups to 1 litre?

gravity is fairly constant all over the world and it hasn't been until the twentieth century that humans have left the Earth to go into space. More importantly though, the concept of mass was not developed until the 1680s when English scientist Isaac Newton proposed a relationship between force and acceleration that profoundly affected the new science of mechanics. This idea will be developed further in Chapter 4.

Mass is a fundamental quantity and the *kilogram* has been adopted as the fundamental unit of mass in the SI or metric system. The standard kilogram is a platinum–iridium cylinder kept at the International Bureau of Weights and Measures near Paris. Accurate copies have been sent to other standardising laboratories in other countries and the masses of other bodies can be determined by balancing them against a copy.

Photo 1.1
The standard kilogram.

Table 1.5 SOME MASSES

OBJECT	KILOGRAMS
Universe	10^{53}
Our galaxy	10^{41}
Sun	10^{30}
Moon	10^{23}
Ocean liner	10^{8}
Human	10^{2}
Grape	10^{-3}
Speck of dust	10^{-9}
Penicillin molecule	10^{-17}
Uranium atom	10^{-26}
Proton	10^{-27}
Electron	10^{-30}
Neutrino	10^{-30}
Uranium atom	10^{-26}

NEI Activity 1.2 BIGGEST, LONGEST AND OLDEST

Use the *Guinness Book of Records*, the Internet or an encyclopaedia to find out the following facts about units of measurement:

1. The highest artificial temperature on Earth was in a fusion reactor in the USA in 1994. How hot did it get?
2. How long can people go without food or water? Has anyone made it past 18 days?
3. What are the masses of the heaviest man and woman ever recorded? How many times greater than that of the lightest person are they?
4. Gold is the most ductile element known — it can be drawn into a very fine wire. How many metres of wire can be produced from 1 g of gold?

NOVEL CHALLENGE

This book is printed on paper classified as 80 gsm (80 grams per square metre). The cover is made of 249 gsm paper. What should the mass of this book be? Check it and see. What went wrong?

Derived units

New quantities can be made up of the base quantities. These are called **derived quantities**. For example, you can have combinations of the base units, such as metres per second and cubic metres or you can have derived quantities that have been given specific names, such as newton, coulomb and watt.

Table 1.6 lists some derived quantities.

Table 1.6 SOME DERIVED QUANTITIES

DERIVED QUANTITY	UNIT	SYMBOL FOR UNIT
Acceleration	metre per second^2	m s^{-2}
Angle	radian	rad
Area	metre^2	m^2
Capacitance	farad	F
Density	kilogram per metre^3	kg m^{-3}
Electric charge	coulomb	C
Energy	joule	J
Force	newton	N
Frequency	hertz	Hz
Momentum	kilogram-metre per second	kg m s^{-1}
Potential difference	volt	V
Power	watt	W
Pressure	pascal	Pa
Resistance	ohm	Ω
Velocity	metre per second	m s^{-1}
Volume	metre^3	m^3

EI Activity 1.3 WORKING SCIENTIFICALLY

Physicists spend their professional lives investigating relationships between physical quantities. In 1665 Robert Hooke described the relationship between the length of a spring and the stress (force) applied to it. Currently, physicists are trying to work out how the fundamental forces of nature are related to each other. In the three activities that follow, you will set up some experiments, collect data and look for relationships between some of the physical quantities mentioned earlier.

Part A: The bent ruler

1. Clamp a 30 cm steel ruler to the edge of a bench leaving most of it overhanging. Measure the distance from the floor to the tip of the ruler. (See Figure 1.2.)
2. Add a 50 g mass to the tip and record how many centimetres the ruler has bent from its unladen position. This is called its **displacement**.
3. Add another 50 g mass and record the total displacement. Then another 50 g and so on until 300 g has been added.
4. Plot a graph of displacement (y-axis) versus mass added (x-axis).
   (a) Does the graph go through the origin (0, 0)? If so, why?
   (b) How many centimetres does the ruler bend per 100 g added? Express this as cm per g.
   (c) Show how the graph would look if you: **(i)** used a thicker ruler; **(ii)** used a wider ruler; **(iii)** used a plastic ruler; **(iv)** allowed only 20 cm to overhang; **(v)** used a frozen ruler; **(vi)** used a steel ruler rapidly heated and cooled (annealed); **(vii)** used a steel ruler heated and cooled slowly. Try it! For all graphs you should provide a theoretical justification of your prediction.

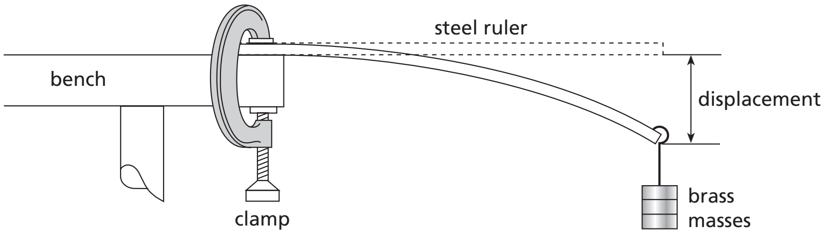


Figure 1.2
The bent ruler set-up for Part A.

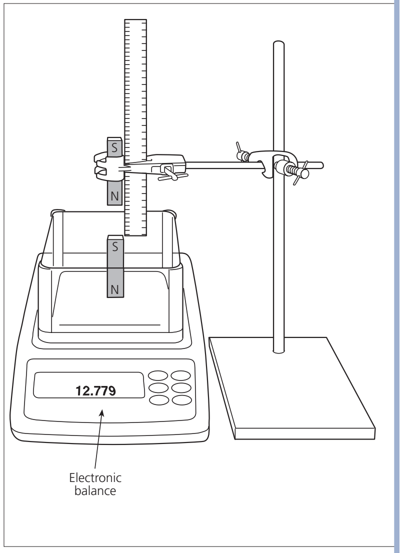
PHYSICS FACT

On his fourth voyage to the New World, Spanish explorer Christopher Columbus was marooned in Jamaica and, after a while, the local Indians refused to provide food. He knew that there would be an eclipse at noon on 29 February 1504 so he summoned the chiefs aboard and told them that unless they gave him food God would blacken the sky. When they refused, the sky went black right on time and when they relented he 'made' the sky go back to normal. Then the Spanish began the systematic plunder and destruction of an entire civilisation. Ah, no wonder science gets a bad name.

NOVEL CHALLENGE

German researcher Günther Bäumler found that people with the surname Smith (Schmidt in German) had, on average, a body mass that was 2.4 kg greater than people with the name Taylor (Schneider).
How would you test his findings in your school? What other variables would affect your results? Check our web page for why this difference occurs.

Figure 1.3
Set-up for measuring the relationship between magnetic force and distance (Activity 1.3, Part B).

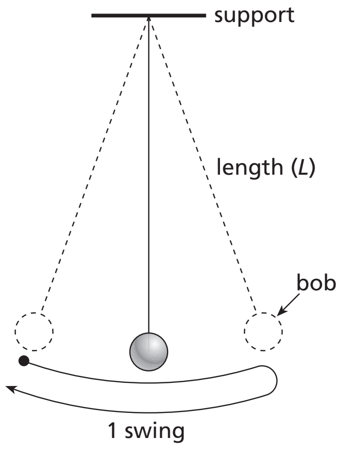


Part B: Magnetic personality

The force between two magnets varies with separation distance.

1. Place a bar magnet vertically upright on the pan of an electronic balance. (See Figure 1.3.) Zero the balance.
2. Place another magnet in a clamp above the first magnet so that the unlike poles face each other. There will be an attractive force, so the scale reading on the balance should be a negative value.
3. Start with the end of the clamped magnet 50 cm from the magnet on the balance and record a reading. If it is not zero, start with a 1 m separation (hold it in your hand).
4. Reduce the separation distance (d) by 5 cm at a time (or less if you like) and take balance readings.
5. Plot the data with separation distance (in cm) on the x-axis, and scale reading (grams) on the y-axis. If you are keen, convert the separation distance to metres; and convert the scale reading to force in newtons (N) by dividing it by 1000 and multiplying by 9.8.
6. Some questions:
   (a) When the distance was halved (from 50 cm to 25 cm), by what factor did the scale reading increase?
   (b) Would you get the same results if you put the magnets into repulsion?
7. Now try plotting $1/d^2$ on the x-axis against the scale readings. Did something magical occur?

Figure 1.4
Characteristics of a pendulum.



Part C: Let him swing!

Three variables you could change about a pendulum are the length, the mass and the distance through which it swings. (See Figure 1.4.) Using a lead fishing sinker or a brass mass tied to a metre of fishing line, construct a pendulum and measure the time for one swing at six different lengths. Keep the mass constant. Plot a graph. Keep your data for Chapter 3.

Converting units

It is often important to convert from one unit to another: for instance, from millimetres to metres or from pounds to kilograms. Two types of conversions are involved:

- From one SI unit to another.
- From a non-SI unit to an SI unit.

The first type will be needed when data are given in one particular unit but the answer has to be given in another form. This might occur when some constant is involved that is in a unit different from that of the data given. For example, if you had to calculate how far you would travel in 10 minutes at a speed of 5 metres per second then you would convert 10 minutes to seconds ($10 \times 60 = 600$) and multiply this number of seconds by the speed ($600 \times 5 = 3000$ metres).

Example

Imagine you have made measurements of a block of wood in a density experiment and need to calculate its volume in cubic metres. Length 35 cm, breadth 2.0 cm, width 1.5 cm.

Step 1: Convert the measurements to SI units (metre):

- length = 35 cm = $35 \times 1 \times 10^{-2}$ m = 0.35 m (3.5×10^{-1} m)
- breadth = 2.0 cm = $2.0 \times 1 \times 10^{-2}$ m = 2.0×10^{-2} m
- width = 1.5 cm = $1.5 \times 1 \times 10^{-2}$ m = 1.5×10^{-2} m.

Step 2: Calculate the volume:

- volume = 0.35 m $\times 2 \times 10^{-2}$ m $\times 1.5 \times 10^{-2}$ m
  = 1.05 m $\times 10^{-4}$ m^3.

Some other simple examples are:

- 25 000 cm = 250 m (2.5×10^2 m)
- 23 km = 23 000 m or 2.3×10^4 m
- 6 hours = 21 600 s or 2.16×10^4 s.

The other type of conversion is from a non-SI unit to an SI unit. This could occur, for instance, when data come from another source such as from some domestic measurement; from another country or from data taken in the past. The United States has yet to adopt SI units for daily use although all science, engineering and medical units throughout the world have been changed to SI. For instance, you may have to convert the mass of a person from pounds to kilograms. The conversion factor is 1 kg = 2.204 622 341 pounds. (See Table 1.7.)

Table 1.7 SOME NON-SI CONVERSION FACTORS

NON-SI UNIT	SI UNIT
Inch (in)	2.54 centimetres
Yard (yd)	0.914 401 8 metre
Gallon (gal)	4.546 litres
Pound mass (lb)	0.453 592 37 kilogram
Pound weight (lb)	4.45 newtons
Mile (mi)	1.609 kilometres
Acre (ac)	0.404 687 hectare
Pound per square inch (psi)	6896 pascals
Horsepower (Hp)	746 watts

Questions

1 From the following, select **(a)** two fundamental quantities; **(b)** two fundamental units; **(c)** two non-SI units: yard, luminous intensity, ampere, year, minute, temperature, force, second, pressure.

2 Convert the speed of light (3×10^8 m s^{-1}) to **(a)** km h^{-1}; **(b)** miles per hour.

3 Convert the following: **(a)** 10.3 m to cm; **(b)** 1.25 cm to m; **(c)** 1120 cm to m; **(d)** 143 367 mm to m; **(e)** 1.8 mm to m; **(f)** 14 cm^2 to m^2; **(g)** 4.8 cm^3 to m^3.

4 **(a)** Japanese sumo wrestlers have to be a minimum of 5 feet 8 inches tall. How many centimetres is this? (1 foot = 12 inches and 1 inch = 2.54 cm.)
(b) The heaviest baby ever born was 23 lb 10 oz. If there are 16 ounces (oz) in 1 pound (lb) and 1 pound equals 0.454 kg, convert the baby's mass to kg.

1.3 SCIENTIFIC NOTATION

Things in the world are not always human-sized. Some are very small; some are huge. The numbers used to express these measurements can get messy. For example, the time taken for light to travel from one side of an atom to the other is about one billion billion billion billionths of a second. The mass of the Sun is two thousand billion billion billion kilograms. In his book *A Brief History of Time*, Stephen Hawking mentions that the publisher told him not to use any numerals. All numbers had to be spelt out because, it was argued, people

NOVEL CHALLENGE

In his 1997 book *Number Sense*, Stanislas Dehaene reported that his tests on brilliant scientists in France showed that it took them longer to say whether 6 was greater than 5 than it did to say whether 9 was greater than 5. Propose a testable hypothesis that could be investigated.

NOVEL CHALLENGE

People shrink in height not only as they get older, but also during each day. Some of our students shrink by $1\frac{1}{2}$ cm between first and last lesson. What is the reason for this? Can you find factual support for your suggestion? Do you think taller people shrink more than shorter ones? Does everyone shrink by a certain percentage? Do younger and older people shrink by the same percentage?

PHYSICS FACT

A very old unit of length was cubit—the length of the arm from elbow to fingertips. It comes from the Latin *cubitum*, meaning 'elbow'. The Egyptian 'royal cubit' was 542 mm long, and a master cubit of black granite was kept in a royal vault. All the cubit sticks in use in Egypt were measured at regular intervals. For example, the Great Pyramid of Giza was 280 royal cubits (RC) high. Other cubits include the biblical cubit of 457.2 mm.

NOVEL CHALLENGE

Humans have 10^{14} cells at a diameter of 0.01 mm each. If they were placed in a line, how many times around the Earth would they go? (The radius of the Earth is 6.38×10^6 m.)

NOVEL CHALLENGE

A brochure for the 1963 Ford Falcon said it averaged 26 miles per gallon of petrol. A 2002 Falcon is reported to use 12 L of petrol per 100 km.

(a) Which is the more economical?

(b) Develop a formula to convert mpg to L/100 km.

(c) In 1963, the standard Falcon engine had a capacity of 170 cubic inches, whereas the 2002 Falcon has a 4.5 litre engine. Which is the bigger?

couldn't understand exponents and wouldn't buy a book with them in. So, the speed of light appears as three hundred million metres per second.The time after the 'Big Bang' that it took for electrons to be created was a thousand billion billion billion billion billionths of a second. You probably know of a simpler way of expressing these values.

A shorthand means of expressing such numbers is called exponential notation. For example:

- 1 million (1 000 000) is written as 10^6.
- 1 billion (1 000 000 000) is written as 10^9.
- 1 millionth (1/1 000 000 or 0.000 001) is written as 10^{-6}.
- 1 billionth (1/1 000 000 000 or 0.000 000 001) is written as 10^{-9}.

Exponents tell us how many times 10 must be multiplied together and hence give the number of zeros. The expression 10^3 means 10 multiplied by itself three times ($10 \times 10 \times 10$); in other words, 1 with three zeros following it (1000).

When writing numbers using exponents, it is common practice to use scientific notation. This involves the following conventions:

- Write numbers in exponential notation with just one numeral before the decimal point, that is, the Earth–Moon distance of 382 million kilometres could be expressed as 382×10^6 km or in scientific notation as 3.82×10^8 km.
- Leave numbers between 0.1 and 100 as they are. There is no need to express 60 seconds as 6.0×10^1 s although you should be guided by your teacher on this matter.

Example

Write the following in scientific notation:

(a) The speed of light — three hundred million metres per second.
(b) The diameter of a red blood cell — 2 millionths of a metre (0.000 002 m).

Solution

(a) Three hundred million is 300×10^6 so the speed of light can be written as 3.00×10^8 m s^{-1}.
(b) 0.000 002 is written as 2.0×10^{-6}.

As you can see, with scientific notation only one numeral appears before the decimal place. The exponent has to be adjusted to allow for this. For example, when the number 300×10^6 became 3.00×10^8, the decimal point in 300 was shifted two places to the left (made smaller) to become 3.00, so to compensate, the exponent has to be increased by two units from 10^6 to 10^8 (made bigger).

Negative exponents are used to indicate numbers less than unity. For example, an electron has a mass of 0.000 549 units. To make this 5.49, we have to shift the decimal point four places to the right (made bigger by 10 000), so an exponent has to be included that compensates for this. In scientific notation it would be 5.49×10^{-4} units.

Further examples

(a) The radius of the Earth is 696 million metres or 6.96×10^8 m.
(b) The diameter of Saturn is 120 thousand kilometres or 1.20×10^5 km.
(c) The diameter of an atom is 0.000 000 000 1 m or 1.0×10^{-10} m.

Care must be taken when entering numbers in exponential notation in a calculator. On a 'scientific' calculator, to enter the number 6.96×10^8, press the buttons 6.96 EXP 8. The display should read 6.96^{08}. A common error is to enter this as 6.96×10 EXP 8. This is wrong. The display would read 6.96^{07}, which means 6.96×10^7. To enter an exponent such as 10^4 by itself you have to imagine that it means 1×10^4 and enter it as such. Remember, the EXP button symbolises the base, which is 10. It is one of the most common mistakes students make and a sure cause of lost marks in tests. When entering negative exponents, the +/– button is pressed after the exponent. A graphing calculator is different, but the problem is the same.

In some computer languages, exponents can be written in a different form. A number such as 6.96×10^7 would be written as 6.96E7 where the E stands for 'exponent'. With an exponent of 10^{-7} this would be written as 6.96E–7.

Questions

5 **(a)** Which is the larger out of **(i)** 'one hundred thousandths of a second' and **(ii)** 'one one hundred thousandth of a second'? **(b)** How can you make it clear whether you are talking about $\frac{100}{1000}$ s or $\frac{1}{100\,000}$ s when you are expressing these numbers in words? **(c)** Write both numbers in scientific notation.

6 Write the following in scientific notation: **(a)** 0.000 552; **(b)** 73 000 000; **(c)** one and a half million; **(d)** 0.000 250.

7 Work out the following on your calculator: **(a)** $1.2 \times 10^{-3} \times 2.2 \times 10^{-4}$; **(b)** $1.8 \times 10^{3} \div (6.4 \times 10^{-8})$.

8 Calculate the volume of an atom of diameter 0.000 000 001 m.

1.4 SIGNIFICANT FIGURES

When you say that it's 100 metres to the shop you are not really saying that this is the distance to the nearest metre. You are being approximate. You have not measured it — it could be 80 m or it could be 150 m. But the distance between the start and finish of a 100 m sprint race has to be 100 m and this has to be to the nearest centimetre. How would you write these distances? They are both 100 m.

It is common practice in science to record all integers that are certain and one more in which there is some uncertainty. The integers known with certainty plus the next figure are called **significant figures** (sf). Imagine you used a metre ruler marked in centimetres and measured the width of a book as 30.4 cm. This number has three significant figures. The first two integers are measured with certainty whereas the third is a mental estimate. The number could also be written as 0.304 m. It still has three significant figures — the first zero is only there to emphasise the location of the decimal point. Imagine you used the same ruler and measured the thickness of a book to be 6.3 cm. There are two significant figures — the .3 cm part is only a best guess, a mental estimate. In metres, this would be written as 0.063 m. There are still only two significant figures — the two zeros only indicate the position of the decimal point and are not significant.

Consider a ruler marked in millimetre divisions as your own ruler probably is. If you drew a line of length 10 mm, you would be drawing a line somewhere between about 9.5 mm and 10.5 mm in length — you probably can't be more accurate than that. In this case, the 1 is significant whereas the 0 is the next uncertain digit. There are two significant figures. But should it be written as 0.01 m or 0.010 m? You should write it as 0.010 m to show that the zero following the 1 is significant but the first two zeros only indicate the decimal point.

If exactly 35 000 tickets to a football grand final were sold then there are five significant figures. This is an exact figure, accurate to the last ticket, so all zeros are significant. In scientific notation it would be written as 3.5000×10^4 tickets. If a crowd commentator estimated the crowd size as 'thirty-five thousand' then the figure is probably an estimate to the nearest thousand. It might be written in the paper as 35 000 but there are only two significant figures — the three zeros are not significant but are there to indicate where the decimal point is located. In scientific notation this would be written as 3.5×10^4 people. Sometimes significant zeros are indicated with a small bar above the numeral. The exact figure of $35\,00\bar{0}$ people has a bar above the final zero whereas the commentator's estimate doesn't. If the crowd was estimated to the nearest hundred it could be written as $35\,\bar{0}00$, which indicates three significant figures. In scientific notation this would be written as 3.50×10^4. This is a better way to specify significant figures.

NOVEL CHALLENGE

Famous biologist Charles Darwin described the size of a canary finch in one of his notebooks as $3\frac{32}{64}$ inches long.

Why didn't he just write $3\frac{1}{2}$ inches? Convert the original measurement to centimetres using the correct number of significant figures.

NOVEL CHALLENGE

In the English translation of a manual on violin playing by the great Hungarian–German teacher Carl Flesch, budding violinists were told to 'lift your fingers 0.3937 inches from the fingerboard'.

Why is this funny? What do you suppose the original measurement was? Rewrite the inches measurement with the correct number of significant figures.

NOVEL CHALLENGE

In a shop in North Walsham, Norfolk, the height restriction to its carpark is written as 2300 mm.

Is there anything wrong with this? Explain!

Rules

- All non-zero figures are significant: 3.18 has three sf.
- All zeros between non-zeros are significant: 30.08 has four sf.
- Zeros to the right of a non-zero figure but to the left of the decimal point are not significant (unless specified with a bar): 109 000 has three sf.

NOVEL CHALLENGE

At a dinosaur exhibit at the Queensland Museum, the attendant said the Muttaburrasaurus was 30 million and 20 years old. 'How can you be that accurate?' asked a student. 'Well I was told it was 30 million years old when I started work here and I've been here 20 years.'
How would you explain to the attendant the folly of his statement?

NOVEL CHALLENGE

The statement '19 is about 20' is reasonable.
Why then can't you say 20 is almost 19? Explain.

NOVEL CHALLENGE

The rate at which hydrogen is consumed on the Sun is proportional to the temperature (in kelvins) raised to the power of 20 (rate $\propto T^{20}$). How much faster is the rate at 6000 K than it would be at 5000 K?

NOVEL CHALLENGE

On the side of 'Tesco' lemonade cans it reads: 8 kJ per 100 mL, 17 kJ per 200 mL.
How can this be?

- Zeros to the right of a decimal point but to the left of a non-zero figure are not significant: in 0.050, only the last zero is significant; the first zero merely calls attention to the decimal point.
- Zeros to the right of the decimal point and following a non-zero figure are significant: 304.50 has five sf.

Some examples of the application of these rules are given in Table 1.8.

Table 1.8 EXAMPLES

NUMBER	NUMBER OF SIGNIFICANT FIGURES	SCIENTIFIC NOTATION
0.003 5	2	3.5×10^{-3}
0.003 50	3	3.50×10^{-3}
0.35	2	3.5×10^{-1}
3.5	2	$3.5\ (\times 10^{0})$
3.50	3	$3.50\ (\times 10^{0})$
35	2	3.5×10^{1}
350	2	3.5×10^{2}
3500.003 5	8	$3.500\ 003\ 5 \times 10^{3}$

Note: normally, numbers between 0.1 and 100 are not written in exponential form but are shown here for clarity.

Multiplying and dividing

A problem arises when performing calculations using significant figures. Imagine you had to calculate the surface area of a road going through a sensitive koala habitat. The traffic engineers said the road easement would be 95.5 m wide and 26 km long. When multiplying $95.5 \times 26\ 000$, the answer of 2 483 000 must show the correct number of significant figures. The rule is: when multiplying or dividing, the answer should contain only as many significant figures as that number involved in the operation that has the least number of significant figures. In this case, 95.5 m has three significant figures and 26 000 m has two. The answer should only have two, so it should be written as 2 500 000 m^2 or 2.5×10^6 m^2. That's a lot of bush.

Other examples are:

- $45.71 \times 34.1 = 1558.711$. This is rounded to 1560 or 1.56×10^3, which has three significant figures (3 sf).
- $365 \div 2.4 = 152.083\ 333\ 3$. This is rounded to 150 or 1.5×10^2 (2 sf).

Rounding-off If you need to round-off you can use this rule: numerals lower than 5 round-off to zero; numbers larger than 5 round-off to 10; when the number to be rounded off is 5 take it up to 10 if the number preceding is even, otherwise take it down to zero. For example: when 16.586 is rounded off to four significant figures it becomes 16.59. When 24.65 is rounded to three significant figures it becomes 24.7 as the 6 is even and hence the 5 is rounded up to 10.

Addition and subtraction

If a 1575 g target is struck with a 2.55 g bullet, which becomes embedded in it, the mass of the target is now 1575 g + 2.55 g = 1577.55 g. Or is it? The final mass has more significant figures than either the target's mass or the bullet's mass. Intuitively this should sound wrong. The final mass should be written as 1578 g. The rule is: calculations are rounded off to the least significant decimal place value in the data.

Examples

(a) 264.68 − 2.4711 = 262.2089 = 262.21.
(b) 2.345 + 3.56 = 5.905 = 5.90.

Questions

9 State the number of significant figures in each of the following: **(a)** 83.83; **(b)** 20.0; **(c)** 5; **(d)** 22 050; **(e)** 100; **(f)** 100.010; **(g)** 1999; **(h)** 2.222 2; **(i)** 40 000; **(j)** 0.050 70; **(k)** 0.000 000 200.

10 For the numbers in Question 9 above, write them out in scientific notation and use the correct number of significant figures.

11 How many significant figures are there in the following: **(a)** 4.6×10^3; **(b)** 1.00×10^5; **(c)** 6.07×10^{-6}; **(d)** 3.300×10^{-10}?

12 Calculate the following and express in scientific notation to the correct number of significant figures: **(a)** 12.3 m × 34.14 m; **(b)** 3.5×10^2 m × 2.18×10^4 m; **(c)** 180 cm ÷ 2.5 s; **(d)** 1.18 cm × 3.1416 cm; **(e)** 2.0×10^{-3} m × 2.0×10^{-4} m.

13 Work out the following: **(a)** 5.2 m + 16.013 m + 24.37 m; **(b)** 2.125 m + 11.473 2 m + 9.0124 m; **(c)** 3.0×10^3 m + 3.0×10^4 m; **(d)** 4.0×10^{-3} cm + 5.0×10^{-2} cm; **(e)** 1.118×10^4 m + 2.34×10^6 m; **(f)** 8.7×10^{-5} m + 3.5×10^{-2} m.

14 Calculate (2.34 kg + 1.118 kg) ÷ (1.05 cm × 22.2 cm × 0.9 cm).

15 A sheet of copper was measured as part of a density experiment. The dimensions were: length 55.5 cm, breadth 2.0 cm, thickness 0.02 cm. Calculate **(a)** the area of the largest surface; **(b)** the volume; **(c)** the perimeter of the largest face.

1.5 ORDER OF MAGNITUDE

When dealing with very large or very small numbers we are often only interested in an approximate figure. For example, the remotest object known is the quasar RDJ030117 located at a distance of 2.8×10^{22} km from Earth. It is just as meaningful to say it is 10^{22} km away. This is said to be its **order of magnitude** (OM). Similarly, the mass of a hydrogen atom is 1.67×10^{-27} kg, so its order of magnitude is 10^{-27}. The order of magnitude is the power of 10 closest to the number. However, when converting a number to its nearest 10, the rule is: numerals greater than 3.16 become 10 and those below 3.16 become zero. The reason for this is that $10^{0.5} = 3.16$.

Table 1.9 ORDER OF MAGNITUDE

MEASUREMENT	DIMENSION	ORDER OF MAGNITUDE
Distance to Andromeda galaxy	1.9×10^{22} m	10^{22} m
Distance to nearest star	4.0×10^{16} m	10^{17} m
Diameter of Earth	1.3×10^7 m	10^7 m
Thickness of a credit card	5.0×10^{-4} m	10^{-3} m
Thickness of a hair	2.8×10^{-5} m	10^{-5} m

Calculations When estimating the order of magnitude of a mathematical calculation, it is convenient to convert each number to its order of magnitude first.

Example

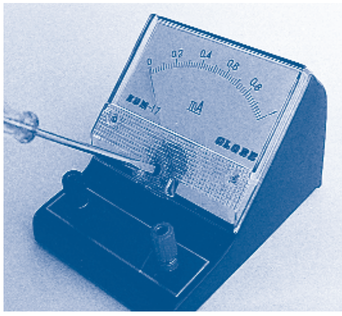
Determine the order of magnitude of this calculation: $(3.0 \times 10^{10}) \times (8.4 \times 10^6)$.

Solution

- 3.0×10^{10} has an OM of 10^{10}; 8.4×10^6 has an OM of 10^7.
- $10^{10} \times 10^7$ equals 10^{17}.

Note: the full answer is 2.52×10^{17}, which does have an OM of 10^{17}.

Photo 1.2
Correcting zero error on an ammeter.



Questions

16 What is the order of magnitude of each of the following: **(a)** 1.8×10^{22}; **(b)** 3.9×10^{12}; **(c)** 2.6×10^{-10}; **(d)** 5.8×10^{-15}; **(e)** 175 000; **(f)** 66 000; **(g)** 0.000 002; **(h)** 0.000 65?

17 Estimate the order of magnitude of the answer for each of the following calculations: **(a)** $(6.2 \times 10^{20}) \times (3.8 \times 10^{-18})$; **(b)** $(600) \times (10 \times 10^{8})$; **(c)** $5.4 \times 10^{-12} \div 3.1 \times 10^{-15}$.

MAKING AND RECORDING MEASUREMENTS 1.6

If you had to count the number of desks in your classroom you would get an exact figure but if you had to measure the width of a desk with a metre ruler your measurement would be an approximation, probably to the nearest millimetre. Measurements, unlike numbers, can never be exact because they all have some amount of error or uncertainty.

You can end up with errors in a measurement because of the limitations of the measuring instrument or the conditions under which it was made. Such errors are not mistakes because they are not someone's fault. Some examples of errors include:

- zero error, for example the pointer or the end of a ruler not on the zero mark to start with (See Figure 1.5.)
- calibration error, for example a stopwatch that runs fast or slow, a thermometer badly graduated, or a metal ruler that has expanded in the heat
- parallax error, for example reading a clock at an angle so that the hand appears to be over another number, reading a thermometer at an angle
- reaction time, for example the delay in starting a stopwatch.

Figure 1.5
Zero error. This voltmeter has a zero error of 0.4 volt. It can be zeroed by adjusting it with a screwdriver (see Photo 1.2).

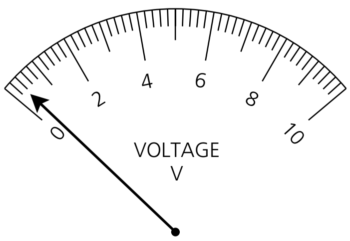


Figure 1.6
(a) A parallax error will occur because there is a gap between the scale and the object being measured. **(b)** There is no parallax error as the scale and object are touching.

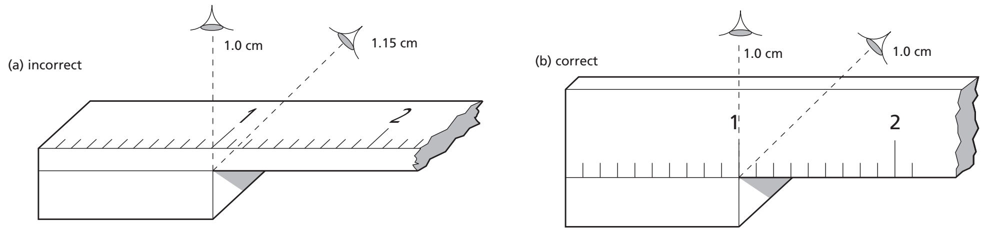
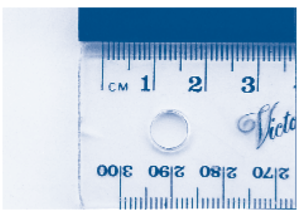


Photo 1.4
If you did this you would have a zero error of 4 mm.



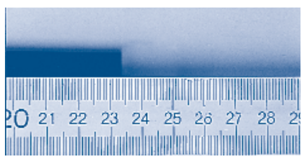


Photo 1.3a
From directly above the reading is 23.35 mm.

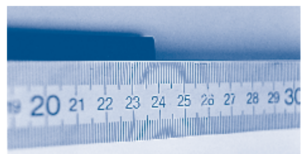


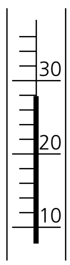
Photo 1.3b
From an angle there is parallax error and the reading is now 23.85 mm.

These errors can be classified into two main types:

- **systematic errors** in which all of the readings are faulty in one direction and can be usually corrected for by a simple calculation or improved experimental technique (Zero errors and calibration errors are of this type.)
- **random errors**, which are irregular errors of observation. Parallax error is an example.

Mistakes are not errors in this context. If you misread a scale (Figure 1.7) by miscalculating the value of each division, this is sometimes called a 'scale reading error' but is really just a mistake.

Figure 1.7
Scale division error on a thermometer. The reading on this thermometer is 28°C not 24°C. Each scale division is 2°C not 1°C as may be thought.



Activity 1.4 PARALLAX ERROR

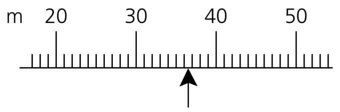
Hold your arm outstretched in front of you with your thumb pointing up. With one eye closed, line your thumb up with some mark on a wall in front of you. Close that eye and open the other and note how many centimetres your thumb has shifted to the side of the mark. Which eye was the more dominant? What are some ways of controlling parallax error?

Scale reading limitations

Students generally read scales to the nearest mark or division. For example, the reading on the ruler shown in Figure 1.8 would generally be stated as 36 mm but it really looks closer to 36.5 mm than to 36.0 mm. A better reading would be 36.5 mm.

Some people would claim to be able to read to the nearest 0.1 mm but this seems overly accurate for the type of scale used. A good rule is that scales should be read to the nearest half of a scale division. Rulers can be read to the nearest half-millimetre and laboratory thermometers to the nearest 0.5°C. An ammeter like the one shown in Figure 1.9 is best read to the nearest 0.05 A.

Figure 1.8
This ruler can be read to the nearest 0.5 mm.



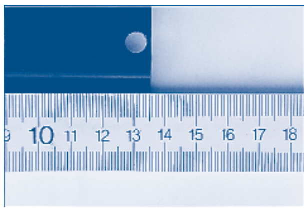


Photo 1.5a
A ruler calibrated in 1 mm divisions can be read to the nearest 0.5 mm. In this case the reading is 135.5 mm.

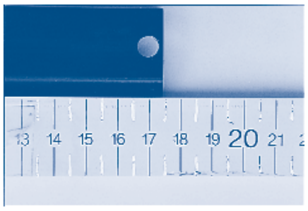
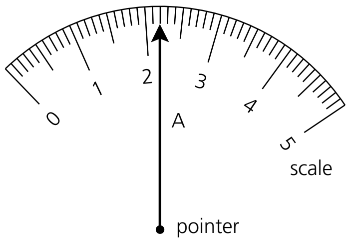


Photo 1.5b
If the ruler was calibrated in 1 cm divisions, then you could read to the nearest 0.5 cm —in this case 17.5 (175 mm).

Figure 1.9
The ammeter scale has 0.1 A divisions, so it can be read to the nearest 0.05 A (half of 0.1).



Uncertainty

You can't measure a physical quantity exactly because all instruments have limitations. These limitations make any reading uncertain. However, some digital instruments appear to give more exact measurements than the manufacturers ever intended. For example, an ammeter with a display of 258 mA seems to be indicating that the current is exactly 258 mA, whereas it may really mean 258 ± 1 mA.

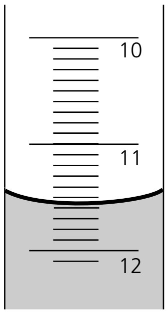
A general rule-of-thumb is that the **uncertainty** in a reading is said to be equal to a half scale division on the instrument. For a ruler marked in millimetres, the absolute uncertainty is ± 0.5 mm so the reading above could have been stated as 36.5 ± 0.5 mm. This absolute uncertainty could be also expressed as a percentage uncertainty:

$$\text{Percentage uncertainty} = \frac{\text{absolute uncertainty}}{\text{observed measurement}} \times 100\%$$

$$= \frac{0.5}{36.5} \times 100\% = 1\%.$$

The uncertainty is a way of expressing how confident you are about the readings provided by the instrument. It is a measure of the limitations of the instrument.

Figure 1.10
This burette shows a reading of 11.55 mL. One half-scale division equals 0.05 mL.



Uncertainty calculations

To add, subtract, multiply or divide numbers, the absolute and relative uncertainties may be required.

- For addition and subtraction, *add absolute uncertainties*.
- For multiplication and division, *add percentage uncertainties*.

Example 1

A container of water rises in temperature from 25.5 ± 0.5°C to 36.0 ± 0.5°C. Calculate the rise in temperature and its percentage uncertainty.

Solution

$$36.0 \pm 0.5°\text{C} - 25.5 \pm 0.5°\text{C} = 10.5 \pm 1.0°\text{C}$$
$$= \frac{1.0}{10.5} \times 100\% = 9.5\%.$$

Example 2

A piece of paper is measured and found to be 5.63 ± 0.05 mm wide and 64.2 ± 0.5 mm long. What is the area of the piece of paper?

Solution

Area = length × width
= (5.63 ± 0.05 mm) × (64.2 ± 0.5 mm)
= 5.63 ± 0.89% × (64.2 ± 0.78%) (convert to percentage uncertainty)
= 361.446 ± 1.67% (add percentage uncertainties)
= 361.446 ± 6.025 mm^2 (convert percentage uncertainty to absolute uncertainty)
= 361 ± 6 mm^2
(Round answer to three significant figures and round the uncertainty to one significant figure as given in the original data.)

Questions

18 A cube of brass was measured and found to have a side of length 13.0 ± 0.5 mm. Determine the volume of the cube.

19 A student made two measurements using a metre ruler calibrated in millimetres.
First measurement = 25.5 mm.
Second measurement = 174.5 mm.
(a) What are the absolute uncertainties for these measurements?
(b) Convert these absolute uncertainties to relative uncertainties.
(c) Add the two measurements and show the absolute uncertainty of the result.
(d) Multiply the two measurements and show the absolute uncertainty of the result.

20 Determine the correct value for the area of a horse paddock 645 ± 5 m long and 345 ± 5 m wide. What is the total length of fencing needed to fence this paddock?

Accuracy and precision

Students often find that despite performing an experiment as accurately as possible and reading the instruments as best as they are able, their results are different from the accepted or textbook result. This difference is called the **error**. The error is a measure of the **accuracy** of a result. Accuracy refers to the closeness of a measurement to the accepted value.

Imagine your group measured the density of water to be 1.02 g/mL when the accepted value was given as 1.00 g/mL at that temperature. Your (absolute) error would be 0.02 g/mL.

Hence:

- **The absolute error** (E_a) = | observed value – accepted value | = |O – A|.

Note: the straight lines (|) in the above equation mean the 'absolute value', that is, the sign (+/–) of the answer is ignored.

- The **relative error** (E_r) is the absolute error expressed as a percentage of the accepted value (A):

$$\text{Relative error } (E_r) = \frac{E_a}{A} \times 100\%$$

In the above example E_r, would equal $\frac{0.02}{1.00} \times 100\% = 2\%$.

This is necessary so that accuracy between different experiments can be compared. Imagine that a student measured the density of lead as 11.29 g/cm^3, while the accepted value was 11.41 g/cm^3. Which result is the more accurate — the density of water or the density of lead? In this case you need to compare relative errors: the error for water was 2% whereas that for lead was 1% and hence was more accurately measured.

Example 1

Calculate **(a)** the absolute error and **(b)** the relative error in a student's measurement of the acceleration due to gravity. They obtained 9.73 m/s^2 whereas the accepted value at their location was 9.813 m/s^2.

Solution

(a) E_a = |O – A|
= 9.73 – 9.813
= 0.08 m/s^2 (to the correct number of significant figures).

(b) $E_r = \frac{E_a}{A} \times 100\%$
$= \frac{0.08}{9.813} \times 100\%$
= 0.8%.

Example 2

When lower profile tyres are fitted to a car in place of the factory fitted ones, a speedometer reading error can occur as the new tyres have a smaller diameter. A table was compiled by a motor magazine during a road test (Table 1.10).

Table 1.10

	SPEED (km/h)			
Observed	60	80	100	110
Actual	57.0	76.0	95.0	104.5

Calculate the relative error at 80 km/h.

Solution

$$\text{Relative error} = \frac{E_a}{A} \times 100\% = \frac{4}{76} \times 100 = 5.2\%$$

Activity 1.5 ERROR IN ESTIMATING PI (π)

Figure 1.11
Experiment to calculate errors in estimating pi.

PHYSICS FACT

In 1783 William Shanks reported a value of pi to 707 places, beating the previous value by 200 places. In 1949 a computer was used for the first time to calculate pi mechanically and they found that Shanks made a mistake at the 528th digit and was wrong from then on. Shanks took 15 years to make his calculation—and he was wrong. What a waste!

1. Draw a line on a piece of paper and place a starting mark at one end (Figure 1.11).
2. Make a mark on the side of a 20 cent coin at the edge.
3. Line up the two marks and roll the 20 cent coin along the line until the mark on the coin touches the paper again, and then put a finish mark.
4. Measure the diameter, d, of the coin with whatever instrument you choose. Measure the length of the line between the start and finish marks. This is the circumference, c, of the coin.
5. Calculate π by using the formula $c = \pi d$ (i.e. $2\pi r$).
6. Knowing that $\pi = 3.141\ 59$, calculate the absolute and relative errors in your estimate of pi.

Summing up:

- *Uncertainty* is a measure of how confidently you can state a measurement or result and is a direct result of the limitations of an instrument. The terms absolute uncertainty and relative uncertainty are used.
- *Accuracy* is a measure of how close a measurement is to an accepted value. The terms absolute error and relative error are used.

Questions

21 Convert the following percentage errors back to absolute errors: **(a)** 27.6 ± 1.5%; **(b)** 10.35 ± 0.6%.

22 Calculate the relative error for the following speeds (as shown in Table 1.10): **(a)** 60 km/h; **(b)** 100 km/h; **(c)** 110 km/h. Does the speedo become more inaccurate at higher speeds?

23 A carbon resistor of nominal resistance 330 ohms is manufactured to a tolerance of 5%. This is, in effect, the maximum relative error. Calculate the range of resistance that this resistor could be.

MEASURING INSTRUMENTS 1.7

Just as units of measurement changed as people's needs changed, so too did the instruments they used for measuring things. Ancient societies achieved incredible accuracy with their primitive devices — rods, string and even line-of-sight. But as precision engineering became vital to industrial society, instruments were developed to achieve such precision.

In this section we will look at the:

- micrometer screw gauge
- vernier calliper
- stroboscope
- digital counter.

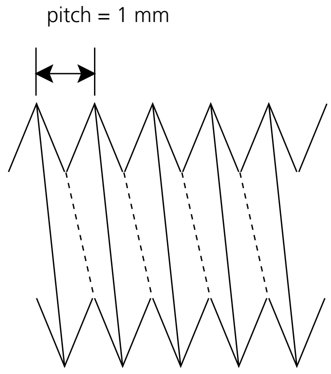
The micrometer screw gauge

To measure really tiny things a **micrometer** can be used. It can measure down to about one-hundredth of a millimetre. The principle behind the micrometer is the screw — one rotation of the screw moves it through a distance equal to the pitch (the distance from one thread to the next) as shown in Figure 1.12. If the screw is rotated only a fraction of a turn, then the screw advances that fraction of the pitch.

A common type of laboratory micrometer has a main scale marked off in half-millimetre divisions. One revolution of the thimble moves the main shaft 0.5 mm. The thimble itself is divided into 50 divisions so that 1 mm equals 100 thimble scale divisions. Hence 1 thimble scale division = 1/100 mm or 0.01 mm. The micrometer in Figure 1.13 shows a reading of 6.5 mm on the main scale and 27×0.01 (= 0.27 mm) on the thimble scale. The final reading is thus 6.77 mm.

There are many types of micrometers available. Your school's could be quite different from the one described here.

Figure 1.12
The pitch of this micrometer is 1 mm.



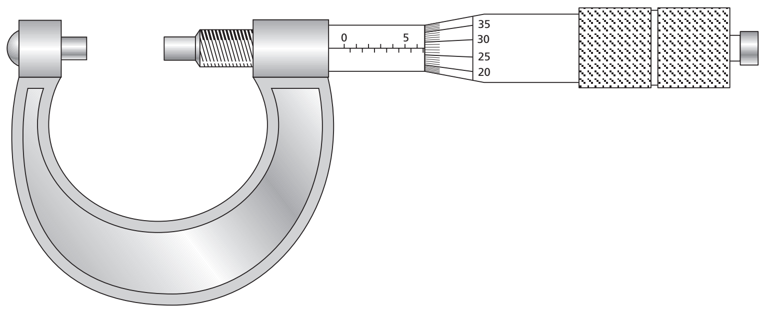


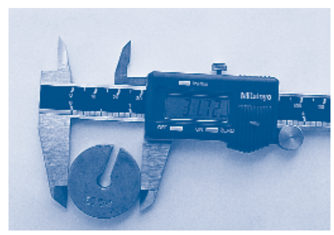
Figure 1.13
This micrometer reads 6.5 mm on the main scale and 0.27 mm on the barrel, making a total of 6.77 mm.

SR Activity 1.6 THE VERNIER CALIPER

Try the following as a good stimulus response task.

The vernier caliper has two jaws that slide together over the object being measured. The caliper was named after the French mathematician Pierre Vernier, who devised the scale. It uses an auxiliary scale (the vernier scale) in conjunction with a main scale to assist in estimating fractions of a main scale division. The main scale is graduated in millimetres (called main scale divisions or MSD) and each centimetre is numbered. The vernier scale is 9 mm long and yet is divided into ten equal divisions (called vernier scale divisions or VSD). It can be shown that the smallest possible division on the vernier scale is one-tenth of 1 mm = 0.1 mm. The procedure is: count the number of complete main scale divisions (MSD) up to the zero line on the vernier scale. Count the number of vernier scale divisions (VSD) to the point where a vernier scale mark and a main scale mark coincide. This will be in 0.1 mm units. For example, in Figure 1.14, the object is 11 mm long plus 5×0.1 mm, which equals 11.5 mm or 1.15 cm.

Photo 1.6
Digital vernier calipers are now becoming more commonplace especially as their price has come down to about $100.



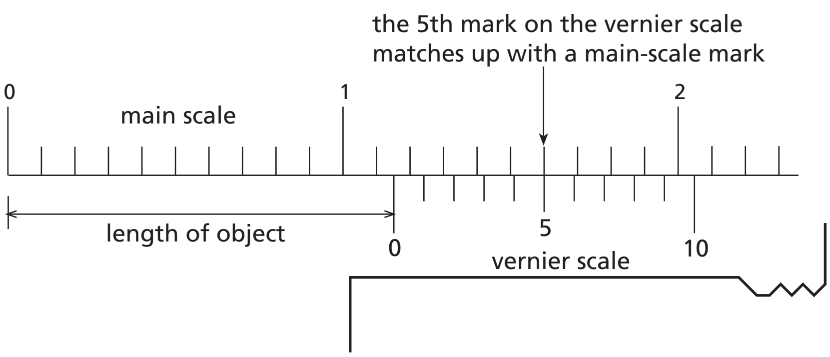


Figure 1.14
This vernier calliper reads (1.1 + 5 × 0.01) cm, i.e. 1.15 cm.

Question

24 What is the reading on the vernier calipers shown in Figure 1.15?

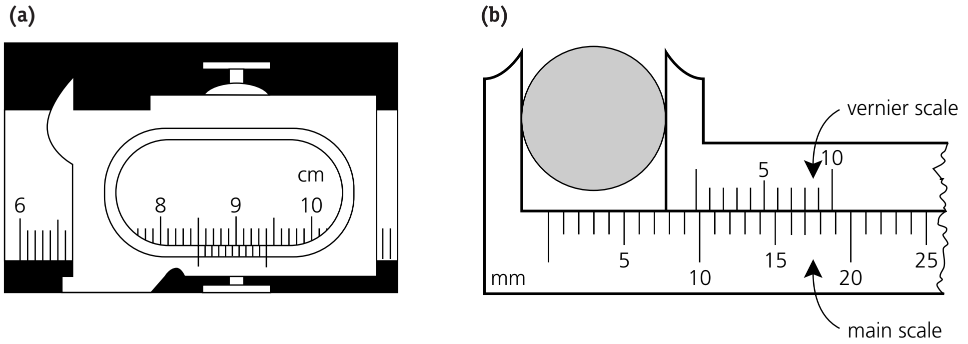


Figure 1.15
For question 24.

The stroboscope

The **stroboscope** owes its name to the Greek *strobos* meaning 'to whirl around' and *skopion* meaning 'see'. The most common form consists of a xenon flash tube similar to that found in a camera flash (Photo 1.8). It can be made to flash at a variable rate from about 1 per second to tens of thousands per second. If a rotating object is in fairly dim conditions and the light flashes when the object is in the same position every time then the object will appear stationary. However, you wouldn't know if the object rotated two or three or a hundred times between flashes so you have to make sure by starting at the lowest strobe frequency and gradually increasing it until motion 'freezes'.

One problem with strobe illumination is that by freezing a rotating object (e.g. a fan blade or a part of a lathe) onlookers may be confused into thinking it is stationary and this would of course be very dangerous. In factories, special precautions are taken with machinery that is illuminated by fluorescent lights. Fluorescent lights flicker at 100 times per second or 100 hertz — once for each crest and trough of an alternating current. Machinery operating at multiple frequencies of this could appear stationary. Such lights have different capacitors added to make them flicker out of synchronisation, which breaks up the strobe effect.

The digital counter

The term '**digital**' conjures up images of modern high-technology but in reality it just means counting in units. This could be like counting 'yes' and 'no' votes in an election; like 'present' and 'not present' when marking a class roll; 'off' and 'on' for an electrical switch or 'light' and 'dark' as cans of soft drink pass a light sensor on a packaging line.

Photo 1.7
A mechanic using a timing light.
(J.A.T. Mechanical, Brisbane)

Photo 1.8
A xenon stroboscope.

1.8 A FINAL NOTE

There is one final caution about measurement and measuring instruments that applies to all devices mentioned throughout this book. An *ideal* measuring device will have no effect on the measurement itself. For instance, when you measure the width of your desk, the desk is unaffected by the measurement. But this is not the case for all measuring devices. When you measure the pressure of a car's tyres some gas is sampled and the tyre has less gas than before you started. The loss is insignificant, however. A voltmeter or ammeter samples electrons from an electrical circuit and will affect the voltages and currents being measured. But again, if the meters are used properly the effect will be minimal. Can you think of other instruments that affect the phenomena being measured and how the effect is minimised?

Practice questions

The relative difficulty of these questions is indicated by the number of stars beside each question number: * = low; ** = medium; *** = high.

Review — applying principles and problem solving

***25** The space shuttle orbits the Earth at an altitude of 300 km. How many millimetres is this?

***26** The Earth is approximately a sphere of radius 6.37×10^6 m. **(a)** What is its circumference? **(b)** What is its volume in cubic metres? **(c)** What is its volume in cubic kilometres?

***27** Submarines typically dive at a rate of 36 fathoms per second. If a fathom is 6 feet and 1 foot is 0.305 m, convert this diving speed to metres per second.

***28** Write the following in scientific notation: **(a)** 3 558.76; **(b)** 40.00; **(c)** 79 000; **(d)** 200 326; **(e)** 1994; **(f)** 20.009; **(g)** 0.050 0; **(h)** 2 500 000; **(i)** 0.000 000 8; **(j)** 5 million.

***29** Do the following calculations on your calculator, using the correct number of significant figures: **(a)** $4.2 \times 10^3 \times 8.1 \times 10^4$; **(b)** $3.7 \times 10^7 \times 4.1 \times 10^{-4}$; **(c)** $7.2 \times 10^4 \div 1.8 \times 10^6$; **(d)** $4.8 \times 10^6 \div 1.6 \times 10^{-3}$; **(e)** $\pi (4.1 \times 10^{-6})^2$; **(f)** $2.8 \times 10^3 \div (\frac{4}{3}\pi (4.7 \times 10^{-5})^3)$.

***30** Express each of the following as an order of magnitude: **(a)** 4.28×10^7; **(b)** 32 000 000; **(c)** 1.2×10^5; **(d)** 1.13×10^{-4}; **(e)** 4.5×10^{-8}; **(f)** 9 192 000; **(g)** 0.000 000 38.

***31** How many significant figures are there in each of the following: **(a)** 95.2 km; **(b)** 3.080×10^5 g; **(c)** 0.0067 L; **(d)** 0.000 670 L?

***32** Convert the following to relative errors: **(a)** 2.40 ± 0.02 V; **(b)** 3.25 ± 0.05 A; **(c)** 25.4 ± 0.4 mm; **(d)** 0.0035 ± 0.0001 T; **(e)** 325 ± 10 cm.

****33** A student is required to determine the density of a particular metal. The object is in the shape of a cylinder. She uses a micrometer calibrated in 0.01 mm (i.e. a limit of reading of 0.01 mm) to measure the diameter of the cylinder and uses a vernier calliper with a limit of reading of 0.1 mm to measure the length. Recall that the error associated with a reading is half the limit of reading. The results are shown in Table 1.11.

Table 1.11

READING	DIAMETER (mm)	LENGTH (mm)
1	16.446	28.4
2	16.444	28.3
3	16.442	28.5

(a) What absolute error is associated with each reading?
(b) Determine the average values for length and diameter.
(c) Determine the value for radius and length. Include the correct error.
(d) Determine the volume of the cylinder, including its error.
(e) If the cylinder has a mass of 56.4 ± 0.2 g, determine the density of the cylinder in g mm^{-3}.

***34** Use your ruler and calculate **(a)** the surface area of the front cover of this book; **(b)** the total external surface area; **(c)** the volume of this book; **(d)** the thickness of one page.

Extension — Complex, challenging and novel

****35** What does the prefix 'micro' signify in the words 'microwave oven'? Does it mean it is a small oven? It has been proposed that food that has been irradiated by gamma rays to lengthen its shelf life should be called 'picowaved'. What do you suppose that means?

*****36** Convert the speed of light (3.0×10^8 m s^{-1}) to furlongs per fortnight. A furlong is one-eighth of a mile; there are 5280 feet in a mile and one foot is 0.305 m.

*****37** A wire of length 756.5 ± 0.5 mm has a mass of 8.5 ± 0.5 g. Calculate the mass per millimetre.

*****38** Isaac Asimov proposed a unit of time based on the highest known speed of light and the smallest measurable distance. It is the light-fermi, the time taken by light to travel a distance of 1 fermi (= 1 femtometre = 1 fm = 10^{-15} m). How many light-fermis are there in 1 second? Recall that light travels at 3×10^8 m s^{-1}.

****39** Some of the prefixes of the SI units have crept into everyday language. What is the weekly equivalent of an annual salary of $36 K (= 36 kilodollars)?

*****40** The hard disk of a particular computer was stated as 200 MB (= 200 megabytes). At 8 bytes per word, how many words can it store? Note that in computerese, kilo means 1024 (= 2^{10}) not 1000 and mega means 2^{20} not 1 million.

****41** When the length of a metre was defined in 1983, the speed of light was accepted as 2 997 924 58 ms^{-1}. Why was it not defined as exactly 3.000×10^8 m s^{-1} to make it simpler?

*****42** The following is an extract from *The Times* newspaper, London. Read it and answer the question below.

Time, gentlemen

The Gregorian calendar, which celebrated its quatercentenary in October 1982, is still working well. So well, in fact, that it will be some time in AD 4316 before it is a complete day out.

The trouble is that when God created, he did so without benefit of digital timekeeping, and a year is currently 365.2422 days long. This leaves a rather useless plane-shaving of time at the end of each year. Julius Caesar was without digitals, too, but his astronomer Sosigenes did a remarkably fair job in 46 BC to produce a year of 365 days and six hours — only a week out every 1000 years. This was perfectly adequate for the ancients, who rose and retired by the sun, but not for those pernickety Christians, who became deeply concerned about Easter being on the correct day.

By the time Pope Gregory XIII wrestled with the problem, the Julian calendar was 10 days adrift. So at midnight on October 5, 1582, he declared the next day to be October 15. It brought the vernal equinox in the northern hemisphere back to March 21 and the peasants never felt a thing.

Britain, having long grown wary of such popish tricks, did not deign to accept the Gregorian calendar until 1752. But it was by no means the last country to abandon old Caesar's almanac. Russia did not go Gregorian until 1918, after the Revolution, and the last country of all to abandon it seems to have been Greece, in 1923.

(a) Why did the Christians need a more accurate calendar?

(b) Russian chemist Mendeléev devised the Periodic Table of the Elements on 1 March 1901 in Moscow. What date would this have been in London?

(c) By 1989 the calendar was only out by 2 hours 49 minutes since 1582. How far out will it be in 2005?

*****43** The size of a molecule can be determined by placing a drop of oil on the surface of water and noting the maximum area of the oil slick which is assumed to be one molecule thick. We tried this and found that one drop spread to a circle with a diameter of 14 cm. We also found that there were 20 drops of oil to the milliliter. Calculate the thickness of the slick.

*****44** There are 6×10^{23} molecules of water in 18 mL of water. If the ocean has a volume of 1.3×10^{18} m^3, how many glasses of water (at 250 mL each) are there in the ocean? Comment on the assertion that 'there are more molecules of water in a glass of water than there are glasses of water in the ocean'.

*****45** Neutron stars have a radius of 20 km and a mass equal to our Sun (2×10^{30} kg). What is the mass of a cubic centimeter of neutron star?