CHAPTER 04

Forces in Action

It seems incredible that a man could start railway wagons moving just by pulling on a rope with his teeth. Robert Galstyan of Armenia did just that in 1992: he set a world record by pulling two carriages a distance of 7 m with his teeth. Question: how could a 100 kg man accelerate 220 t of railway carriages from rest? Had he studied Newton's laws of motion?

4.1 SOME WRONG IDEAS ABOUT FORCES

People have often been baffled by other questions about forces:

- After you shoot an arrow does it keep going until the force runs out?
- If it takes a force to keep a thing moving, why doesn't the Moon crash into the Earth?
- Why do racing cars have 'spoilers' to increase wind resistance when really they want to go faster?
- Are there any forces acting on you if you're weightless?
- Cream seems more dense than milk so how come it floats on top of the milk?
- Cork is very lightweight — but could I lift a 1 metre diameter ball of it?
- Can rockets take off faster if they have a concrete launch pad?
- Which weighs more — a tonne of feathers or a tonne of lead?

Every one of these statements is based on a misconception about forces. Many of them go back 2000 years to Aristotle's idea that a moving thing had an internal source of 'impetus', which it was given when first thrown or moved. Such an idea acted as an obstacle to the understanding of motion for 1500 years and it still persists in students and others even today. Other *wrong* ideas are:

- If a body is not moving there is no force on it.
- The speed of an object depends on the amount of force on it.
- When the force stops, motion stops.

It's hard to convince people that these are wrong because they do sound 'right' — they seem to agree with what we see. But two cases should help to clear up misunderstandings.

NOVEL CHALLENGE

The average mass of Sumo wrestlers in 1974 was 126 kg. In 2003 their average mass had risen to 157 kg. If this trend continues, when will they no longer be able to stand up? (The maximum mass that two legs can carry is 180 kg.)

Case 1: Space travel

Objects travelling in space keep going at constant velocity when there is no external force acting on them. The *Voyager* spacecraft left our solar system several years ago and is travelling on long after the jets ran out of fuel. On the other hand, a hockey ball rolls to a stop because frictional forces act on it and slow it down.

Photo 4.1
Voyager spacecraft.

Case 2: Ice skater

An ice skater will continue on at constant velocity until she tries to turn. The turning is a change in direction and hence a change in velocity. She will slow down unless she pushes off again.

Italian scientist **Galileo** (1564–1642) used the same logic to conclude that it is unbalanced forces that cause objects to slow down and stop. We call this force **friction**, a force that resists motion between two surfaces in contact. He took the word from the Latin *frictaire*

Photo 4.2
Spring balances.

meaning 'to rub'. Galileo's ideas were very bold for his time because he was not able to verify them experimentally. He ended up in hot water with the Church when he asserted that other planets were much the same as Earth and revolved around the Sun, whereas the Church taught that the Sun revolved around the Earth.

MEASURING FORCE 4.2

Simply stated, a force is a push or a pull and it is fitting that the unit of force be named after one of the world's greatest physicists, Isaac Newton.

The **newton** (N) is commonly measured in the laboratory with a device called a spring balance. This has a spring that extends when masses are hung on it or when other forces are applied. The scale is calibrated in grams for mass or in newtons for force. Because the direction of the force is important, **force is a vector quantity**.

EI Activity 4.1 FEELING A NEWTON

The size of 1 newton is not familiar to most people. The 'feel' of a newton helps you in your problem solving.

1. Obtain a spring balance calibrated in newtons and check that it reads zero when held vertically. Adjust it if it doesn't. Pull gently to feel forces of 1 N, 2 N, 3 N etc.
2. Hang masses of 100 g, 200 g etc. on the hook to see what force is needed to hold them up.
3. Hold a 100 g mass stationary in your hand. This requires a force of about 1 N.
4. When you sit on a bicycle, what force does your total mass exert on the bike?
5. Use bathroom scales under the front and rear wheel of your bike to see how this force is distributed.

BALANCED AND UNBALANCED FORCES 4.3

To study the effect of forces acting on an object we need to distinguish between **balanced** and **unbalanced** forces. When spring balances are hooked onto either end of a cart and given equal pulls in opposite directions (Figure 4.1), the carts remain at rest because the forces are balanced — they are **equal and opposite**.

Figure 4.1
The forces on the cart are balanced — they are equal and opposite in direction.

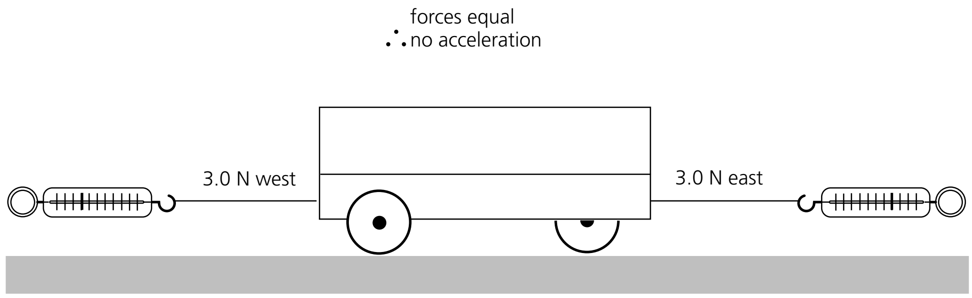
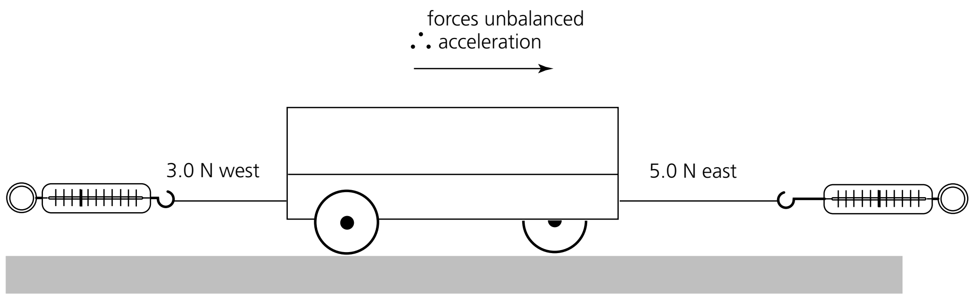


Figure 4.2
The force on the cart is unbalanced — it is greater to the right than to the left.



If the pull on the balance to the right was increased to 5 N (Figure 4.2), then the forces would become unbalanced and the cart would move off in the direction of the larger force.

Example 1

The resultant force acting on the cart in Figure 4.2 can be calculated. It seems obvious but it is important to get the setting-out correct.

Solution

Finding the resultant force is a vector addition thus:

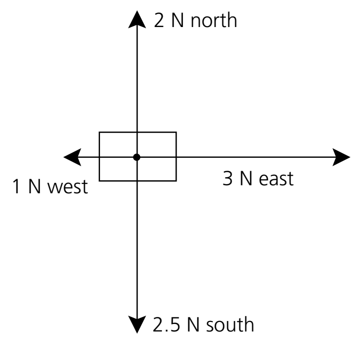
$$\begin{aligned} \boldsymbol{F}_R &= \boldsymbol{F}_1 + \boldsymbol{F}_2 \\ &= 5.0 \text{ N east} + 3.0 \text{ N west} \\ &= 5.0 \text{ N east} + (-3.0 \text{ N east}) \\ &= 2.0 \text{ N east} \end{aligned}$$

If there are several forces acting on an object you should try to reduce them to a simpler case by adding pairs in opposite directions first before combining with forces at angles.

Example 2

Calculate the resultant force acting on the object shown in Figure 4.3.

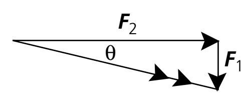
Figure 4.3



Solution

$$\begin{aligned} \boldsymbol{F}_1 &= \boldsymbol{F}_S + \boldsymbol{F}_N \\ &= 2.5 \text{ N south} + (2.0 \text{ N north}) \\ &= 2.5 \text{ N south} + (-2.0 \text{ N south}) \\ &= 0.5 \text{ N south} \\ \boldsymbol{F}_2 &= \boldsymbol{F}_E + \boldsymbol{F}_W \\ &= 3.0 \text{ N east} + (-1.0 \text{ N east}) \\ &= 2.0 \text{ N east} \\ \boldsymbol{F}_R &= 0.5 \text{ N south} + 2.0 \text{ N east (see Figure 4.4)} \\ &= \sqrt{0.5^2 + 2.0^2} \\ &= 2.1 \text{ N} \\ \theta &= 14° \text{ so the direction is E}14°\text{S} \end{aligned}$$

Figure 4.4

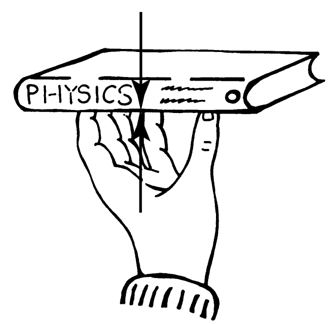


Questions

1 Calculate the resultant force when the following forces act on the same object: **(a)** 2.4 N north, 1.8 N south, 1.9 N north; **(b)** 65 N down, 92 N up and 74 N up; **(c)** 50 N north, 30 N west, 60 N south; **(d)** 26 N west, 20 N east, 30 N north, 15 N south.

2 Figure 4.5 shows a physics book held at rest in a person's hand. Two forces are shown in the diagram. One is the weight of the book pushing down and the other is the force of the hand pushing up.
(a) Are the forces balanced? Explain.
(b) Assume the hand was suddenly removed. Are the forces now balanced? What would you observe?

Figure 4.5
'Are the forces balanced?'



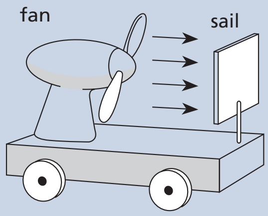
4.4 NEWTON'S FIRST LAW OF MOTION

Sir Isaac Newton was the first scientist to put Galileo's ideas into the form of a universal physical law, that is, one obeyed throughout the universe. In 1688, Newton proposed the first law of motion:

> An object maintains its state of rest or constant velocity motion unless it is acted on by an external unbalanced force.

NOVEL CHALLENGE

A fan blows a cart with a sail attached.



If the fan and the sail are on the same cart, what happens? Explain why.

PHYSICS FACT

At 5.00 pm Houston time on 17 July 1969, the *Apollo 11* spacecraft was 50 000 km from the Moon on its return journey to Earth with its engines off. Its speed was 4740 km/h at 4.30 pm and at 6.00 pm its speed was still the same. With no net force its velocity remained constant.

✔ TEST YOUR UNDERSTANDING

Why does the agitator in a washing machine go back and forth instead of going steadily in one direction? Explain in terms of Newton's first law.

The following four examples show Newton's law applied to real life.

At rest and stays at rest

Some magicians can jerk a tablecloth out from under a dinner setting of glasses and cutlery, leaving them at rest on the table.

In motion and stays in motion

In a head-on car crash, the occupants tend to continue in their state of motion and move forward towards the dashboard. It is usually the seat belts that restrain them.

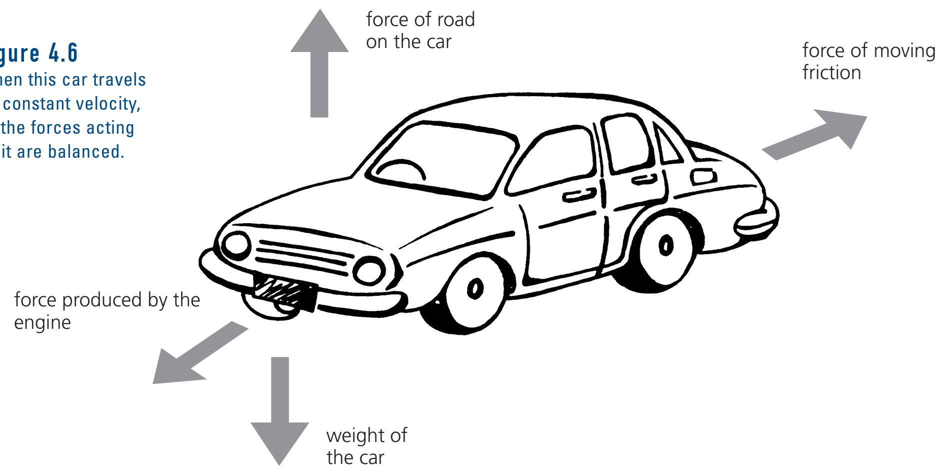
Not wanting to change direction

As a car goes round a corner, your body wants to continue in a straight line so the car door presses against you as it moves sideways. People often say that they get flung against the car door. It is actually the door that gets flung against them.

Balanced forces, constant velocity

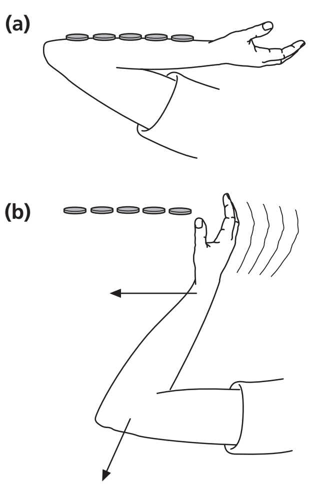
Consider a diagram of the forces acting on a car travelling along a road at constant velocity (Figure 4.6).

Figure 4.6
When this car travels at constant velocity, all the forces acting on it are balanced.



The downward force of the car on the road is balanced by the upward force of the road on the car. The force produced by the engine is balanced by the friction of the tyres on the road and the air resistance. As long as these forces remain balanced the car will not accelerate.

Figure 4.7
Coins stay still long enough for you to catch them.



EI Activity 4.2 INERTIA

Two tricks you can do involve Newton's first law.

1. Place a row of coins along your forearm as shown in Figure 4.7 **(a)**.

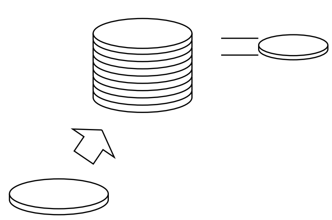
   With practice, as you fling your arm down (Figure 4.7**(b)**), the coins should stay motionless long enough for you to catch them in your hand.

2. Make a pile of 20 cent coins on a smooth surface as shown in Figure 4.8.

   Flick another 20 cent coin towards the stack and with practice you should be able to knock the bottom coin out without disturbing the rest.

Describe how Newton's first law applies in these situations.

Figure 4.8

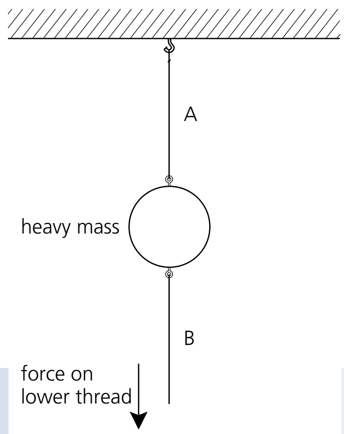


Questions

3 A thread supports a mass hung from the ceiling. Another identical string is tied to the bottom of the mass (Figure 4.9). Which thread is likely to break if the bottom thread is pulled **(a)** slowly; **(b)** quickly? Explain your prediction.

4 Explain how Newton's first law applies in the following cases:
- **(a)** You flick your hands after washing them, before you use a towel.
- **(b)** You spin your wet umbrella to remove excess water before folding it up.
- **(c)** You can't stay upright on a stationary bicycle without putting your feet down but you have no problem while you ride along.
- **(d)** Falling off a building and accelerating is not dangerous but the deceleration bit at the end is.
- **(e)** Boxers get 'punch drunk' after too many blows to their head.

Figure 4.9
For question 3.



4.5 MASS

Everyday experience tells us that a given force will produce different accelerations in different objects. Kick a football and it moves off quickly. Kick a car and it hardly moves. The difference is their **mass**. Obviously, the car has a greater mass than the ball. The word 'mass' was first used in the fourteenth century in this sense. It comes from the Greek *maza* meaning 'barley cake', hence 'lump or mass'. Mass is measured in units of **kilograms** although grams and tonnes are widely used.

What is mass?

Since the word mass is used in everyday language we should have some understanding of it. Is it a body's size, weight or density? The answer is no, none of these, though these characteristics are sometimes confused with mass. The **mass** of a body is a characteristic of its resistance to motion. This is also called its **inertia**. It was astronomer Johannes Kepler who first used the term in physics in the seventeenth century. At the time, in Latin it merely meant 'lack of art' (*in* = not, *ert* = art), 'no skill' or 'idleness'. It was Kepler's wit that saw the term added to our language. Newton's first law of motion may also be called the 'law of inertia'.

Measuring mass

Mass can be defined and measured in two main ways: as inertial mass or as gravitational mass.

Inertial mass is a measure of resistance to motion. If a known force is applied to different objects, then the resultant acceleration is directly related to mass. A 1 kg object will accelerate at twice the rate as a 2 kg object. An 'inertial balance' (Figure 4.10) can be used to measure inertial mass. The object to be measured is placed on the outer section, which is then given a push and allowed to vibrate back and forth. The greater the mass, the slower the rate of vibration. Such a balance is used in the space shuttle to measure astronauts' mass. The laboratory version is sometimes called the 'wig-wag' machine.

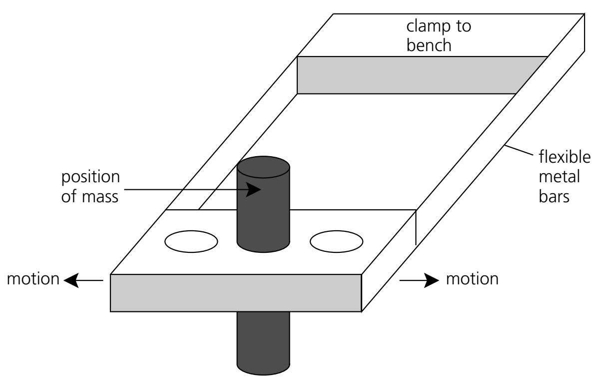


Figure 4.10
The wig-wag machine (inertial balance).

Gravitational mass is a measure of the pull of gravity on an object. A spring balance is often used to measure gravitational mass. It works on the principle that the force of gravity (weight) of an object is proportional to the mass of an object. A beam balance can also be used to compare weights and hence masses of objects (Figure 4.11).

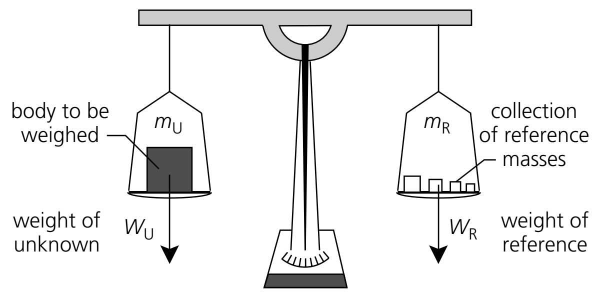


Figure 4.11 An equal-arm balance. When the device is in balance, the masses on the left and the right pans are equal.

NOVEL CHALLENGE

Isaac Newton's mother said that he would fit into a 'quart pot' at birth. If the density of a baby is 1020 kg m^{-3}, calculate his mass.

Gravitational mass and inertial mass are equivalent. (See Chapter 30.)

Density

Which is more dense — milk or cream? Cream is certainly thicker but it floats on the top of milk so it is less dense than milk. You can't go around floating objects on top of each other to compare their densities. This may work for two liquids but what about two solids? A standard definition is needed.

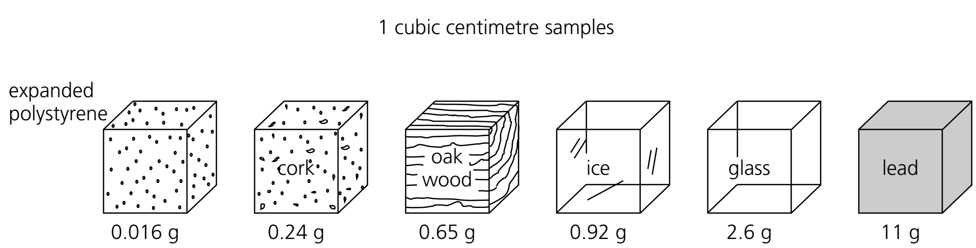


Figure 4.12 Comparing the masses of 1 cubic centimetre of several substances illustrates the different densities.

Density is the mass per unit volume

$$\text{Density} = \frac{\text{mass}}{\text{volume}} \text{ or } D = \frac{m}{V}$$

The units for density will be kg/m^3 or **kg m^{-3}**. Sometimes the unit **g cm^{-3}** is used. In this book we will use both.
Note: the SI symbol for density is the Greek letter 'rho' (ρ) and is preferred to 'D'. It is up to you and your teacher which to use. You may find that in chemistry class the symbol D is used. Density will be dealt with in more detail in the chapter on fluids and buoyancy.

Table 4.1 SOME DENSITIES

MATERIAL	DENSITY (kg m^{-3})	DENSITY (g cm^{-3})
Air (at STP)	1.3	0.0013
Cork	240	0.24
Petrol	800	0.8
Ice	920	0.92
Water	1 000	1.0
Sea water	1 030	1.03
Milk	1 030	1.03
Sand	1 600	1.6
Pyrex	2 230	2.23
Carbon	2 300	2.3
Aluminium	2 700	2.7
Diamond	3 500	3.5
Iron	7 900	7.9
Brass	8 400	8.4
Copper	8 960	8.96
Silver	10 300	10.3
Lead	11 300	11.3
Gold (9 carat)	11 300	11.3
Mercury	13 600	13.6
Pure gold (24 carat)	19 300	19.3

Example
Calculate the density of a cube of copper that has a side of 2.00 cm and a mass of 71.68 g. Give your answer in **(a)** g cm^{-3}; **(b)** kg m^{-3}.

Solution
(a) Volume = $l \times b \times h = (2.00 \text{ cm})^3 = 8.00 \text{ cm}^3$.
Density = mass/volume = 71.68/8.00 = 8.96 g cm^{-3}.
(b) Volume = $(2.00 \times 10^{-2} \text{ m})^3 = 8.00 \times 10^{-6} \text{ m}^3$.
Mass = 71.68×10^{-3} kg.

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{71.68 \times 10^{-3} \text{ kg}}{8.00 \times 10^{-6} \text{ m}^3} = 8960 \text{ kg m}^{-3} \text{ (or } 8.96 \times 10^3 \text{ kg m}^{-3}\text{)}.$$

Questions

5 The density of iron is 7.86 g cm^{-3}. What is the mass of a cube of iron whose side is 2.50 cm long?

6 Could you lift a ball of cork of diameter 1.5 m? Cork has a density of 0.24 g cm^{-3} (240 kg m^{-3}).

4.6 NEWTON'S SECOND LAW

Newton's first law deals with cases where the forces are balanced, so no acceleration occurs. His second law deals with unbalanced forces and hence acceleration will occur. Here are some examples:

- *Dropping a rock over a cliff.* There is no upward force to balance the force of gravity (assuming we neglect air resistance).

NOVEL CHALLENGE

Isaac Newton's early childhood was marked by rejection and hatred. His step-father refused to allow him into the house. He lived a life of long battles with von Liebnitz about who invented calculus. Robert Hooke hated him. As Master of the Royal Mint Newton frequented brothels and bars trying to catch counterfeiters. From 1678 to 1696 he conducted experiments by heating up heavy metals such as lead and mercury and breathing in the vapour (sweet, saltish, vitriolic) so it was no surprise that he developed mental illness by 1693. Samples of his hair had 200 parts per million mercury where 5 ppm is normal and 40 ppm dangerous. He has been ranked as the second most influential person in the world (influential, not important). *Develop an argument for who might be first and third?*

NOVEL CHALLENGE

(a) *If you took a beaker of wet sand from the beach, would it weigh more or less than the same volume of dry sand?*
(b) *Stand on some wet sand at the beach and it goes dry around your feet. Why is this?*

NOVEL CHALLENGE

Some people say that Newton discovered the laws of motion but others say he invented them. *Who is right?*

- *A bullet travelling up a rifle barrel.* The force due to the pressure of hot expanding gases is greater than the friction from the walls of the barrel.
- *Driving away from traffic lights.* The force produced by the engine is greater than the friction of the tyres and air resistance, so a car will accelerate.
  - The heavier the car the more force is needed to accelerate away from the traffic lights.
  - The faster you want to accelerate, the greater the force needed.
  - The acceleration occurs in the direction of the unbalanced force.

Newton's second law summarises these facts:

> The acceleration of an object varies in direct proportion to the external unbalanced force applied to it and inversely proportional to its mass.

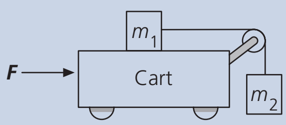
Mathematically: $\boldsymbol{a} \propto \boldsymbol{F}$ and $\boldsymbol{a} \propto \frac{1}{m}$ or $\boldsymbol{a} = \frac{\boldsymbol{F}}{m}$.

This can be rearranged to: $\boldsymbol{F} = m\boldsymbol{a}$.

As stated earlier, the unit of force is the newton (N) so we can define a newton as the force needed to give a 1 kg object an acceleration of 1 m s^{-2}.

NOVEL CHALLENGE

For m[1] and m[2] to remain in the same positions relative to the cart, what force (***F***) has to be applied?



Example

An unbalanced force of 48 N west is applied to a 6.0 kg cart. Calculate the cart's acceleration.

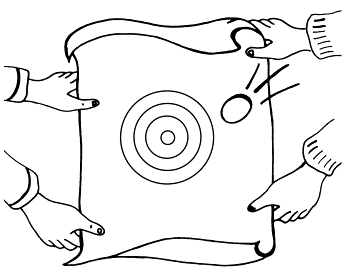
Solution

$$\boldsymbol{a} = \frac{\boldsymbol{F}}{m} = \frac{48 \text{ N west}}{6.0 \text{ kg}} = 8.0 \text{ m s}^{-2} \text{ west}$$

Activity 4.3 THROWING EGGS

Throw some raw eggs full-force at a sheet held as shown in Figure 4.13. Have the bottom curled up to catch the eggs once they have hit the sheet. Explain the result in terms of Newton's second law. If the sheet was wet what do you think would happen? We're not game to try it.

Figure 4.13
A soft landing for an egg stops the shell cracking.



Making your bike go faster

A practical example of Newton's second law involves bicycle racing. In 1986, aerospace titanium–aluminium–vanadium alloys were introduced. These have a density of about 60% of that of steel and so a 300 g steel chain can be replaced by a 130 g one. Hubs can drop by 34 g, and wheels by an amazing 800 g. With over 60 replaceable parts, cyclists can drop about 3 kg of mass from their bikes, which is about 3.5% of total mass of bike plus rider. Tests show that paring just 1 kg from a bicycle can save 2.5 s in a 3 km circuit. That's 16 m at an average speed of 26 km h^{-1}. It also enables them to climb hills at two gears higher than normal and if you consider that the difference between 1st and 10th place is about 0.5% in performance, a 3.5% mass loss can be a winner. The savings are not cheap. A Trek 5500 bicycle with carbon fibre frame (7 kg) and Ti–Al–V alloy throughout is about $4000.

Loss of consciousness

It has been known for a long time that rapid acceleration or deceleration can severely affect the human body. Too high a deceleration can cause loss of consciousness, for example in a sharp loop-the-loop by a jet fighter pilot and crew. Alternatively, it can result in death. Smashing into a power pole can kill a car driver and of course that is why cars have 'crumple zones' to slow the rate of deceleration in an accident.

Questions

7 Calculate the missing quantities in Table 4.2 (do not write in this book).

Table 4.2

	F	*m*	*u*	*v*	*a*	*t*
(a)		1000 kg	rest	25 m s^{-1}		8.5 s
(b)	25 N	15 kg	rest			2.0 s
(c)	1000 N		10 m s^{-1}	40 m s^{-1}	10 m s^{-2}	
(d)		200 g	0.85 m s^{-1}	0.60 m s^{-1}		1.5 min
(e)	150 N			rest	−2.2 m s^{-2}	4 s

NOVEL CHALLENGE

Quick now, could you lift a ball of cork 1.5 m in diameter? Now work out its mass.

8 A car of mass 2000 kg decelerates from 30 m s^{-1} to rest in a distance of 100 m. Calculate the retarding force required to stop the car.

9 Racing driver David Purley (1945–85) survived a deceleration from 173 km/h to zero in a distance of 66 cm in a crash at Silverstone, UK in 1977. He suffered 29 fractures, three dislocations, six heart stoppages and made the *Guinness Book of Records*. Calculate the net horizontal force acting on him in the crash. His body mass at the time was 55 kg.

10 Comment critically on the following claims:

(a) It requires a greater force to accelerate a 2000 kg car from rest to 15 m s^{-1} than from 15 m s^{-1} to 30 m s^{-1} in the same time.

(b) Twice the force is needed to accelerate a 1.5 t car from rest to 60 km^{-1} over 100 m than is required over 200 m.

(c) An object always accelerates in the direction of the net force.

(d) A lower net force is needed to accelerate an object from rest to 10 m s^{-1} than is required to accelerate it from rest to 20 m s^{-1} irrespective of the time taken.

11 In an experiment to find out how the motion of a trolley was related to the force acting on it, a 1.5 kg trolley was accelerated by various forces. The results are summarised below:

Force (N)	0.00	0.10	0.20	0.30	0.40	0.80
Acceleration (m s^{-2})	0.00	0.07	0.13	0.20	0.27	0.53

(a) Plot the data with ***F*** on the *x*-axis.

(b) What relationship is suggested by the data?

12 In an experiment to verify Newton's second law, the equipment shown in Figure 4.14 was set up.

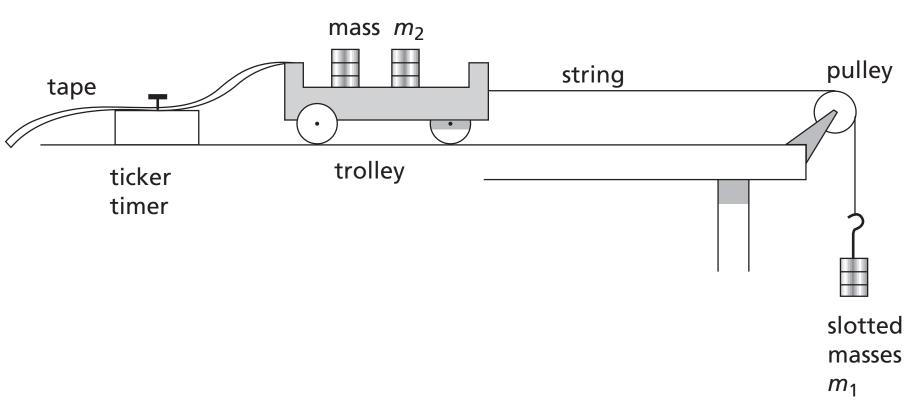


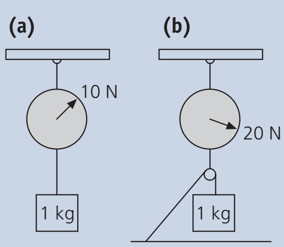
Figure 4.14
As masses are transferred from the trolley to the carrier, the force exerted on the trolley is increased while the mass of the whole system remains constant.

NOVEL CHALLENGE

For most cars, the rear tyres support more weight than the front tyres. For example a Toyota Corolla has 43% of its weight supported by the front tyres and 57% by the rear. When a Corolla brakes, the weight on the front increases to about 69% and reduces to 31% on the rear. Why is this, and why do cars dip when the brakes are applied?

NOVEL CHALLENGE

If a 1 kg mass hanging on a spring balance shows a weight of 10 N, as in diagram **(a)**, will diagram **(b)** be correct? The second diagram shows a 1 kg mass suspended over a pulley by a string tied to the table. Explain.



The mass m_1 hangs vertically and its weight (the force of gravity) is responsible for providing the accelerating force that causes the mass m_2 to move in a horizontal direction. The weight of m_1 is equal to 9.8 N for every 1 kg of hanging mass. As both masses were connected by a light string, the total mass being accelerated by the weight of m_1 is equal to the sum of m_1 and m_2. To keep the total mass of the system constant but to vary the accelerating force (the weight of m_1), the brass masses were shifted from the trolley to the hanger. This made m_1 heavier and m_2 lighter by the same amount. The acceleration was measured by a ticker timer. The results were as shown in Table 4.3.

Table 4.3

HANGING MASS m_1 (g)	MASS OF GLIDER AND m_2 (g)	ACCELERATION a (m s^{-2})
100	750	1.15
200	650	2.45
300	550	3.46
400	450	4.52
500	350	5.76
600	250	6.90

(a) For each case, calculate the force applied ($\boldsymbol{F}_A$) by multiplying the hanging mass (kg) by 9.8 and expressing the answer in newtons (N).
(b) Plot acceleration vs force and comment on whether Newton's second law is confirmed.
(c) How are total mass and the ratio $\boldsymbol{F}/\boldsymbol{a}$ related?
(d) Predict what would happen to the shape of the graph if there was some friction present.
(e) How would you modify the experiment to keep the force the same but vary the total mass?

NEWTON'S THIRD LAW OF MOTION 4.7

PHYSICS FACT

The main gun on the British warship HMS Invincible had a bore (internal diameter) of 16 inches (40 cm) — and that's huge! Its shells could penetrate 24 inches (60 cm) of steel armour on enemy ships. The recoil of the gun was so great that it would buckle up the wooden deck and peel off the paint.

Forces come in pairs.

- If a hammer exerts a force on a nail, the nail exerts an equal and opposite force on the hammer.
- If you lean against a wall, the wall pushes back on you.

These situations can be summed up by the words 'you cannot touch without being touched'. They are examples of **Newton's third law of motion**, which states:

> To every action there is an equal and opposite reaction.

In other words, '*if a body A exerts a force on another body B, then body B exerts an equal and opposite force on body A*'. This second way of expressing the law seems more wordy but is more precise. There can be problems working out the **action** and **reaction** pairs in some situations.

Examples of other action–reaction pairs are given in Table 4.4.

Table 4.4 ACTION–REACTION PAIRS

ACTION	REACTION
• Exhaust gases are pushed out of the rocket	• The rocket pushed forward by the gas
• A sprinter pushes on the starting blocks	• The blocks push on the sprinter
• A vase of flowers presses down on a table	• The table pushes upward on the vase
• A tyre pushes on the road	• The road pushes on the tyre
• A softball is hit by a bat	• The bat slows down as the ball pushes back
• An orbiting satellite is attracted to the Earth	• The Earth is attracted to the satellite

Consider a packet of biscuits resting on a table. The two forces can be labelled as $\boldsymbol{F}_W$, the weight of the biscuits; and $\boldsymbol{F}_N$, the **normal reaction** force that the table pushes up normal to the surface. 'Normal' means at right angles (Figure 4.15).

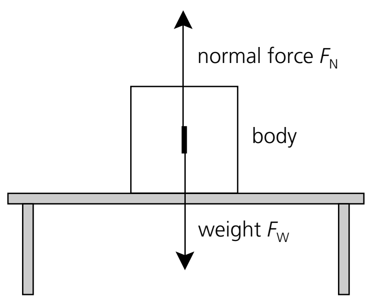


Figure 4.15

In this case they are equal and opposite because no acceleration is occurring. If the mass of the biscuits was 125 g (0.125 kg) for instance, then the weight would be 1.25 N down and the normal reaction force would be 1.25 N upward.

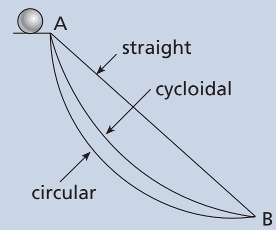
Questions

13 What are the reactions to the following actions: **(a)** a tennis ball is hit by a racquet; **(b)** a horse walks along a road; **(c)** a horse drags a log along a dirt track; **(d)** a click beetle jumps into the air; **(e)** a man falls out of a tree?

14 A vase of mass 2.5 kg rests on a table.
(a) What is the normal reaction force exerted by the table on the vase?
(b) What if a very large mass such as 500 kg is placed on the table? Explain.

NOVEL CHALLENGE

Brachistochrone



In the summer of 1693, John Bernoulli posed the following problem, which still hadn't been solved 6 months later. The day Newton heard it he solved it. *The problem: you have three paths for a ball to roll down (see figure) — which is the fastest?* The device is called a *Brachistochrone* (Greek *brachy* = 'short', *chronos* = 'time'). Newton didn't give his name but Bernoulli said *'Tanquam ex ungue leonem'* — Latin for 'as the lion is known by his claw'. He recognised the genius of Newton. If you put a ball anywhere on the cycloid it will take the same time to reach B. *Strange — Why? The cycloid is called a* tautochrone *(Gk* tauto *= 'same').*

4.8 FORCE, WEIGHT AND GRAVITY

Objects held above the Earth's surface are attracted towards the Earth's centre by a pulling force called the **force of gravity**. This pulling force causes freely falling objects to accelerate downward. **Gravitational attraction** occurs between all bodies but a general discussion is left until a later chapter.

If you drop a rock, it would accelerate downward at 9.8 m s^{-2} provided that no other external forces interfere with its motion. This is called its **free-fall** acceleration. The force due to gravity is given by $\boldsymbol{F} = m\boldsymbol{a}$. For a 1 kg rock the force of gravity equals 1 kg × 9.8 m s^{-2} or 9.8 kg m s^{-2} directed downward. This is 9.8 newtons. To hold the rock steady in your hand requires an equal and opposite force. This is a measure of the **weight** of the rock. It has the symbol $\boldsymbol{F}_W$ although some teachers and texts prefer to use just $\boldsymbol{W}$. **The SI convention is to use *F* for force and different subscripts for specific types of force.** The word 'weight' has been around for a long, long time, hence the confusion when a common term is given a specific meaning in physics. It comes from pre-historic German where *wegan* meant 'to carry'. Its secondary use meaning 'heaviness' was an Old English adaptation.

NOVEL CHALLENGE

In astronaut Neil Armstrong's biography, it says that on Phobos — one of the two potato-shaped moons of Mars — he would weigh only 3 ounces. *If 'g' on Phobos is one-thousandth that of its value on Earth, what would his mass be in kg?*

INVESTIGATING

You have been provided with a ball, a stopwatch and a tape measure. *How many different ways can you think of to measure the distance from a top floor verandah to the ground below? List sources of error and comment on the most accurate method.*

INVESTIGATING

If you weigh yourself on bathroom scales the reading is greater if the scales are on carpet rather than on a hard floor. Now why is this? It would make a great investigation — in fact it was reported in *New Scientist*.

The formula relating mass to weight is then:

$$F_W = mg \quad \text{(where } \boldsymbol{g} = \text{acceleration due to gravity)}$$

On Earth, $g = 9.8\ \text{m s}^{-2}$ but does vary from location to location. The value of acceleration due to gravity on other planets depends largely on the planet's mass and is shown in Table 4.5.

Table 4.5 FREE-FALL ACCELERATIONS

BODY	ACCELERATION DUE TO GRAVITY (m s^{-2})
Jupiter	24.9
Neptune	11.8
Saturn	10.5
Earth (average)	9.8
Earth (poles)	9.83
Earth (equator)	9.78
Venus	8.8
Uranus	7.8
Mercury	3.7
Mars	3.7
Moon	1.6

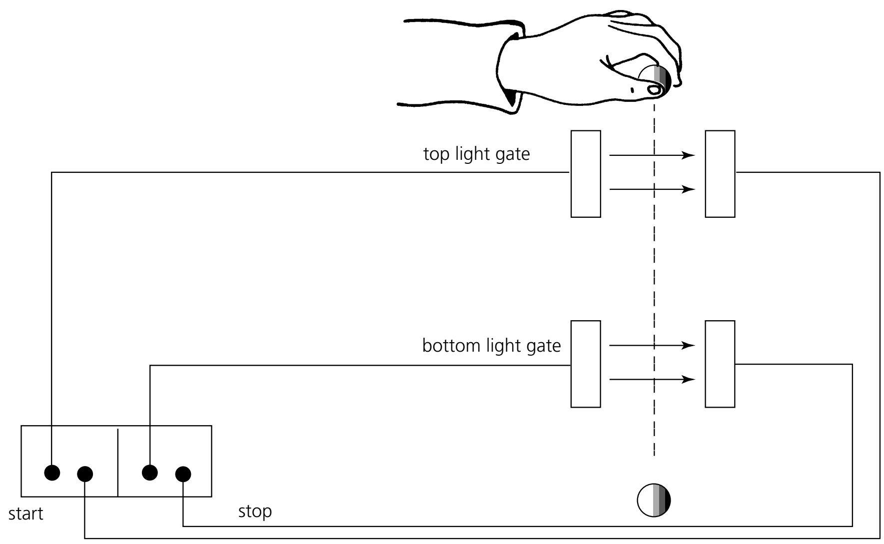
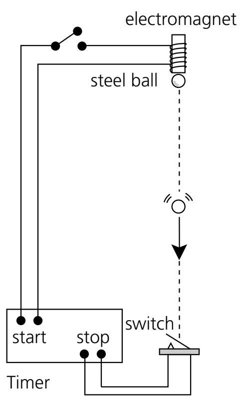


Figure 4.16 Light gates can record the time interval of a falling ball.

Figure 4.17 The ball falls when the electromagnet turns off. This starts the timer.



Several devices are available in the laboratory for measuring acceleration due to gravity. Because you are limited to a vertical distance of about 2 m, the timing of the fall is very short and can't be done by hand with a stopwatch.

Two common methods to measure the time are with:

- light gates (Figure 4.16)
- a mechanical switch (Figure 4.17).

A light gate consists of a light source and a detector. When an object is dropped through the top gate, the counter-timer is started and when the object passes through the lower one the timer stops. Alternatively, a striped piece of plastic can be used as part of a computer interfacing package.

The other method consists of an electromagnet holding a ball bearing high up. When the switch is opened the electromagnet turns off and the ball falls. At the same instant, the opening of the switch starts a timer. When the ball hits the lower plate a switch is closed and the timer stops.

Activity 4.4 BODY MASS INDEX

SR

Body mass index (BMI) is a mathematical formula that correlates mass and height to determine how much body fat you have. See Table 4.6 below. BMI is a better predictor of health risk than simple body mass measurements. BMI should not be used to assess competitive athletes or body builders because of their relatively larger amount of muscle. Neither should pregnant or lactating women, children, or frail sedentary elderly people use BMI.

1 What is your BMI? (BMI = mass/height^2, with mass in kilograms and height in metres.)

2 Is there any evidence that BMI varies during different stages of a woman's menstrual cycle?

3 Are there any other reasons *not* to use BMI as a health index?

Table 4.6

MASS CATEGORY	BODY MASS INDEX
'Underweight'	<18.5
Healthy	18.5 – 24.9
'Overweight'	25 – 29.9
Obese	>30

NOVEL CHALLENGE

A man comes up to a bridge that can just support his weight and 1 of the 4 balls he is carrying. He decides to juggle as he crosses so that only 1 ball will be in his hands at any one time. *What do you think of his solution?*

Weight and mass

Weight is different from mass. *Mass* is a measure of an object's resistance to motion and doesn't vary no matter where the object might be taken to in the universe. *Weight* is a measure of the force of gravity acting on that object and will vary depending which planet it is on or what gravitational forces it is being subjected to. Refer back to Table 4.5. For a body of mass m located at a point where the free-fall acceleration is $\boldsymbol{g}$, then the magnitude of the weight (force) vector is given by $\boldsymbol{F}_W = m\boldsymbol{g}$.

You could measure the weight of a body by placing it on some bathroom scales or by hanging it from a spring balance. With bathroom scales, gravity pulls downwards with a force we've called weight $\boldsymbol{F}_W$ (= $m\boldsymbol{g}$) and the scales push up with a reaction force normal to the surface ($\boldsymbol{F}_N$). $\boldsymbol{F}_N$ could also be called the 'scale reading'.

In this case, as there is no acceleration, the magnitude of the two forces are equal in magnitude, so $\boldsymbol{F}_W = \boldsymbol{F}_N$ and the scales read the true weight of the body.

For example, if you have a mass of 60 kg then your weight on Earth is 60 × 9.8 or 588 N. It is fairly usual to approximate the free-fall acceleration due to gravity on Earth as 10 m s^{-2} so the relationship is simplified to: weight (N) = mass (kg) × 10 m s^{-2}. Your teacher will tell you if you are to use 10 or 9.8 in problems. Certainly for experiments you would use 9.8 m s^{-2}.

The free-fall acceleration on the Moon is 1.6 m s^{-2} so on the Moon $\boldsymbol{F}_W = m \times 1.6$ N = 96 N. If you want to lose weight, fly to the Moon.

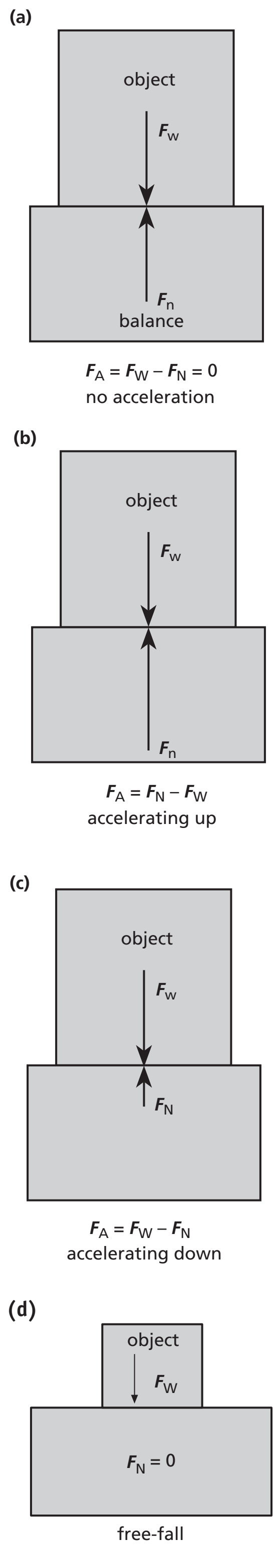
Example

A man has a mass of 65 kg. Calculate his weight on **(a)** the Earth; **(b)** the Moon; **(c)** Jupiter.

Solution

(a) $\boldsymbol{F}_W$ = mg = 65 × 10 = 650 N.
(b) $\boldsymbol{F}_W$ = mg = 65 × 1.6 = 104 N (100 N).
(c) $\boldsymbol{F}_W$ = mg = 65 × 24.9 = 1618 N (1600 N).

Figure 4.18
Accelerated frame of reference.



Apparent weight

In the example above, the body was at rest on the surface of Earth or a planet. The resultant force is zero ($F_W = F_N = 0$) and this would hold as long as the body was not accelerating (Figure 4.18**(a)**). If it was accelerating, the resultant force would not be zero, according to Newton's second law. This is considered below.

Your weight (mg) can be considered constant at the surface of the Earth. This is sometimes called your true weight but is really just 'weight'. There are slight variations in $\boldsymbol{g}$ due to the different composition of rocks below but it is on average about 9.8 m s^{-2}. Any effects due to the rotation of the Earth will not be considered until Chapter 6.

Your weight (mg) can appear to change in two main ways:
- by being in an accelerated frame of reference (skydiving, or going up or down in an elevator)
- by buoyancy effects (floating in a swimming pool).

Accelerated frame of reference

Going up in an elevator

If you were standing on some bathroom scales in an elevator and it accelerated upwards, the reading on the scales would increase (Figure 4.18**(b)**). The reason: as with a non-accelerated (or 'inertial') frame of reference, such as a stationary elevator or one travelling at constant velocity as described above, you are acted on by two forces. One is gravity, which pulls down with a force F_W (= mg) called your weight; the other is the normal reaction force (F_N), which pushed the scales upwards onto the soles of your feet. But the body is accelerated upwards, so F_N and F_W cannot be equal; in fact, F_N must be greater than F_W. The difference in magnitude is the resultant or applied upward force (F_A):

$$F_A = F_N - F_W$$
$$F_N = F_W + F_A$$
Apparent weight = weight + applied force or
$$F_N = mg + ma = m(g + a)$$

The bathroom scales push upward with a force F_N whose magnitude is the reading on the scales. It is called the apparent weight.

Going down in an elevator

In this case the upward force (F_N) is less than the downward force of gravity (F_W) (Figure 4.18**(c)**). The difference is equal to the applied force (F_A):

$$F_A = F_W - F_N$$
$$F_N = F_W - F_A$$
Apparent weight = weight − applied force
$$F_N = mg - ma = m(g - a)$$

Free-fall

Free-fall is another accelerating frame of reference. Here the upward force of the scales is zero so your acceleration is equal to $\boldsymbol{g}$. If you were standing on some bathroom scales on a trap door in a floor and the door opened, the scales would not register a reading so your apparent weight would also be zero as you fell (Figure 4.18**(d)**).

Free-fall is really quite a different case from the accelerated frames of reference mentioned in the above cases of the accelerator going up and down. Floating in a pool is an inertial (non-accelerated) frame of reference. There is a buoyant force acting upward that is equal to your weight so that your resultant downward force is zero. This is a completely different case from free-fall. Buoyancy acts as someone grabbing you under the arms and lifting you off the scale — it has nothing to do with acceleration. The only thing in common is the term 'apparent weight', which is zero in both cases.

Example

What is the weight and what is the apparent weight of a 50 kg person in a lift that is **(a)** accelerating upwards at 1.5 m s^{-2}; **(b)** accelerating downwards at 1.5 m s^{-2}; **(c)** in free-fall?

Solution

The weight in all cases is equal to mg (that is, $\boldsymbol{F}_W = m\boldsymbol{g}$): $50 \times 10 = 500$ N.
Case **(a)**: Apparent weight ($\boldsymbol{F}_N$) = $m\boldsymbol{g} + m\boldsymbol{a} = m(\boldsymbol{g} + \boldsymbol{a}) = 50\ (10 + 1.5) = 575$ N.
Case **(b)**: Apparent weight ($\boldsymbol{F}_N$) = $m\boldsymbol{g} - m\boldsymbol{a} = m(\boldsymbol{g} - \boldsymbol{a}) = 50\ (10 - 1.5) = 425$ N.
Case **(c)**: Apparent weight ($\boldsymbol{F}_N$) = $m\boldsymbol{g} - m\boldsymbol{a} = m(\boldsymbol{g} - \boldsymbol{a}) = 50\ (10 - 10) = 0$ N.
Note: as bathroom scales are calibrated in kg, the scale reading in Case **(a)** would be 57.5 kg; in Case **(b)** 42.5 kg; and in Case **(c)** 0 kg.

Questions

15 **(a)** How many times heavier by weight would a person be on Saturn than on Earth?
(b) Why does Table 4.5 have three values for Earth?

16 In hospitals, newborn babies have their 'weight' recorded in grams but the nurses usually convert this to pounds and ounces for the parents' benefit.
(a) If a physicist had a 7 lb 8 oz baby, what would its weight be? (1 kg = 2.2 lb; 16 oz = 1 lb.)
(b) If the baby was born in the 'weightless' conditions of outer space, how could the parents measure the baby's Earth weight for the benefit of relatives at home?

17 Calculate the apparent weight of a 70 kg person under each of the following conditions: **(a)** Floating in water; **(b)** Free-falling off the stage at a concert; **(c)** Accelerating upward in a lift at 0.5 m s^{-2}; **(d)** Accelerating downward in a lift at 0.5 m s^{-2}.

4.9 TERMINAL VELOCITY

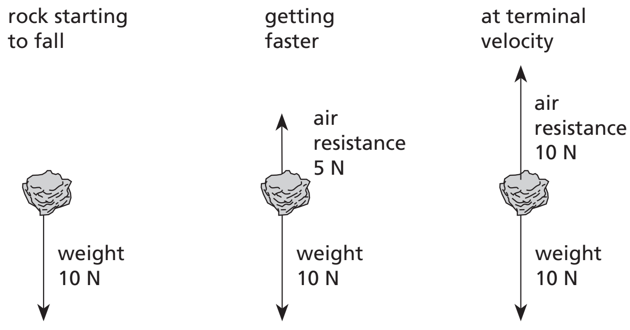
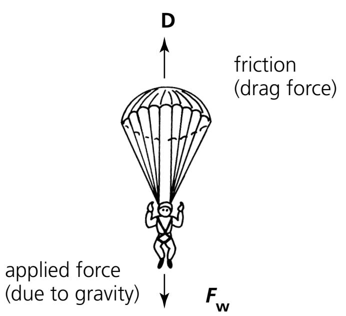


Figure 4.19
Forces acting on a rock falling freely to the ground.

Figure 4.20
Forces acting on a parachutist.

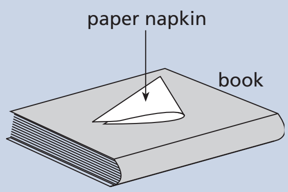


If you drop a rock off a cliff, it gets faster and faster as gravity causes it to accelerate towards the ground. However, as it speeds up, air resistance also increases so eventually the force upwards due to the air equals the force of gravity (down) and the rock stops accelerating and falls at a constant velocity thereafter. This is called its **terminal velocity**. 'Terminal' merely means 'the end' (Latin *terminus* means 'limit').

Without air resistance, objects would accelerate at 10 m s^{-2} toward the ground. In general, the frictional force between an object and the medium through which it is moving is called the **drag force** (Figure 4.20).

NOVEL CHALLENGE

Imagine you were to drop a book that has a paper napkin resting on the top (see Figure). *How will they both fall? Wrong! Try it. Why?*



NOVEL CHALLENGE

If you dropped a marble, a big styrofoam ball and a small one (from a bean bag) together from chest height, two hit the ground at the same time. *Which two and would they be faster or slower than the other one? Why? Try it and see!*

NOVEL CHALLENGE

Two skydivers are freefalling and before their 'chutes open they try to throw a tennis ball back and forth. Propose some reasons why it won't be possible. *Once their parachutes opened would it then be possible?*

Table 4.7 TERMINAL VELOCITY OF SOME OBJECTS

OBJECT	TERMINAL VELOCITY ($m\ s^{-1}$)
6 kg steel shot	150
Skydiver (typical)	50
Cricket ball	40
Tennis ball	30
Rock (1 cm)	25
Basketball	20
Ping-pong ball	10
Raindrop (3 mm)	7
Insect	6
Parachutist (typical)	5

The drag force (***D***) depends on three factors: the cross-sectional area (*A*), the velocity (***v***) and the density (ρ) of the fluid in which the object is moving. In air you will free-fall for about 12 s and reach 190 km h^{-1} in about 370 m if you are spreadeagled. If you fall head-first you will reach about 300 km h^{-1} in that time.

The world record for free-fall is held by Joseph Kittinger from the US Air Force, who jumped out of a balloon at 31.3 km altitude. He reached a velocity of 988 km h^{-1} before his parachute opened at 5300 m above the ground. His 26 km free-fall lasted 4 minutes 37 seconds. That would be some stunt for your school fête.

Surviving free-fall

- In April 1987, during a jump, parachutist Greg Robertson noticed that fellow parachutist Debbie Williams had been knocked unconscious in a collision with a third skydiver and was unable to open her parachute. Robertson, who was well above Williams at the time and who had not yet opened his parachute for the 13 500 foot (4100 m) plunge, reoriented his body head-down so as to minimise his area (*A*) and maximise his downward speed. Reaching an estimated speed of 200 miles per hour (90 m s^{-1}), he caught up with Williams and then went into a horizontal spread-eagle to increase drag (***D***) so that he could grab her. He opened her parachute and then, after releasing her, his own, with a scant 10 s before impact. Williams received extensive internal injuries due to her lack of control on landing but survived.
- While flying in a British bomber in March 1944, tail-gunner Nick Alkemade bailed out after being hit by enemy fire. With no parachute he fell 5500 m to the snow-covered ground in Germany. He was uninjured.
- Hostess Vesna Vulovic fell 10 000 m after her DC9 plane blew up over Czechoslovakia in 1972. She hit the ground and fractured her spine, legs and arms; she was in a coma for three days and had amnesia for three weeks. She has since completely recovered.
- In June 1985 a teenage boy, while mountain climbing in California, fell 45 m into a pool of water 1.2 m deep. He walked away (to the applause of his mates, no doubt).
- A chimney sweep in 1952 in London fell 45 m into rubble and sustained a deceleration of 162 '*g*'s.

SR

Activity 4.5 DRAG COEFFICIENTS

The drag force (***D***) depends on three factors: the cross-sectional area (*A*), the velocity (***v***) and the density of the fluid (ρ). The formula relating them is: $\boldsymbol{D} = \frac{1}{2}C\rho A\boldsymbol{v}^2$.

The drag coefficient *C* is the constant of proportionality and has values ranging from about 0.4 to 1.0.

As an object falls, its acceleration starts at 10 m s^{-2} but gets smaller and smaller until it is zero at the terminal velocity. At this point the weight of the object equals the drag force. The object of this activity is to plot the acceleration against velocity for a tennis ball under free-fall (Figure 4.21).

1. Measure the mass of a tennis ball and its cross-sectional area. Tables of drag coefficients suggest that a tennis ball has a coefficient of 0.7 and the density of air (ρ) is 1.2 kg m^{-3}.

   Note: the resultant force = $m\boldsymbol{a}$ = weight − drag.

$$\boldsymbol{F}_R = m\boldsymbol{a} = \boldsymbol{F}_W - \tfrac{1}{2}C\rho A\boldsymbol{v}^2$$

2. Use a computer spreadsheet or manually calculate the acceleration for every value of $\boldsymbol{v}$ from 0 to its terminal velocity. If you are doing it manually, perhaps you should increase $\boldsymbol{v}$ in units of 5 m s^{-1}.
3. Graph the results.
4. Would it be possible to prepare a table (or spreadsheet) showing $\boldsymbol{a}$ and $\boldsymbol{v}$ as time elapsed increases from 0 to 100 seconds? Pretty difficult huh?

SR Activity 4.6 CATS AND BAD PHYSICS

Investigating cats falling out of high-rise buildings may sound interesting, but if your results seem odd then maybe your data are incomplete. Here's a review of a non-experimental investigation by two veterinarians, Wayne Whitney and Cheryl Mehlhaff, in a report for the *Journal of the American Veterinary Association*. Read it and answer the questions below.

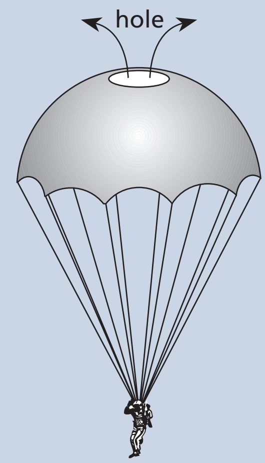
Cats falling from more than seven storeys of high-rise buildings in Manhattan sustained fewer fractures than those that fell from lower levels, according to the researchers. They speculated that a cat relaxes and spreads its limbs more horizontally, like a flying squirrel, after it reaches terminal velocity in a fall. They calculated that the average 4 kg cat reached terminal velocity of about 100 km/h after falling about five storeys. At lower levels, and therefore below terminal velocity, the sense of acceleration may cause them to curl up, making them more prone to injury, Whitney and Mehlhaff suggest.

Only one of 22 cats which fell from more than seven storeys died from its injuries, and there was only one fracture among 13 cats that fell more than nine storeys. In comparison, almost all human falls from over six storeys onto a hard surface are fatal.

Once a cat reaches its terminal velocity, it then begins to slow down. This is because the cat relaxes, changing its position from back arched, head down, and legs pulled tightly underneath its body, to resemble a spreadeagle cat, which slows it down. The reason for this is that our bodies are only sensitive to acceleration, and when you feel acceleration you get scared and curl up in a defensive (foetal) position.

Critics of the study argue that cats do die from great heights and cite the case of Pamela Marx from Brooklyn, New York, who wrote, 'I have had two cats fall from both tenth-floor and fourteenth-floor terraces and both unfortunately died. I never reported these incidents to any medical centre and believe that other people probably don't report their cats' deaths, either. You can add my two cats to your list and report that at least two cats died in fifteen falls over nine storeys.'

NOVEL CHALLENGE



If the objective of a parachute is to slow the descent of a falling object in air, why do parachutes have a hole (the *apex vent*) in the top allowing air to escape? In the Second World War parachutes did not have apex vents and they swung like pendulums as they descended (watch an old war movie, you'll see).
What is the physics behind this?

NOVEL CHALLENGE

If an elephant, a man and a mouse fell from the twentieth storey of a high-rish building, the elephant would splatter on impact and die, the human would be crushed and die, and the mouse would walk away.
Why is this?

PHYSICS FACT

It is often said: 'If a 20 cent coin fell on your head from a high-rise building, it would go through your skull and kill you'. But this is rubbish; it would merely bounce off. Another urban myth bites the dust.

Question

In terms of how you'd carry out a non-experimental investigation, what deficiencies in the data collection and analyses were there? How would you rectify them? Comment critically on the original study.

SR Activity 4.7 RAINDROPS KEEP FALLING …

The following information enables you to calculate the terminal speeds of raindrops of varying sizes. Raindrops vary in diameter from a maximum of 6.35 mm to a minimum of 0.51 mm.

The drag force is given in Activity 4.5: $\boldsymbol{D} = \frac{1}{2}C\rho A\boldsymbol{v}^2$, where C = the drag coefficient and is 1.2 for a sphere; ρ_a = density of air = 1.2 kg m^{-3}; A = the cross-sectional area of the falling object = πr^2; and $\boldsymbol{v}$ = velocity of the falling object. The radius (r) must be in metres.

At terminal velocity, the drag force equals the weight of the raindrop, and the weight is given by $\boldsymbol{F}_W = m\boldsymbol{g}$. In this equation, $\boldsymbol{g}$ is the acceleration due to gravity (9.8 m s^{-2}) and m is the mass of the raindrop in kilograms. This can be calculated by letting mass = volume x density ($m = \frac{4}{3}\pi r^3 \rho_W$) where the density of water is 1000 kg m^{-3}. Hence:

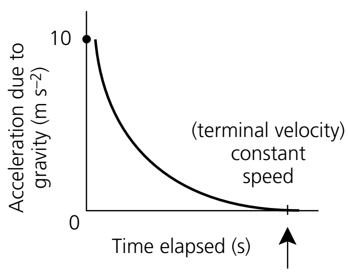
$$\boldsymbol{D} = \boldsymbol{F}_W$$

$$\frac{1}{2}C\rho A\boldsymbol{v}^2 = \frac{4}{3}\pi r^3 \rho_W \boldsymbol{g}$$

1. Rearrange and simplify this equation to calculate the terminal velocity of a raindrop 5 mm in diameter.
2. How long would it take all possible raindrops to fall from a cloud 2 km above the ground?
3. Draw a graph of raindrop diameter (x-axis) against terminal velocity.

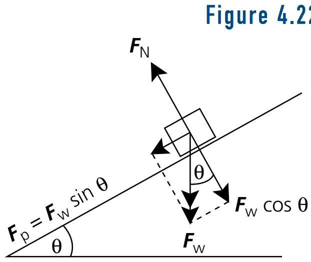
SR Activity 4.8 BOATS AFLOAT

Figure 4.21



The hull speed of a boat is the maximum speed it can reach before drag increases dramatically. For a ship 30 m long it is 25 km h^{-1}; for a 3 m boat it is 9 km h^{-1}; and for a 30 cm hull (e.g. a duck) it is 2.4 km h^{-1}. However, if hull size is too small, surface tension becomes a significant factor. For example, a whirligig beetle has a hull size of 1 cm; its hull speed should be 12 cm s^{-1} (based on extrapolation from the above data) but is actually 25 cm s^{-1} because of surface tension. For a 0.5 cm hull the speed is 30 cm s^{-1} instead of 9 cm^{-1}. But if the hull is too small, the surface tension is too great and the animal can't move. Plot the data and calculate the hull speed of some common animals such as a mouse and a rat. Comment on the accuracy of your analysis.

Figure 4.22



INCLINED PLANES 4.10

An object placed on a smooth inclined plane will accelerate down it. The accelerating force is provided by the component of the object's weight in a direction down the plane (Figure 4.22).

In the diagram, the weight of the object ($\boldsymbol{F}_W = m\boldsymbol{g}$) has been resolved into two components at right angles — one perpendicular to the plane ($\boldsymbol{F}_\perp = \boldsymbol{F}_W \cos\theta$) and the other parallel to the plane ($\boldsymbol{F}_P$ (or $\boldsymbol{F}_{||}$) $= \boldsymbol{F}_W \sin\theta$). This can be summarised as:

> Parallel component: $\boldsymbol{F}_P$ (or $\boldsymbol{F}_{||}$) $= \boldsymbol{F}_W \sin\theta = m\boldsymbol{g} \sin\theta$
> Perpendicular component $\boldsymbol{F}_\perp = \boldsymbol{F}_W \cos\theta = m\boldsymbol{g} \cos\theta$
> Normal force: $\boldsymbol{F}_N = \boldsymbol{F}_W \cos\theta$

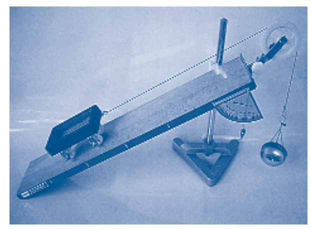
The normal force is equal and opposite to the force perpendicular to the plane because there is no acceleration in that direction.

Using Newton's second law, the resultant force equals $m\boldsymbol{a}$.

Hence: $m\boldsymbol{a} = m\boldsymbol{g} \sin\theta$, so $\boldsymbol{a} = \boldsymbol{g} \sin\theta$.

Note: the mass term cancels out, so we can say the acceleration is independent of the mass of the object in this very specific case. Notice also that this component of gravitational acceleration is down the plane. See Photo 4.3 of an inclined plane apparatus.

Photo 4.3
Demonstration inclined plane used in physics classes.



Limiting cases

From your knowledge of trigonometry you should see that:

- as θ approaches 0°, $\boldsymbol{F}_P$ approaches zero and $\boldsymbol{F}_\perp$ approaches $\boldsymbol{F}_W$
- as θ approaches 90°, $\boldsymbol{F}_P$ approaches $\boldsymbol{F}_W$ and $\boldsymbol{F}_\perp$ approaches zero.

Example 1

A 15 kg wedding cake is allowed to slide freely down a smooth 30° incline. Find **(a)** the resultant force down the incline; **(b)** the acceleration of the object.

Solution

(a) $\boldsymbol{F}_P = m\boldsymbol{a} = m\boldsymbol{g} \sin\theta = 15 \times 10 \times \sin 30° = 75$ N.

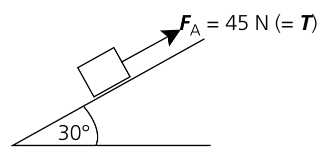
(b) $\boldsymbol{a} = \boldsymbol{g} \sin\theta = 10 \times \sin 30° = 5$ m s^{-2}.

Example 2

An 8 kg carton of soft drink is being pulled up a frictionless 30° incline using a rope and an applied force ($\boldsymbol{F}_A$) of 45 N (Figure 4.23). This applied force through the rope is often called the rope **tension** and can be given the alternative symbol $\boldsymbol{T}$.

Calculate the acceleration, if any, up the incline.

Figure 4.23



Solution

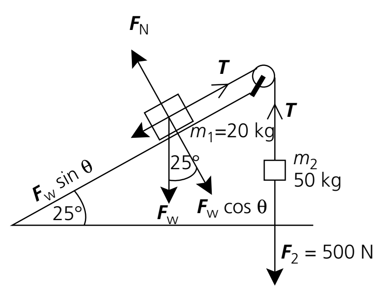
> $\boldsymbol{F}_P$ (down) $= m\boldsymbol{g} \sin\theta = 8 \times 10 \times \sin 30° = 40$ N
> $\boldsymbol{T}$ (up) $= 45$ N
> $\boldsymbol{F}_R = 45 - 40 = 5$ N up the incline (resultant force or net force)
> $\boldsymbol{F} = m\boldsymbol{a}$ $a = \boldsymbol{F}/m = 5/8 = 0.6$ m s^{-2}

Note: in the above case an applied force (tension) in the rope of exactly 40 N would not cause acceleration. The carton would travel at constant velocity or remain at rest. Students often forget that in cases where there is no net force there is no acceleration but that an object can keep travelling at constant speed. In the above case you would need more than 40 N in the rope to start the carton moving up the incline from rest; but once it was moving, a force of 40 N would keep it at constant speed. Less than 40 N in the rope would cause it to slow down, stop and start to move down the incline. In all of these cases friction has been neglected. This, of course, is unrealistic and will be dealt with in Section 4.12.

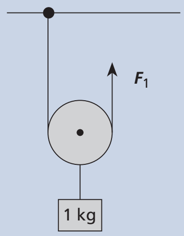
Example 3

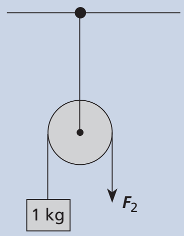
A 20 kg object is attached by a thin cord to a 50 kg mass that hangs over a frictionless pulley at the top of a 25° incline (Figure 4.24). Calculate **(a)** the acceleration, if any, of the object; **(b)** the tension in the string.

Figure 4.24



NOVEL CHALLENGE





(a) *Which box requires the smaller force to lift it?*
(b) *Which box requires less work to raise it 1 metre?*

Solution

(a) Let m_1 be the 20 kg mass on the incline; m_2 is the 50 kg hanging mass.

$$\boldsymbol{F}\text{ (down the incline)} = \boldsymbol{F}_P = m_1\boldsymbol{g}\sin\theta = 20 \times 10 \times \sin 25° = 85\text{ N}$$

$$\boldsymbol{F}\text{ (up the incline)} = \boldsymbol{F}_A = m_2\boldsymbol{g} = 50 \times 10 = 500\text{ N}$$

$$\boldsymbol{F}_R = \boldsymbol{F}_A - \boldsymbol{F}_P = 500 - 85 = 415\text{ N up the incline}$$

$$\boldsymbol{F}_R = m\boldsymbol{a} \quad \boldsymbol{a} = \frac{\boldsymbol{F}_R}{m_{(\text{total})}} = \frac{415}{20 + 50} = 5.9\text{ m s}^{-2}\text{ up the incline}$$

(b)

$$\boldsymbol{T} = \boldsymbol{F}_W \sin\theta + m\boldsymbol{a}$$
$$= m\boldsymbol{g}\sin\theta + m\boldsymbol{a}$$
$$= 20 \times 10 \times \sin 25° + 20 \times 5.9$$
$$= 203\text{ N}$$

Note: some teachers prefer to use an alternative solution, which uses the tension in the rope ($\boldsymbol{T}$) and produces simultaneous equations in terms of $\boldsymbol{T}$ for each object (m_1 and m_2) separately:

For object m_1:

$$m_1\boldsymbol{a} = \boldsymbol{T} - \boldsymbol{F}_{W1}\sin\theta = \boldsymbol{T} - m_1\boldsymbol{g}\sin\theta$$

$$\boldsymbol{a} = \frac{\boldsymbol{T} - m_1\boldsymbol{g}\sin\theta}{m_1}$$

For object m_2:

$$m_2\boldsymbol{a} = \boldsymbol{F}_{W2} - \boldsymbol{T}$$

$$\boldsymbol{a} = \frac{m_2\boldsymbol{g} - \boldsymbol{T}}{m_2}$$

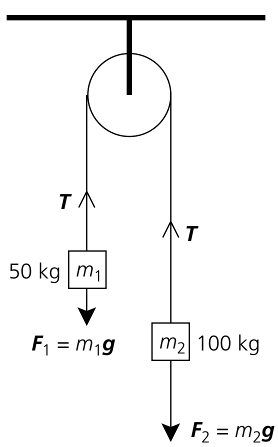
Solving simultaneous equations gives $\boldsymbol{T}$ = 203 N as before.

Be guided by your teacher and/or what you may learn in maths. Both methods as you can see produce the same answer.

PULLEYS 4.11

Other questions of great interest to physicists and engineers are those about **pulleys**. Pulleys are essential components of many of the machines used in industry and in the home. They reduce friction between two surfaces by converting sliding friction into rolling friction. The word 'pulley' came to the English language in the fourteenth century from medieval Greek — *polos* meant 'pivot'. An understanding of the forces involved enables design engineers to specify sizes and strengths of materials with the knowledge that any loads imposed will be within safe working limits. Typical application examples follow.

Figure 4.25



Example

An object of mass 50 kg and another of mass 100 kg are tied to the ends of a light inextensible (non-stretching) string. The string passes over a smooth pulley (see Figure 4.25). Determine **(a)** acceleration of the system (magnitude only); **(b)** the tension in the string.

Solution

Again this problem can be solved in two ways: **1.** by developing equations involving the tension in the string on each mass separately and equating the tension ($\boldsymbol{T}$) in each equation; **2.** alternatively, the forces can be considered as a whole. Either method is suitable.

The diagram can be rearranged into a linear form (Figure 4.26).

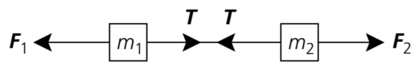


Figure 4.26

$$F_R = F_1 + F_2 = m_1\boldsymbol{g} + -m_2\boldsymbol{g} = 50 \times 10 - 100 \times 10 = 500 \text{ N to the right}$$
$$F_R = (m_1 + m_2)\boldsymbol{a}$$
$$500 = 150\boldsymbol{a}$$
$$\boldsymbol{a} = 3.3 \text{ m s}^{-1} \text{ (the 100 kg mass moves down)}$$

The tension in the string can be calculated by considering just one side: on the left the tension in the string has to equal the weight of the 50 kg object (pulling down) plus the accelerating force on the 50 kg force upward:

$$\boldsymbol{T} = m_1\boldsymbol{g} + m_1\boldsymbol{a} = 50 \times 10 + 50 \times 3.3 = 665 \text{ N upward on the left}$$

Note: the tension in the string on the right has to equal the same value (665 N). You can prove this by showing that the tension has to equal the weight of the 100 kg object (pulling down) *less* the accelerating force (because it is acting in the same direction as the weight):

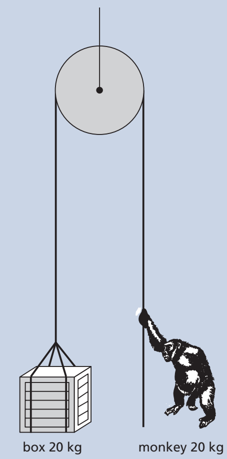
$$\boldsymbol{T} = m_2\boldsymbol{g} - m_2\boldsymbol{a} = 100 \times 10 - 100 \times 3.3 = 665 \text{ N upward on the right}$$

There are several ways of approaching these pulley problems. You may find the following method more useful:

$$m_1\boldsymbol{a} = \boldsymbol{T} - m_1\boldsymbol{g}$$
$$m_2\boldsymbol{a} = m_2\boldsymbol{g} - \boldsymbol{T}$$
$$\frac{T - m_1\boldsymbol{g}}{m_1} = \boldsymbol{a} = \frac{m_2\boldsymbol{g} - \boldsymbol{T}}{m_2}$$
$$\boldsymbol{T} = 665 \text{ N}$$

NOVEL CHALLENGE

A monkey has the same mass as a box. He climbs a rope.
Who will reach the pulley first?



Questions

18 A 30 kg box of vegetables moves down a 35° frictionless incline. Find **(a)** the normal reaction; **(b)** the resultant force down the incline; **(c)** the acceleration down the incline.

19 Two blocks of masses 2 kg (A) and 3 kg (B) respectively rest on a smooth horizontal surface and are connected by a taut string of negligible mass. A force of 10 N is applied to the 3 kg mass as shown in Figure 4.27. Calculate **(a)** the tension in the string between them; **(b)** the acceleration of the system.

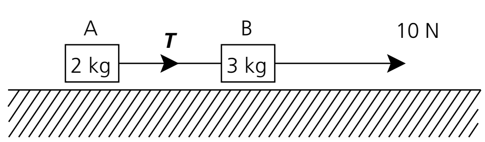
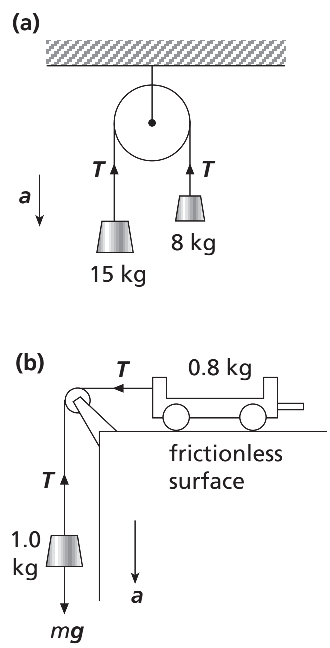


Figure 4.27
For question 19.

20 For each situation shown in Figure 4.28, find **(a)** the acceleration and **(b)** the tension in the string.

Figure 4.28
For question 20.



FRICTION 4.12

In Grade 10 you were probably asked to write about what life would be like without friction. It should have been apparent that it would be a nightmare — in fact, impossible.

- You couldn't swallow, walk, hold a pen or do your homework.
- Your clothes would unravel and your shoe laces would come undone.
- Nails and screws would be useless — your house would fall down.
- Mountains would crumble but TV cameras couldn't record it.
- Childbirth would be by caesarean because uterine pushing would be useless and forceps wouldn't grip.
- Life would last about one day before everyone died. The farewell party would be a disaster because you couldn't hold on to your cup, you couldn't swallow and you couldn't stand up. Ever been to a party like that?

Friction of course is absolutely necessary but it is also a hindrance. The search for ways of altering it has gone on for thousands of years. The search to *understand* it has only gone on for a few hundred years — only since the birth of physics.

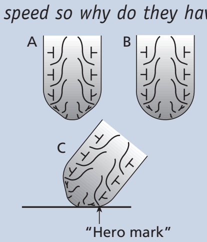
NOVEL CHALLENGE

Motorcycle tyres are of two main types — sport and touring. Sport tyres are v-shaped (A), touring are rounded (B). The figure below shows the profiles of Dunlop's mega-successful D207GP race tyre, which has been cleaning-up in supersport competitions for some time now. On the right (B) is the road version of the tyre (the D207). The unusual tread pattern is called a **cosecant groove**. In maths you may have learnt about the cosecant or cosec trigonometric function, which equals 1/sin. Dunlop scientists found that the wear on a tyre is proportional to the cosec of the lean angle (the angle of the bike to the road). Dunlop engineers incorporated cosecant grooves to minimise the tyre wear problems (see figure).

(a) Plot a graph of the lean angle (from 5° to 50° in 5° increments) against the cosec of the lean angle. *Note*: when θ = 30°, cosec 30° = 1/sin 30° = 2. A computer spreadsheet may speed things up. *How does the curve compare with the groove shape in the D207GP tyre?*

(b) As riders using the v-shaped tyre go into a corner they flip the bike on to its side rather than doing a uniform lean. *Why is this? The upper limit of contact between the tyre and the road is called the 'hero line'. What do you suspect this means?*

(c) Cruising bikes like the Harley Davidson have tyres with a flat profile. *This must impair their cornering at speed so why do they have it?*



What causes friction?

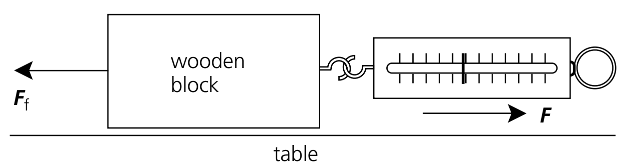
Friction is a force acting between the surface atoms of one body and those of another. If two highly polished and carefully cleaned metal surfaces are brought together in a very good vacuum, they cannot be made to slide over one another. Instead they **cold-weld** together instantly, forming a single piece of metal. Under ordinary circumstances, however, such close atom-to-atom contact is not possible. Even a highly polished metal surface is far from being flat on the atomic scale. Moreover, the surfaces of everyday objects have layers of oxides and other contaminants that reduce this cold-welding effect.

When two surfaces are placed together, only the high points touch each other. It's like a mountain range turned upside down on top of another. The actual area in contact may be only one-thousandth of the total surface area. Many points cold-weld together. When the surfaces are pulled across each other, there is a continuous breaking and remaking of the welds. Strangely enough, it is not always the softer material that gets worn away by friction. In machinery driven by rubber belts, it is often the metal pulley that wears away before the soft rubber belt. Grit becomes impregnated in the soft rubber and grinds away at the metal of the pulley. Pulleys are often replaced before the belts.

EI Activity 4.9 FRICTION IS A DRAG

1. Take a block of wood, note its mass, connect a spring balance to it and place it on the table. Gently pull on the balance horizontally and note the maximum force on the scale before it starts to move (Figure 4.29). This is called the limiting friction, starting friction or static friction.

Figure 4.29



2. Continue to pull it at constant speed across the bench and note the reading. This is called the **sliding friction**. Double check your results.
3. Add a known mass to the top of the block and repeat.
4. Repeat with additional masses, noting the spring balance reading each time.

5 Convert the spring balance reading to an equivalent sliding frictional force (F_f) in newtons.

6 Calculate the normal reaction (F_N) in newtons of the block in each case.

7 Plot F_f (y-axis) against F_N (x-axis) and comment on the relationship.

8 Which is greater, sliding friction or limiting friction?

9 Leave a block on the bench overnight and measure limiting friction the next day. Has it changed? Why?

In the above activity you should have established that F_f was directly proportional to F_N, that is, $F_f \propto F_N$. A constant of proportionality (μ) can be included in the relationship and the formula becomes:

$$F_f = \mu F_N \text{ or } \mu = \frac{F_f}{F_N}$$

The symbol μ is called the **coefficient of sliding friction** and is usually less than 1.0 but can range as high as 7 or 8. It is a ratio — it has no units.

$$\text{Coefficient of friction} = \frac{\text{frictional force}}{\text{normal contact force}}$$

Table 4.8 COEFFICIENTS OF FRICTION

SURFACES IN CONTACT	STARTING FRICTION	SLIDING FRICTION
Steel on ice (ice skates)	0.02	0.01
Teflon on teflon	0.04	0.04
Waxed skis on wet snow	0.14	0.10
Wood on Laminex	0.40	0.30
Glass on glass	0.94	0.40
Steel on steel	0.78	0.42
Wood on wood	0.62	0.48
Rubber tyre on wet road	0.70	0.50
Rubber tyre on dry road	0.90	0.70
Steel on lead	0.95	0.95
Foam rubber on foam rubber	8.0	7.0

Rolling friction

If you've ever tried to slide down a grassy slope on a piece of cardboard you know that you'd go faster on a go-kart with wheels. Wheels and bearings replace sliding friction with rolling friction and this is much lower. Imagine a car parked on a slope. The friction between the brake pads and the wheels prevents the car rolling. Without rolling being available, the tyres would have to slide, and sliding friction between the road and the tyres is great enough to keep the car from moving downhill.

Properties of friction

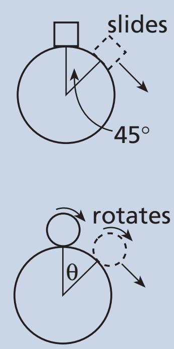
- Starting friction is always more than sliding friction.
- Rolling friction is always less than sliding friction.
- Friction is always in the direction opposing motion.
- Sliding friction is practically independent of surface area.
- $F_f = \mu F_N$

NOVEL CHALLENGE

A piece of pine dowel is placed in an electric drill and rotated against a piece of hardwood. *Which will catch on fire first? Try it.*

NOVEL CHALLENGE

A block of wood is placed on top of a smooth cylinder (see figure).



When the block gets to 45° it slides off. *If a ball was used instead of the block and allowed to roll down the surface, would it fall off at a bigger or smaller angle than 45°?*

NOVEL CHALLENGE

A cubical block of mass 10 kg is placed on a surface where μ=1.0. *Where would you need to push on the block so that it was on the verge of tipping and sliding?*

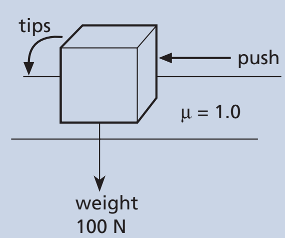


Photo 4.4
Friction modifier for a car.

Figure 4.30
Bearings.

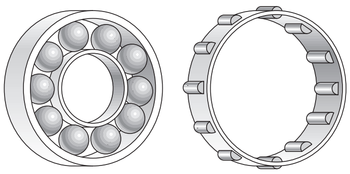


Figure 4.31
A typical joint in the human body.

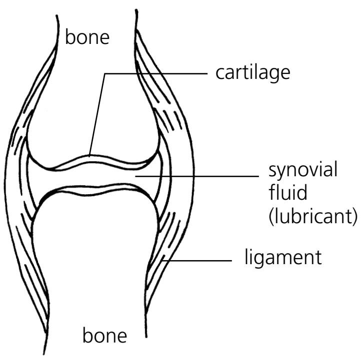
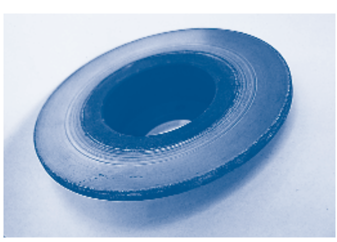


Photo 4.5
Disc rotor from a car. When you put your foot on the brakes, pads are pressed against this rotor to slow the car down. The grooves in this one show that the pads wore down to bare metal — dangerous but can be machined flat again.



Reducing friction

About 40% of the fuel used in a car is to overcome friction. This friction may be between the body and the air or between the mechanical components (pistons and cylinders, bearings or gears). Obviously, motor engineers have attempted to reduce those types that are a hindrance by either streamlining the body design or by use of **lubricants** between moving mechanical parts. Lubricants such as the one pictured in the photo may contain teflon (polytetrafluoroethylene — PTFE), which is added to the engine oil lubricant. Another friction modifier is the oil additive molybdenum disulfide (MoS_2). If the oil level in a car gets too low, friction increases, which produces more heat and can damage the engine.

Friction modifiers are not new. The Egyptians built enormous pyramids 4500 years ago using huge stone blocks that were difficult to move by sliding. To move them they used log rollers underneath and took advantage of the fact that rolling friction is lower than sliding friction.

Friction can also be reduced by choosing suitable materials for the contact surfaces; for example, a steel shaft should rotate in a bronze or white-metal bearing. In hi-fi equipment, bearings usually have a nylon 'bush' — a low friction insert inside the bearing. Sometimes bearings are impregnated with graphite granules. Another common bearing is the ball or roller bearing. They can reduce friction by up to 100 times as they convert sliding friction to rolling friction (Figure 4.30). Strangely enough, under load, they work even better.

Human joints are lubricated by synovial fluid (Figure 4.31) between layers of cartilage lining the joint. When you move the efficiency of this lubricant increases. What a piece of work is man (and woman).

Not all lubricants are liquids. For example, hovercrafts float on a cushion of air. A linear air track in a physics laboratory works in the same way. Air can be an excellent lubricant.

EI Activity 4.10 VEHICLE FRICTION

To carry out this activity you need the help of a licensed driver.

Part A: Rolling friction

1 The coefficient of rolling friction of a car can be found by pushing a car along a flat, horizontal surface. Have a person sit in the driver's seat ready to apply the brakes. The car should be put in neutral gear and the engine turned off.

2 Put a set of bathroom scales on the bumper bar and note the reading (in kg) needed to keep the car rolling at a slow constant speed. It will probably be between 10 and 20 kg.

3 Stop the car and push from the opposite direction. Average the results and convert to newtons.

4 Look in the owner's manual to find out the mass of the car.

5 Use the friction formula to calculate the coefficient of rolling friction.

6 What is the source of this rolling resistance?

Part B: Engine friction

1 Repeat the above experiment, still with the engine off, but put the car in top gear and let the clutch out. The bathroom scale reading will then be the sum of engine friction and rolling friction. You should get values around 50 kg — a typical value for a 1989 Honda Civic.

2 Subtract this value from the rolling friction (Part A) to get engine drag. Convert to newtons.

3 Comment on the source of engine drag.

Part C: Air drag

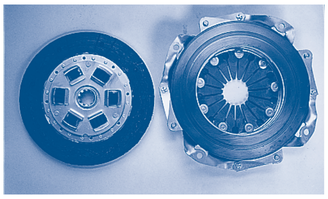
This part requires two people inside the car — a licensed person to give full attention to driving and the other to time and record results.

1. Find a straight, smooth, level road with little traffic at the time. Be careful not to interfere with other vehicles and don't do it at night.
2. Drive at a steady 40 km/h in top gear, push the clutch in and measure the time it takes to slow down to 30 km/h. This is called the 'coast-down' time. We found times of 24 s were average.
3. Use a formula to calculate the deceleration.
4. Use Newton's second law to compute the total frictional force.
5. Subtract rolling resistance and engine resistance to give air drag.

Summary: Total friction = rolling resistance + engine drag + air drag.

Express the three types of friction as a percentage of the total friction.
Comment on your results and comment further on how the activity could be improved or extended.

Photo 4.6
Clutch assembly out of a manual car. The clutch plate (on the left) provides a smooth coupling between the flywheel (not shown) and the pressure plate (on the right). When the clutch fingers in the centre of the assembly are pressed in, the pressure plate moves away so the gears can be changed.



NOVEL CHALLENGE

In an extreme skiing competition in Alaska in 1995, a New Zealand woman tumbled 400 m down a 50° slope and ended up with severe head trauma. (The slope used in the Olympics is 35°.) How fast would she be going if the coefficient of friction was **(a)** zero, **(b)** 0.10?

Increasing friction

Although friction can be a nuisance (as seen in the previous activity), it is also necessary and may even need to be increased. Car tyre tread patterns are designed to increase rather than decrease friction although other factors come into the design as well. Factors such as rate of wear, flexibility, strength, dispersion of road moisture and cost are just as important. The same considerations go into the design of running shoe soles. Some questions at the end of this chapter deal with design of consumer products.

4.13 EXAMINING FRICTION

There are three main situations that you should be familiar with when examining friction between surfaces. They can be best thought of as: horizontal applied force; angled applied force and inclined plane. Important: the term 'at constant speed' means that the applied force in the direction of motion is equal to the friction. If it is greater than friction the object will accelerate. It can never be less than friction!

Horizontal applied force

This is the simplest case. The applied force and friction are in a line and oppose each other.

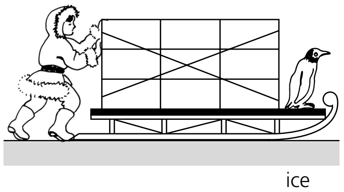
Example 1

What horizontal force has to be applied to a 50 kg sled on an ice surface ($\mu = 0.3$) as shown in Figure 4.32 to make it move at constant speed?

Solution

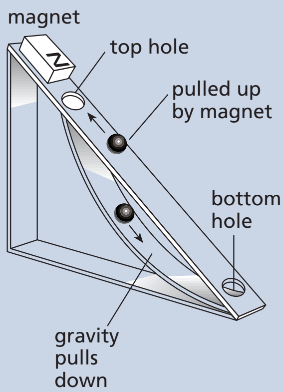
$$F_W = mg = 50 \times 10 = 500 \text{ N down}$$
$$F_N = 500 \text{ N up}$$
$$F_f = \mu F_N = 0.3 \times 500 = 150 \text{ N}$$
$$F_A = 150 \text{ N to right (constant speed)}$$

Figure 4.32



INVESTIGATING

Many people claim to have made perpetual motion machines. Find a description of one and say why it couldn't work. Would the one shown here work?



Example 2

A spring balance reads 300 g as it is used horizontally to drag a 750 g block of wood along a laboratory bench. Calculate the coefficient of friction.

Solution

$$F_N = 0.75 \times 10 = 7.5 \text{ N up}$$

$$F_A = F_f = 0.3 \times 10 = 3 \text{ N}$$

$$\mu = \frac{F_f}{F_N} = \frac{3}{7.5} = 0.4$$

Questions

21 A butcher pulls on a freshly cleaned 40 kg side of beef with a horizontal force of 220 N and it slides across the boning table at constant speed. Calculate the coefficient of friction.

22 A horizontal steel cable is used to drag a bucket filled with coal along the ground at constant speed. If the mass of the bucket and coal is 6.1 t and the coefficient of friction is 0.58, calculate the tension in the cable.

Angled applied force

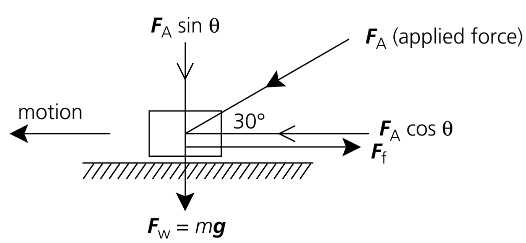
The complication with this case is that the applied force has to be resolved into two components at right angles. The vertical component will change the normal reaction whereas the horizontal component will be responsible for motion along the surface.

Angled forces can be of two types: pushing or pulling.

Example 1: Pushing

A child is pushing a 25 kg box of toys along a carpeted floor at constant speed.

Figure 4.33



If the child's arms make an angle of 30° to the horizontal and she pushes with a force of 100 N, calculate **(a)** the vertical component of the applied force; **(b)** the horizontal com ponent of the applied force; **(c)** the normal reaction; **(d)** the force of friction; **(e)** the coefficient of friction.

Solution

(a) Vertical component = $F_A \sin\theta = 100 \times \sin 30° = 50$ N.

(b) Horizontal component = $F_A \cos\theta = 100 \times \cos 30° = 87$ N.

(c) Normal reaction = weight + vertical component pushing down $= mg + F_A \sin\theta$
$= 250 + 50 = 300$ N.

(d) Friction = horizontal component (because of constant speed) = 87 N.

(e) $\mu = \frac{F_f}{F_N} = \frac{87 \text{ N}}{300 \text{ N}} = 0.3.$

Note: if the applied force is a pulling force at an angle then the normal reaction is the weight *minus* the vertical component. See the example that follows.

NOVEL CHALLENGE

A wooden block is put on an electronic balance, which then reads 100 g (weight = 1 N). A string is attached to the block and a spring balance is attached to the string. A force of 0.4 N is applied at an angle of 45°. *What will the balance read? Try it to check.*

Example 2: Pulling

A boy drags a 15 kg box across a concrete floor at constant speed by means of a cord at an angle of 20° to the floor. If the force applied is 100 N, calculate the coefficient of friction.

Solution

- Vertical component = 100 sin 20° = 34 N.
- Normal reaction = weight − vertical component = 15 × 10 − 34 = 116 N.
- Horizontal component = 100 cos 20° = 94 N.
- Friction ($\boldsymbol{F}_f$) = 94 N.

$$\mu = \frac{\boldsymbol{F}_f}{\boldsymbol{F}_N} = \frac{94\ \text{N}}{116\ \text{N}} = 0.8$$

Example 3: Pulling

What force is needed to be applied to the handle of a 10 kg sled to drag it at constant speed across a horizontal sandy beach? The handle of the sled is at an angle of 30° to the horizontal and the coefficient of friction is 0.5.

Solution

$$\begin{aligned}
\boldsymbol{F}_f &= \mu \boldsymbol{F}_N \\
\boldsymbol{F}_A \cos\theta &= \mu \boldsymbol{F}_N \\
&= \mu(\boldsymbol{F}_W - \boldsymbol{F}_A \sin\theta) \\
\boldsymbol{F}_A \times \cos 30° &= 0.5\ (10 \times 10 - \boldsymbol{F}_A \sin 30°) \\
0.87\ \boldsymbol{F}_A &= 50 - 0.5 \times \boldsymbol{F}_A \times 0.5 \\
0.87\ \boldsymbol{F}_A &= 50 - 0.25\ \boldsymbol{F}_A \\
1.12\ \boldsymbol{F}_A &= 50 \\
\boldsymbol{F}_A &= 45\ \text{N}
\end{aligned}$$

> **NOVEL CHALLENGE**
>
> Here's another Fermi question: What force is required to break a blade of grass by pulling at each end?

> **NOVEL CHALLENGE**
>
> A 10 kg block and a 5 kg block sit side-by-side on a benchtop, just touching each other. *Which is the easier way to push them along: using a 100 N force at 30° to the top edge of the 10 kg block or to the top edge of the 5 kg block? Assume the coefficient of friction is 0.5 for these surfaces.*

Questions

23 A worker drags an 80 kg crate across a factory floor at constant speed by pulling on a rope tied to the crate. The worker exerts a force of 350 N on the rope, which is inclined at 38° to the horizontal. Calculate **(a)** the frictional force; **(b)** the normal reaction; **(c)** μ.

24 A lawn roller of mass 200 kg is being pushed at a constant speed by the handle, which is inclined at 40° to the horizontal. If the coefficient of friction is 0.12, calculate the force being applied to the handle by the pusher.

Inclined planes

Again, with this sort of problem the normal reaction is not equal to the weight but the component of the weight at right angles to the incline (= $\boldsymbol{F}_W \cos\theta$). If the object slides down the plane then the friction acts up the plane, but if the object is dragged *up* the plane, then the friction acts down the plane. There are two situations: sliding down and sliding up.

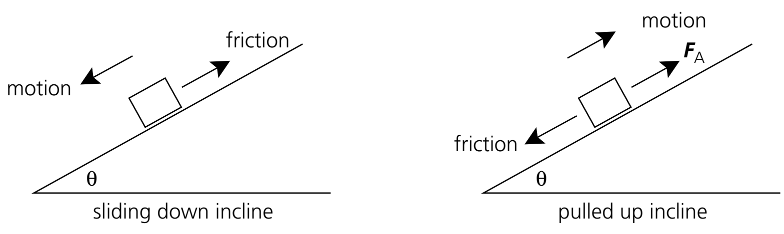


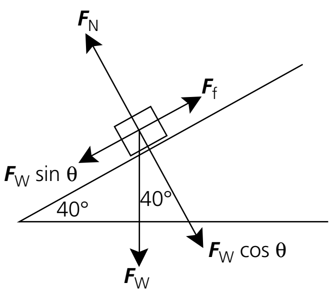
Figure 4.34

Example 1: Sliding down

A 14 kg toolbox is placed on a plank of wood. When one end of the plank is raised, the toolbox begins to slide down the incline at a uniform speed when the angle reaches 40°. Calculate the coefficient of friction.

Solution (See Figure 4.35)

Figure 4.35



$$F_f = F_W \sin\theta$$

$$F_N = F_W \cos\theta$$

$$\mu = \frac{F_f}{F_N} = \frac{F_W \sin\theta}{F_W \cos\theta} = \frac{14 \times 10 \times \sin 40°}{14 \times 10 \times \cos 40°} = \frac{90}{107} = 0.84$$

Note: the previous equation can be simplified. The ratio $\frac{\sin\theta}{\cos\theta} = \tan\theta$, so the coefficient of friction is merely the tan of the angle at which an object slides at constant speed down an incline (tan 40° = 0.84). This is a simple method for finding μ in the classroom. Note also that the value is independent of mass. The mass term cancels out.

Example 2: Sliding up

A bricklayer's apprentice is dragging a tray of bricks up a 50° inclined plank by pulling on a rope attached to the tray. The rope is parallel to the plank. If the load of bricks has a mass of 40 kg and the coefficient of friction is 0.6, calculate the force in the rope.

Solution

As the bricks are moving up the incline, friction acts down the incline. The applied force F_A in the rope equals the component of the weight down the incline plus the frictional force:

$$\begin{aligned} F_A &= F_W \sin\theta + F_f = F_W \sin\theta + \mu F_N \\ &= 40 \times 10 \times \sin 50° + 0.6 \times 40 \times 10 \times \cos 50° \\ &= 306 + 154 \\ &= 460 \text{ N} \end{aligned}$$

ROAD ACCIDENT INVESTIGATION 4.14

Traffic accident investigation is not just about examining the damage to cars and people; it also involves applying physics principles to determine how the accident happened. One of the key pieces of evidence comes from the tyre skid marks.

The speed of a vehicle prior to skidding to a halt can be deduced from the skid mark length and the coefficient of friction between the tyre and the road surface.

NOVEL CHALLENGE

With the automatic gearbox in 'drive' a Toyota 1200 kg RAV 4 will remain stationary facing uphill on a 5° slope. What would its initial acceleration be on the flat (assuming the driver's foot was not on the accelerator)? We got 0.87 m s^{-2}.

— Horizontal surface

Suppose a vehicle of mass m travels on a level road at a speed u prior to skidding. During the skid, the only horizontal force acting on the car is friction: $F_f = \mu F_N = \mu m g$. As this is the net force, the deceleration of the car is given by Newton's second law: $F_{net} = ma$ and is equal to F_f. Hence, combining the two equations gives us $\mu mg = ma$. This cancels down to: $a = \mu g$.

If we use our kinematic formula $v^2 = u^2 + 2as$, and assume the vehicle came to rest (i.e. $v = 0$), acceleration a is equal to $u^2/2s$, hence $u^2/2s = \mu g$, or $u = \sqrt{2\mu g s}$

Note that the deceleration is independent of the mass of the vehicle.

Example

Calculate the initial speed of a 1500 kg car that skidded 40 m to a halt on a level road where the coefficient of friction was 0.65.

Solution

$$u = \sqrt{2\mu \boldsymbol{g} s} = \sqrt{2 \times 0.65 \times 10 \times 40} = 22.8 \text{ m s}^{-1} \text{ (82.1 km h}^{-1}\text{)}$$

Different surfaces

When a car skids across different surfaces, the starting speed can be calculated using:

$$u = \sqrt{2\mu_1 \boldsymbol{g} s_1 + 2\mu_2 \boldsymbol{g} s_2 + 2\mu_3 \boldsymbol{g} s_3 + \ldots}$$

where μ_1 is the coefficient of friction on surface 1, s_1 is the length of this surface, and so on.

Example

A car skids with all four wheels locked and leaves skid marks of 19.3 m on dry bitumen, 5.6 m on concrete pavement and 15.4 m on grass. The μ values are 0.74, 0.82 and 0.46 respectively. Calculate the speed of the car at the start of the skidding.

Solution

$$u = \sqrt{2\mu_1 \boldsymbol{g} s_1 + 2\mu_2 \boldsymbol{g} s_2 + 2\mu_3 \boldsymbol{g} s_3 + \ldots}$$

$$u = \sqrt{2 \times 0.74 \times 10 \times 19.3 + 2 \times 0.82 \times 10 \times 5.6 + 2 \times 0.46 \times 10 \times 15.4}$$

$$u = 22.8 \text{ m s}^{-1} \text{ (82.0 km h}^{-1}\text{)}$$

Inclined surfaces

When the skid marks are on a road that has a slope of θ degrees, the simplest way to calculate the starting speed is to use an 'effective' value for μ:

- **up slope:** $\mu_{us} = \mu + \sin\theta$
- **down slope:** $\mu_{ds} = \mu - \sin\theta$

Questions

25 Calculate the skid-to-stop distance of a car travelling on a road at 100 km h^{-1} with a coefficient of friction of 0.68.

26 A car skidded to a stop, producing skid marks of 5.6 m on bitumen (μ = 0.61) and 3.2 m on concrete (μ = 0.79). Estimate the speed of the car prior to skidding.

27 A car skidded to a halt down a road of gradient of 13.8°, producing skid marks of 17.4 m. The coefficient of friction was 0.73. Calculate the speed of the car prior to skidding.

Activity 4.11 SPEED AND STOPPING DISTANCE

Does it take twice as long to stop a car if its speed is doubled? Let's find out.

A light gate attached to a computer timer is useful for this experiment. The TI graphing calculator and a CBL with a light gate work fine. You will need the 'Data Gate' program.

1. Glue a small piece of card 5 cm long to the top of a small toy car.
2. Construct a ramp and place the light gate at the bottom. (See Figure 4.36.)
3. Allow the car to run down the incline, through the light gate where its speed is measured, and let it run across the floor to a halt.
4. The light gate will measure the time interval for the 5 cm card to pass through. (Calculate $\boldsymbol{v}$ by dividing 10 cm by the time taken.)
5. Measure the distance the car moves before stopping.
6. Repeat with different height inclines. What did you conclude?
7. Plot speed (x-axis) against stopping distance.

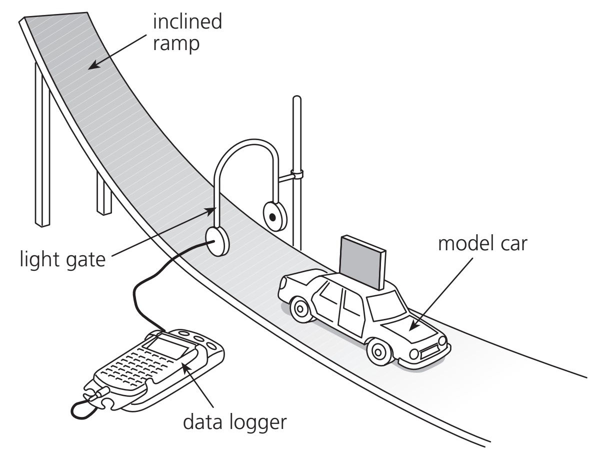


Figure 4.36 Using a data-logger to measure car speeds.

Questions

28 In Figure 4.37, the block weighs 50 N and the applied force $\boldsymbol{F}_A$ is 20 N. Calculate the normal force in each case.

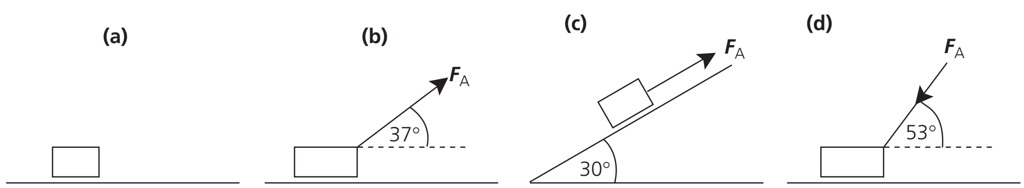


Figure 4.37 For question 28.

29 When a laboratory inclined plane is raised at one end, a small wooden block of mass 80 g slides down the slope at constant speed when the angle reaches 25°. Calculate the coefficient of sliding friction.

30 A block of mass m_1 = 3.70 kg rests on a 30° incline and is held stationary by a mass (m_2) of 1.3 kg hanging vertically over a frictionless pulley at the top (Figure 4.38). Calculate the value of μ.

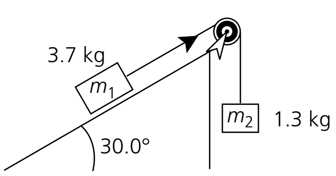


Figure 4.38 For question 30.

Practice questions

The relative difficulty of these questions is indicated by the number of stars beside each question number: * = low; ** = medium; *** = high.

Review — applying principles and problem solving

***31** Calculate the resultant force when the following forces act on the same object:
(a) 24 N north, 18 N south, 19 N north. **(b)** 6.5 N down, 9.2 N up and 7.4 N up.
(c) 55 N north, 35 N west, 65 N south. **(d)** 6 N west, 5 N east, 3 N north, 1.5 N south.

***32** The density of aluminium is 2.7 g cm^{-3}. Calculate the mass of a sheet of the metal 25 cm long by 10.0 cm wide and 3.0 mm thick.

***33** Which of Newton's laws of motion best describes the following situations?
(a) When a car accelerates the occupants feel pressed back into their seats.
(b) When you turn on the garden hose it moves around if it is not held.
(c) A nailfile, when thrown vertically, undergoes uniform deceleration.
(d) A sandwich, when dropped off a cliff, travels straight down.

***34** A handbag is sliding down a 30° incline at constant speed. Which one of the following relationships is false about the situation?
(a) $\boldsymbol{F}_W \sin 30° > \boldsymbol{F}_f$. **(b)** $\boldsymbol{F}_N = m\boldsymbol{g} \cos 30°$. **(c)** $\boldsymbol{F}_f = \mu\boldsymbol{F}_N$. **(d)** $\boldsymbol{F}_W = m\boldsymbol{g}$.

***35** What force is necessary to uniformly accelerate:
(a) a 6.4 kg mass at 2.4 m s^{-2} east; **(b)** a 0.16 kg mass from rest to 2 m s^{-1} in 3 seconds; **(c)** an object weighing 25 N at 9.8 m s^{-2}; **(d)** a 0.50 kg object from rest to 5.0 m s^{-1} over 4.0 metres; **(e)** a 75 kg object from 40 m s^{-1} to 60 m s^{-1} in 5 milliseconds?

***36** A wooden box of bolts has a mass of 250 kg and requires a horizontal force of 2100 N to slide it along a horizontal wooden surface at a constant speed.
(a) Calculate the coefficient of friction.
(b) If the box were to be kept moving constantly at twice this speed what force would be needed to maintain this constant speed?

****37** In a TV tube an electron experiences an unbalanced force of 8.0 femtonewtons over a distance of 20 mm. (Look in the Appendix for the mass of an electron and the meaning of the prefix 'femto'.)
(a) Calculate the electron's acceleration.
(b) Calculate the electron's speed at the end of the 20 mm (starting from rest).

****38** The graph shown in Figure 4.39 is an acceleration–force graph for an experiment with a loaded cart pulled by rubber bands:
(a) What does the intercept of the graph with the force axis measure?
(b) What acceleration would an applied 2.5 N force produce?
(c) What net force would produce an acceleration of 2.0 m s^{-2} and what applied force is this equal to?
(d) What is the mass of the loaded cart?

****39** A bicycle and rider have a combined mass of 65 kg. When travelling at 5.0 m s^{-1} on a level road, the cyclist ceases to pedal and comes to rest in 255 m. What frictional forces must have been acting on the cyclist?

***40** Consult a table of densities to find out:
(a) which, if any, metals are lighter than water, and list their densities;
(b) whether any liquids are heavier than mercury at room temperature.

***41** You are standing on the edge of a frozen pond where friction is negligible. In the centre is a blue circle 1.0 m in diameter. There is a prize of 4.0 L of icecream if anyone can apply all three of Newton's laws of motion to get there. How could you do it?

****42** If friction is independent of surface area, why is it that racing car drivers use wide tyres to improve the grip?

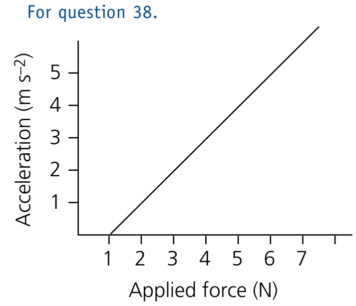
TEST YOUR UNDERSTANDING

When the upward force of air resistance on a parachutist equals his weight (down), shouldn't he be stationary? Explain.

NOVEL CHALLENGE

In 1999, a 19-year-old Gold Coast man tried to stop a car that was starting to roll down a driveway slope. He didn't succeed and was run over. Why couldn't he stop the car — after all, it was only on a 10° slope? How much force can you push with? Try this. Have someone hold a set of bathroom scales against the wall at about hip height while you push the scales with your hands as hard as you can. You'll probably only push to a scale reading of 40 kg (400 N). Calculate the maximum angle of an incline that you could stop a 1500 kg car from rolling down. Surprising, huh?

Figure 4.39
For question 38.



TEST YOUR UNDERSTANDING

(Answer true or false)
- Forces are needed for motion with constant velocity.
- Objects stop moving when the force is removed.
- Inertia is the force that keeps things in motion.
- The normal force on an object always equals the weight.
- Objects thrown in the air start to fall when they run out of force.

NOVEL CHALLENGE

I live at the top of a road that has a 5° downhill slope. When I let my car roll down the slope it reaches 25 km h^{-1} by the time it gets to the bottom, 400 m away. What frictional force must be acting?

**43 Is it better to have high friction between a car tyre and the ground to get good grip or better to have low friction to reduce drag forces? Examine critically.

**44 During an experiment, a linear air track glider was subjected to a single force whose magnitude could be varied. Assume the friction was negligible. The acceleration from various forces was measured and the results tabled as shown below:

Force (N)	5	10	15	20	25
Acceleration (m s^{-2})	1.5	3.0	4.5	6.0	7.5

(a) Draw a graph.
(b) What is the mass of the trolley?
(c) If friction was present how would the shape of the graph have changed?

**45 An experiment was conducted to find the relationship between force, mass and acceleration. A stretched rubber band was used to provide constant force on a trolley to which different masses were added (see Figure 4.40). The trolley was released from rest and timed to move 50.0 cm. A student made the following notes:

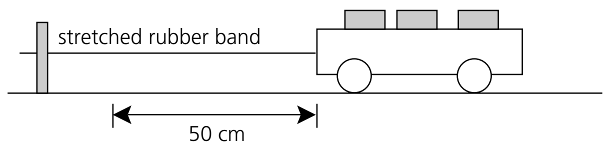


Figure 4.40
For question 45.

'When the trolley had no masses on it, it took 2.00 s. With 300 g added it took 2.39 s and with 900 g added it took 3.02 s. I measured the mass of the trolley by itself and it was 700 g.'

(a) Draw up a data table to show the results.
(b) Calculate the acceleration for each trial and add this to the table.
(c) Draw a graph of total mass (x-axis) versus acceleration.
(d) What relationship is suggested by this graph?
(e) Use Newton's second law of motion formula to calculate the force provided by the rubber band in each case and add to the table.
(f) The student was supposed to have measured the time with 600 g added but forgot. **(i)** What acceleration would the student have calculated? **(ii)** How long would the trolley have taken to cover the 50 cm in this case?
(g) Name one factor that would have had to remain constant during the experiment.

NOVEL CHALLENGE

A lawyer emailed me (RW) wanting to know what was meant when someone was 'in an accident and suffered high-g forces'. I told him in a few paragraphs. How would you explain it? By the way, he never offered to pay, but I bet he charged his client. Hmmm!

**46 Figure 4.41 has been taken from *Scientific American*, December 1994. It shows the fuel needs of the various stages of a car's propulsion system. (One US gallon equals 4.0 L.)

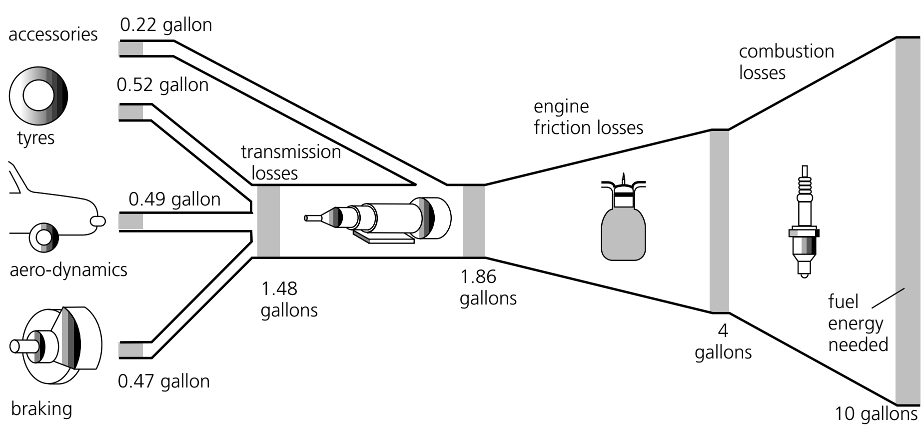
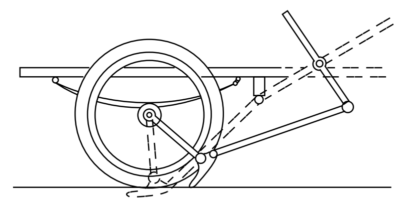


Figure 4.41
Energy losses in a typical automobile.

(a) Make a table to show the percentage of fuel use for each stage.
(b) State which stage the following modifications would affect and say whether it would increase or decrease fuel consumption: **(i)** PTFE oil additive; **(ii)** proper engine tuning; **(iii)** low gearbox oil level; **(iv)** glovebox light that stayed on all the time; **(v)** sleeker body shape (of car, not driver); **(vi)** out-of-round disk brake rotors; **(vii)** hood rack and surfboard.
(c) If two cars were identical except that one had a six cylinder engine and the other a four cylinder engine (like the two VB Commodore models of the early 1980s), which stage would be affected (if any)?

****47** A British engineer, Mr Ralph Jackson, was awarded Patent No. 858 in 1901 for his 'Brake for Wheeled Vehicles' (Figure 4.42).
In his patent application he said that 'by a system of levers, the wheel may be raised from contact with the ground'. Explain the physical principles of his device and some good and bad points about it.

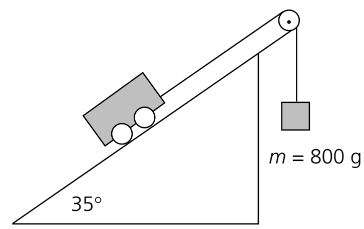
Figure 4.42
Brake for wheeled vehicles.



Extension — complex, challenging and novel

*****48** Consider the system shown in Figure 4.43. The trolley has a mass of 1000 g and is stationary when placed on a slope of 35° under the conditions shown.
(a) Determine the frictional forces acting in this system. **(b)** Calculate μ.

Figure 4.43
For question 48.



*****49** A sphere of mass 0.3 g is suspended from a 30.0 cm cord. A steady horizontal breeze pushes the sphere so that it makes an angle of 37° with the vertical. Find the magnitude of the wind force and the tension in the cord.

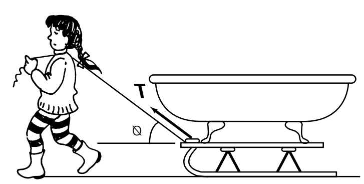
*****50** A cable used to pull mine cars vertically to the pit head has a breaking strain of 3×10^4 N. If the mine shaft is 500 m deep and a full mine car has a mass of 2.5 t, calculate: **(a)** the maximum acceleration the car can attain without breaking the cable; **(b)** the shortest time in which the car can be pulled from rest to the surface in the event of an accident.

*****51** A toboggan of mass 1000 kg starts to move down a 30° slope at an amusement park. In addition to the friction between the runners and the track (μ = 0.2) there is air resistance that has been shown to be equal to 500 N + 80 × (number of people in the toboggan) N. Which would get to the bottom of the slope first — a toboggan with one person of mass 60 kg or a toboggan with two passengers, each of mass 60 kg? Show your working.

*****52** If a car's wheels are 'locked' during emergency braking, the car slides along the road leaving bits of ripped-off tyre and small melted sections of road from the skid marks that reveal the cold-welding during the slide. The record for the longest skid marks on a public road was set in 1960 by a Jaguar on the M1 in England. The marks were 290 m long. Assuming that the coefficient of friction was 0.60, how fast was the car going when the wheels were locked?

*****53** A woman pulls a sled carrying a bath tub along a horizontal surface at constant speed (Figure 4.44). If the mass of sled and bathtub was 75 kg and μ was 0.10 and the angle θ as shown was 42°, calculate the tension in the rope.

Figure 4.44
For question 53.



*****54** A 1500 kg sled is coasting at 20 m s^{-1} on ice where friction is negligible. Suddenly it hits a 22.0 m long rough patch used for ice cricket, which creates a frictional force of 6×10^3 N. With what velocity does the sled leave the end of the rough patch?

*****55** A falling cat reaches a terminal speed of 60 km h^{-1} while it has its legs and head tucked in. When it stretches out its cross sectional area (A) doubles. Calculate its new terminal speed. Refer to Activity 4.5 (page 89) for the appropriate formula.

*****56** An electron is projected horizontally at a speed of 1.2 megametres per second into an electric field that exerts a vertical force of 450 attonewtons on it. The mass of an electron is 9.11×10^{-31} kg. Determine the vertical distance the electron is deflected during the time it has moved 30 mm horizontally. Atto (a) = 10^{-18}.

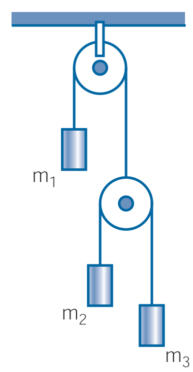
*****57** A crate of tiles of mass m_1 = 14 kg moves up a 30° incline at constant speed when pulled by a crate of cement of equal mass. The crates are connected by a taut, massless cord over a frictionless, massless pulley. Calculate the frictional force and the value of μ.

*****58** A van skidded to a halt up a road which had a slope of 7° and ran into the back of a parked car. The van produced skid marks of 13.7 m on a surface with a μ of 0.71. From the damage to the parked car, police estimated that the van's impact speed was 25 km h^{-1}. Estimate the speed prior to skidding.

*****59** In an accident, a car skidded 9.6 m over bitumen (μ = 0.66) and 2.6 m on concrete (μ = 0.76) before smashing into a fire hydrant. Police estimated the crash speed to be 30 km h^{-1}. Estimate the speed of the car prior to skidding.

*****60** A mass m_1 hangs over a frictionless pulley and attached to the other end is another frictionless pulley with masses m_2 and m_3 arranged as shown in Figure 4.45. Calculate the acceleration of the three masses and the tension in the strings.

Figure 4.45



*****61** A man is hauling a box of mass 100 kg up a 35° incline by a rope attached to the top of the box (Figure 4.46). If the rope makes an angle of 20° to the incline and the coefficient of friction between the box and the incline is 0.65, calculate the force applied by the man to keep the box moving at constant speed.

Figure 4.46