UNIT 03

Energy & Momentum

CHAPTER 08

Momentum

8.1 EXPLOSIONS, COLLISIONS AND BALLET DANCING

Ballet, bullets, bombs, baseball, boxing and binary stars have something in common. They all involve the combination of mass and velocity. They all involve **momentum**.

These are some questions that a study of momentum can help answer:

- Would you rather be hit by a 1 g ball-bearing travelling at 100 m s^{-1} or by a 100 g ball travelling at 1 m s^{-1}?
- Which would hurt more — being tackled by a lightweight footballer travelling at high speed or by a big fat one travelling at low speed?
- Police bullets are designed to stay inside their targets and not go through them. How?
- You are standing in the middle of a frictionless frozen pond. Someone with a little knowledge of physics says that you can't get to the edge because of the laws of momentum. What can you do to prove him wrong?
- A cat that falls out of a window upside-down can right itself and land on its feet. How can this be if it has nothing to push against while falling? What's the advantage of landing on its feet anyway?
- When you shoot a bullet at a watermelon suspended on a string, the melon moves towards you. How can this be and what has it to do with the assassination of JFK?

8.2 CENTRE OF MASS

Physicists love to look at something complicated and find in it something simple and familiar. If you toss a cricket bat into the air its motion as it turns is more complicated than that of a cricket ball. A diver who executes a somersault has an even more complicated motion still. Every part of the body moves in a different way from every other part, so you cannot represent the body as a single particle as you can with a ball. However, if you look closely, you will find that there is one special point that moves in a simple path — a parabola — much as the ball does. This point is called the **centre of mass**. It is the point at which the whole mass of an object is considered to be concentrated for the purpose of applying the laws of motion.

EI Activity 8.1 CENTRE OF MASS

You can locate the centre of mass of a bat by finding the point at which it balances on an outstretched finger.

1 If you can get hold of some different bats or racquets, find the balance point of each and mark it with a felt pen. Draw diagrams to show the location.

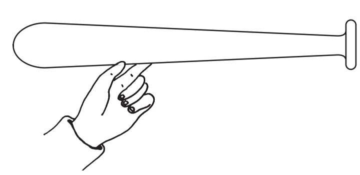
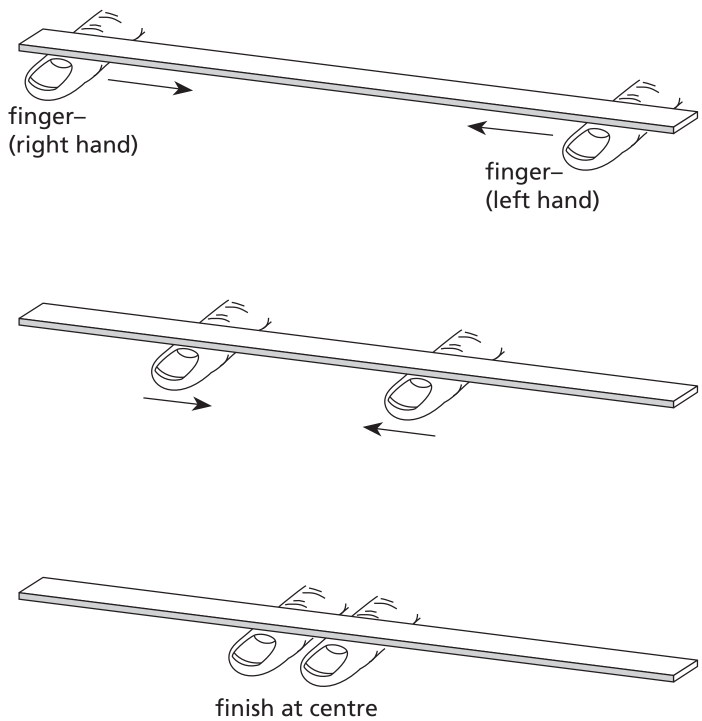


Figure 8.1
The balance point of a baseball bat.

2 How does the centre of mass compare with the centre of percussion as discovered in Chapter 6? (Recall letting the bat swing like a pendulum and finding its effective length.)

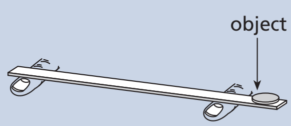
3 Hold a ruler horizontally on the outstretched index fingers of both hands as in Figure 8.2. Slowly bring your fingers in from the ends of the ruler and note where they end up. Does it matter where you start your fingers from? Why?

Figure 8.2
Where do your fingers meet?



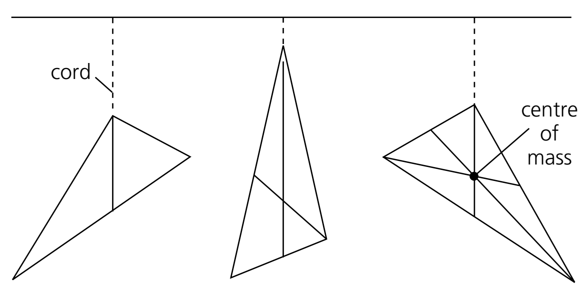
NOVEL CHALLENGE

Imagine you place a finger under each end of a ruler and a coin is placed on one end. You pull away both fingers and the ruler and coin fall together staying in contact. But if you just pull away the finger under the coin something odd happens. *What and why? Try it.*



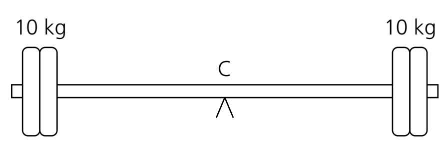
For regular shaped objects like a metre ruler, the centre of mass is at the midpoint. That's why you pick up a plank of wood in the middle. For irregular objects, though, the centre of mass can be found by letting the object hang from a pivot hole or point and drawing a vertical mark on the object. When this is done several times, the point at which the lines cross marks the centre of mass.

Figure 8.3
Locating the centre of mass.



For more regular rigid bodies, some simple principles can be used to find the centre of mass mathematically. Consider a set of weights on the ends of a steel bar (Figure 8.4).

Figure 8.4



Intuitively, the centre of mass is at the centre of the bar. In this case the products of each weight and its distance from the pivot point are equal. When the masses are unequal, obviously the centre of mass is closer to the heavier mass. Children use the ideas of centre of mass when operating a seesaw. The seesaw has a fixed pivot point. If two children of very different mass get on, the heavier child has to sit closer to the pivot point. This positions the centre of mass of the system of two children at the pivot point.

This suggests an inverse relationship between the weights or masses of the children and their distance from the centre of mass:

$$\frac{\boldsymbol{F}_{w1}}{\boldsymbol{F}_{w2}} = \frac{s_2}{s_1}$$

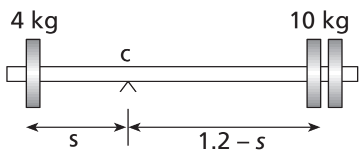
or, in general:

$$\boldsymbol{F}_{w1} \times s_1 = \boldsymbol{F}_{w2} \times s_2$$

Example

Masses of 4 kg and 10 kg are on the ends of a 1.2 m long bar as shown in Figure 8.5. Determine the centre of mass of the system.

Figure 8.5



Solution

Point C is located s metres from the 4 kg mass and (1.2 – s) metres from the 10 kg mass. For the bar to balance:

$$\begin{aligned} \boldsymbol{F}_1 \times s_1 &= \boldsymbol{F}_2 \times s_2 \\ 40 \times s &= 100(1.2 - s) \quad \text{using } \boldsymbol{g} = 10 \text{ m s}^{-2} \\ s &= 0.86 \text{ m} \end{aligned}$$

Later in this chapter, the product of $\boldsymbol{F} \times s$ in similar situations will be defined as **torque** and discussed in detail as it applies to rotating bodies.

Motion of the centre of mass

Knowledge of the motion and properties of the centre of mass gives some good insights into everyday phenomena.

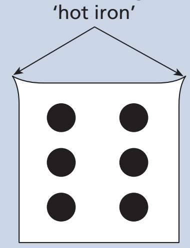
The grand jeté

When you do a long jump, chances are that your body will follow a parabolic path like a baseball thrown in from the outfield. But when a skilled ballet dancer does a split leap across the stage in a grand jeté, the path taken by her head and torso is nearly horizontal during much of the jump. She seems to be floating across the stage. The audience may not know Newton's laws of motion, but they can always sense that something magical has happened. The secret is that she raises her arms and legs as she jumps upward. These actions shift her centre of mass upward through her body. Although the shifting centre of mass faithfully follows a parabolic path across the stage, its movement relative to the body decreases the height that would be attained by her head in a normal jump. The result is that the head and torso follow a nearly horizontal path.

NOVEL CHALLENGE

There are several types of 'crooked' dice used by cheats. For each one described, deduce why they are crooked:

1 Green's Load (1880) — two spots drilled out and mercury added.
2 Tapping dice — hollow centre filled with mercury but with a small tube to one corner. Tap to make them crooked.
3 Bevelled — rounded on some edges.
4 Slick — one surface highly polished.
5 Hot iron — a ridge along one edge.
6 Capped — one face capped with rubber.

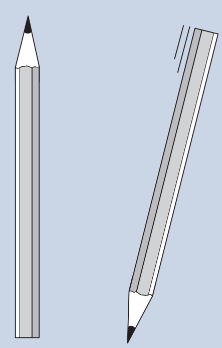


NOVEL CHALLENGE

It is easy to stand a pencil up on its base but impossible to stand it up on its point.

But why? What if you could put it in a sealed container free of air currents and arranged it so that its centre of mass was exactly over the point — could you do it then? Still no!

But what is the physics behind the failure?



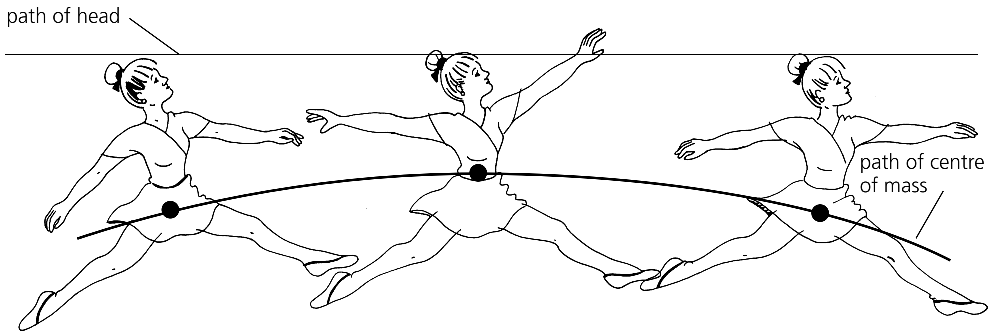


Figure 8.6
A grand jeté.

The Fosbury flop

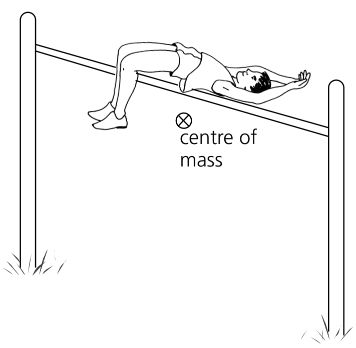


Figure 8.7
In a Fosbury flop, the centre of mass may actually pass under the bar.

The most successful high jumpers are tall, long-legged athletes because their centre of mass is further off the ground and so does not have to be lifted as far as that of a shorter jumper when clearing the bar. The high jumper must try to adopt a body position at take-off that keeps the centre of mass as high as possible. The momentum acquired during the run-up is modified in the last two steps before take-off. The jumper sinks down on the second last step and then comes erect on the take-off step so that the body has an initial upward velocity. The time that the jumper's foot is in contact with the ground on this last step is called the take-off time and is of the order of 0.12 to 0.17 s. The jumper has to also rotate so that the body is horizontal as it goes over the bar. This is the Fosbury flop, a technique popularised by American high jumper Dick Fosbury who developed the style and used it to win the 1968 Olympic gold medal. The technique has the advantage that the centre of mass passes under the bar even though the jumper curls over the top. For example, if the high jump bar was set at 2.07 m it is estimated that the jumper's centre of mass was only lifted from 1.27 m at take-off to 1.95 m when clearing the bar. Jumpers using the older scissor jump or western-roll style would have had to jump at least an extra 12 cm to clear the bar.

Questions

1 A 2.5 kg mass and a 4 kg mass are placed 1.5 m apart. Where is their centre of mass?

2 Where is the centre of mass of the Earth (mass 6×10^{24} kg) and the Moon (mass 7.4×10^{22} kg) when they are 3.8×10^{8} m apart?

MOMENTUM 8.3

'Momentum' is another term like 'velocity' that gets used in newspapers and magazines in strange ways. Journalists write that 'protests against whaling are gaining momentum' or about a truckies' blockade having a momentum of its own. However, what physicists mean by momentum is to do with mass and velocity.

The product of mass and velocity is called **momentum** (Latin *momentum* = 'movement'). It is a useful quantity to describe the motion of an object.

$$\text{Momentum} = \text{mass} \times \text{velocity}$$
$$\mathbf{p} = m\mathbf{v}$$

Momentum is a vector quantity, being the product of a scalar (mass) and a vector (velocity). The unit of momentum does not have a special name. The unit is kg m s^{-1} and is the same as N s, and this is often used. **The direction of momentum is the same as the direction of velocity.**

Example 1

Calculate the momentum of a 2 kg bowling ball travelling at 8 m s^{-1} south.

Solution

$$\boldsymbol{p} = m\boldsymbol{v} = 2 \times 8 = 16 \text{ kg m s}^{-1} \text{ south}$$

Example 2

A proton of mass 1.67×10^{-27} kg is accelerated from 3×10^4 m s^{-1} north to 3×10^5 m s^{-1} north. Calculate the **change** in momentum.

Solution

- Change in momentum = final momentum − initial momentum.

$$\begin{aligned}\Delta \boldsymbol{p} &= \boldsymbol{p}_\mathrm{f} - \boldsymbol{p}_\mathrm{i} \\ &= m\boldsymbol{v} - m\boldsymbol{u}\end{aligned}$$

- Final momentum = $m\boldsymbol{v} = 1.67 \times 10^{-27} \times 3 \times 10^5 = 5.0 \times 10^{-22}$ kg m s^{-1} north.
- Initial momentum = $m\boldsymbol{u} = 1.67 \times 10^{-27} \times 3 \times 10^4 = 5.0 \times 10^{-23}$ kg m s^{-1} north.
- Change in momentum = 5.0×10^{-22} kg m s^{-1} − 5.0×10^{-23} kg m s^{-1}
  = 4.5×10^{-22} kg m s^{-1} north.

Questions

3 Calculate the magnitude of the momentum of the following moving objects:
(a) a 1000 g bowling ball moving at 1.6 m s^{-1}; **(b)** a 2.0 t car moving at 15 m s^{-1};
(c) the Earth in its journey around the Sun. The Earth's mass is 6×10^{24} kg and its radius of orbit is 1.5×10^{11} m.

4 A cricket ball of mass 200 g travelling at 20 m s^{-1} east is struck directly back at 30 m s^{-1} west. Calculate the change in momentum. *Hint:* the change in velocity is *not* 10 m s^{-1}. Can you see why?

8.4 ACTING ON IMPULSE

In 1687 Isaac Newton wrote that the force on an object determined the 'rate of change of the quantity of motion'. He expressed this in his second law of motion, which can be written as $\boldsymbol{F} = m\boldsymbol{a}$. But it can also be expressed in terms of momentum.

The acceleration can be replaced by $\frac{\boldsymbol{v} - \boldsymbol{u}}{t}$ to give:

$$\boldsymbol{F} = \frac{m(\boldsymbol{v} - \boldsymbol{u})}{t} = \frac{m\boldsymbol{v} - m\boldsymbol{u}}{t} = \frac{\text{change in momentum}}{\text{time}}$$

Hence the rate of change of momentum is equal to the external force causing the change. This can be rearranged as:

$$\boldsymbol{F}t = m\boldsymbol{v} - m\boldsymbol{u} \quad \text{or} \quad \boldsymbol{F}t = \Delta\boldsymbol{p}$$

The product $\boldsymbol{F}t$ is called the **impulse** (Latin *pulsus* = 'to beat' or 'drive'). Impulse depends on the size of the force and for how long it is applied. It is also equal to the **change** in momentum. The unit for impulse is Newton second (N s), which is the same as kg m s^{-1}. Some books use the symbol I for impulse but we will leave it as $\boldsymbol{F}t$.

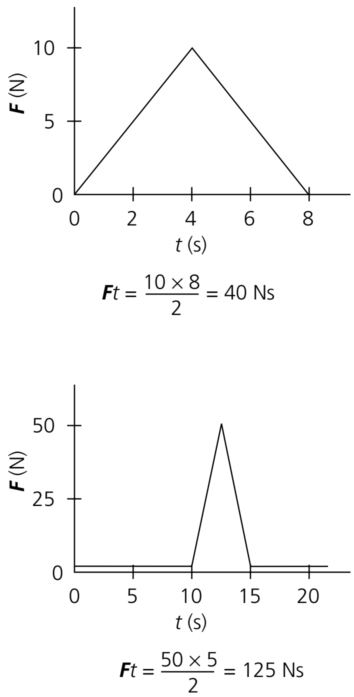
NOVEL CHALLENGE

Can you jump off a chair onto the floor while holding a cup full of water without spilling any? Plan how you should land to do this. Hmmm! It sounds good in theory but …

NOVEL CHALLENGE

Imagine you are standing on some bathroom scales and you bend your knees quickly. *Predict what will happen to the scale reading. But why?*

Figure 8.8
Impulse is the area under an F/t graph.



Force–time graphs

Impulse is also equal to the area under the graph (see Figure 8.8).

Most impacts involve forces that do not remain constant.

Example

A graph showing how force varies with time as a stationary 57 g ball is struck by a racquet is shown in Figure 8.9.

Calculate **(a)** the impulse; **(b)** the final velocity of the ball.

Solution

(a) The area under the graph is a measure of $\mathbf{F} \times t$, that is, impulse. In this case the impulse is approximated by the dotted triangle:

$$A = \frac{b \times h}{2} = \frac{3.2 \times 10^{-3} \times 2.5 \times 10^{3}}{2} = 4.0 \text{ N s (south)}$$

(b)

$$\mathbf{F}t = m(\mathbf{v} - \mathbf{u})$$
$$4 = 0.057(\mathbf{v} - 0)$$
$$\mathbf{v} = 70 \text{ m s}^{-1} \text{ south.}$$

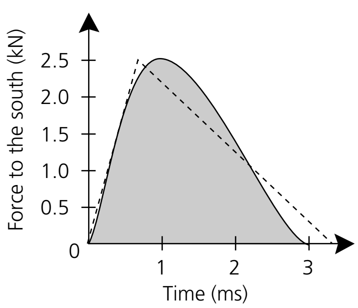
Questions

5 For how long must a frictional force of 5.6 N act in order to bring to rest a mass of 2.4 kg moving at 6.0 m s^{-1} north?

6 A car of mass 1200 kg accelerates at 5.5 m s^{-2} for 12 s. Determine the impulse imparted to the car.

CONSERVATION OF LINEAR MOMENTUM 8.5

Figure 8.9
The force exerted on a tennis ball during a serve can be represented graphically.

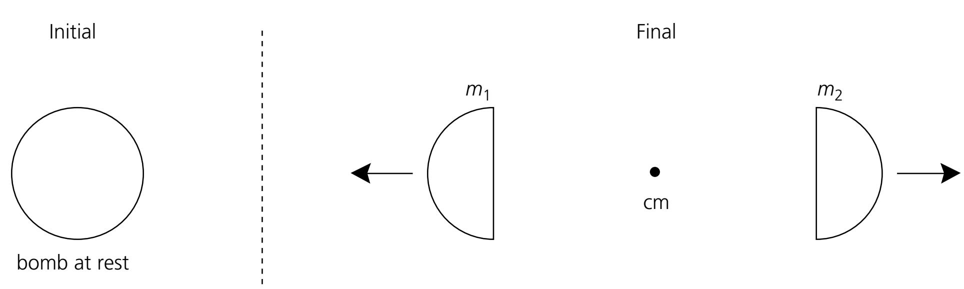


When you throw a ball, shoot a bullet or give someone a push you tend to move backward. Newton's third law of motion explained that action and reaction were equal and opposite forces. A study of momentum can describe the motion of interacting bodies mathematically. The two most common interactions we can study are **explosions** and **collisions**. We'll start with explosions because they are a bit simpler.

Explosions

An explosion can be thought of as a single object separating into two or more fragments. The word 'explode' was first used to mean 'burst with destructive force' in the nineteenth century when a mathematical treatment of explosions became necessary. Prior to that, the Latin verb *explodere* meant 'to drive off the theatre stage with hisses, boos, loud noises and claps'. It came from *ex-* meaning 'out' and *plaudere* meaning 'clap'. Many scientific words started off meaning something else.

Figure 8.10



Consider a 10 kg bomb at rest that explodes into two fragments (Figure 8.10). If a 4 kg piece (m_1) travels west at 15 m s^{-1} ($\boldsymbol{v}_1$), then the 6 kg piece (m_2) would have moved in the opposite direction (at a speed $\boldsymbol{v}_2$). As there was no external unbalanced forces acting on the bomb (all forces were internal), we have a closed system and there would be no change in the total momentum of the system. This is called the **law of conservation of momentum. In a closed system, the change in momentum is zero**. That is,

$$\begin{aligned}\Delta \boldsymbol{p} = 0 \text{ or } \boldsymbol{p}_{\text{initial}} &= \boldsymbol{p}_{\text{final}}\\ (m_1 + m_2)\boldsymbol{u} &= m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2\\ 10 \times 0 &= 4 \times 15 + 6 \times \boldsymbol{v}_2\\ \boldsymbol{v}_2 &= -10 \text{ m s}^{-1} \text{ (the negative sign means east)}\end{aligned}$$

Example

A boy on rollerskates is travelling along at 8 m s^{-1}. He has a mass of 60 kg and is carrying his school bag of mass 10 kg. He throws the bag directly forward at 20 m s^{-1} relative to the ground. Calculate the boy's speed after the 'explosion'.

Solution

The boy and the bag have initial velocities in the positive direction. The final velocity of the bag is also positive.

$$\begin{aligned}\boldsymbol{p}_{\text{initial}} &= \boldsymbol{p}_{\text{final}}\\ (m_1 + m_2)\boldsymbol{u} &= m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2\\ (60 + 10) \times 8 &= 60 \times \boldsymbol{v}_1 + 10 \times 20\\ 560 &= 60 \times \boldsymbol{v}_1 + 200\\ \boldsymbol{v}_1 &= 6 \text{ m s}^{-1}\end{aligned}$$

The positive direction means that the boy would continue to move forward.

Relationships such as this can be applied to all sorts of explosions — a cannon or rifle being fired, a bomb exploding, a heart pumping a pulse of blood, a hose squirting water and even a nucleus giving off radioactive particles.

Cases in which the bodies explode in a straight line are not that common, however. Explosions in two dimensions will be dealt with later.

Questions

7 Two children at rest push off from each other in a swimming pool. One with a mass of 50 kg moves east at 1.5 m s^{-1} and the other who has a mass of 45 kg moves to the west. What is the second child's velocity?

8 A girl of mass 50 kg is stationary on an ice rink. She throws a 1.0 kg parcel horizontally at 5.0 m s^{-1}. At what velocity does the girl move?

Collisions

In everyday language, a collision occurs when objects crash into each other. Although we will refine that definition, it conveys the meaning well enough. Some familiar collisions are:

- the Creek meteorite crater in Australia
- a billiard ball being struck by a cue
- a boxer punching a body bag
- hammering a nail into a piece of wood
- gas molecules bouncing off each other.

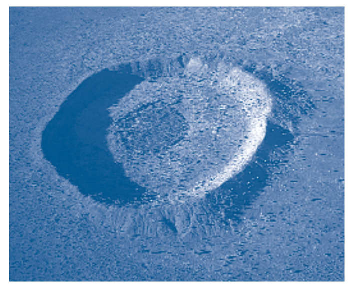
Car collisions

The safety of the passengers in a car during a collision also depends on the time interval in which the moving car is brought to a stop. To reduce the force of the impact, we have to

NOVEL CHALLENGE

A very lightweight boat 24 m long and mass of 30 kg lies still on a quiet pond. A 90 kg man walks from bow (front) to stern. *How far does the boat move relative to the pond? The answer is not 72 m.*

Photo 8.1
Meteorite crater.



NOVEL CHALLENGE

Shooters who want to reduce the recoil of their rifles use a variety of anti-recoil devices. The simplest is to vent the exhaust gases out sideways instead of leaving them trapped in the barrel. One effective method involves drilling a hole in the rifle butt (the wooden shoulder piece) and inserting a rod of steel about 2 cm in diameter. Better still, inventive shooters use a length of steel water pipe $\frac{3}{4}$ filled with mercury and capped. So how does this help?

increase the time it takes. Manufacturers do this by making the front and rear of cars collapsible. These 'crumple zones' must be neither too hard nor too soft. They must progressively collapse so that the time of the collision is made as long as possible. Other safety features in a car are:

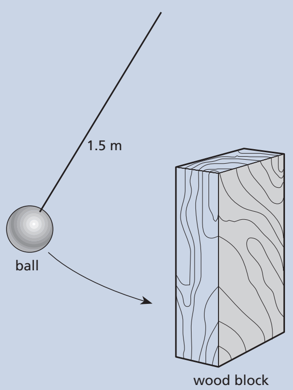
- air bags
- safety belts: inertia reel and self-tensioning
- antilocking brakes
- impact-absorbing bumper bars
- a collapsible steering column
- a rigid cabin compartment
- a soft dashboard instead of metal or wood.

Most of these are based on the principle that the longer it takes for your body to come to rest, the smaller the force your body has to stand. While the change in momentum is usually the same no matter how you crash, it is better to suffer a small force for a long time than a large force for a short time.

Does your family own a large four-wheel drive vehicle? It might be interesting to apply your knowledge of physics to the comparative safety of these vehicles in a collision, given the fact that they are very solid and rigid with only limited crumple zones.

NOVEL CHALLENGE

A superball is tied to a 1.5 m string and suspended vertically from a hook. It is pulled back and allowed to strike a wooden block standing on the floor. The experiment is repeated with a lump of plasticine of the same mass as the ball. One knocks the block over, one doesn't. *Which is which and why?*



Types of collisions

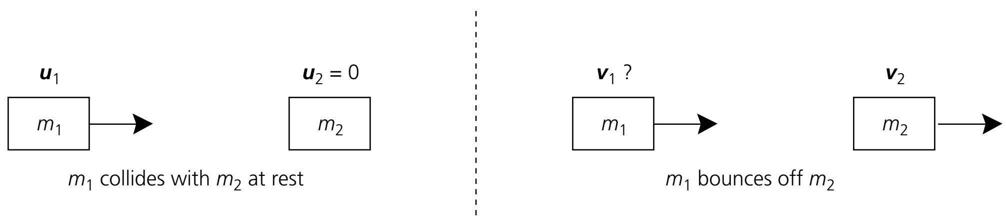
Collisions can be grouped into two types:

- Rebound, where objects bounce off each other (e.g. gas molecules or billiard balls).
- Coupled, where objects remain locked together (e.g. a bullet in a target).

Rebound

Consider a collision between two masses m_1 and m_2 with initial velocities $\boldsymbol{u}_1$ and $\boldsymbol{u}_2$ respectively.

Figure 8.11



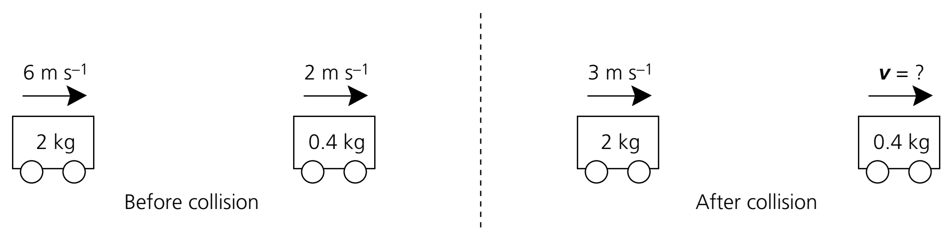
For the law of conservation of momentum to hold, the initial momentum must equal the final momentum:

$$m_1\boldsymbol{u}_1 + m_2\boldsymbol{u}_2 = m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2$$

Example

A cart with a mass of 2 kg travelling at 6 m s^{-1} collides with another cart of mass 0.4 kg travelling in the same direction at 2 m s^{-1}. It bounces off as shown in Figure 8.12. After impact, the 2 kg cart travels at 3 m s^{-1} in the same direction. Calculate the velocity of the 0.4 kg trolley after the collision.

Figure 8.12



Solution

$$m_1\boldsymbol{u}_1 + m_2\boldsymbol{u}_2 = m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2$$
$$2 \times 6 + 0.4 \times 2 = 2 \times 3 + 0.4 \times \boldsymbol{v}_2$$
$$\boldsymbol{v}_2 = +17 \text{ m s}^{-1}$$

(the positive sign indicates that the trolley is moving in the same direction as before).

Coupled or sticking together

When objects stick together or are joined together they are said to be coupled (Latin *copula* = 'to bond'). In a collision where the objects become coupled, the law of conservation of momentum still holds but the mass of the combined body after the collision is equal to the sum of the individual masses of the colliding bodies.

Some examples of coupled collisions are:

- an arrow sticking into its target
- two cars colliding head-on.

Example

A supermarket trolley loaded with shopping has a mass of 60 kg. It rolls across the floor at 4 m s^{-1} and collides with an empty trolley of mass 25 kg, which was stationary. They become fastened together and roll on as one. Calculate the velocity of the two trolleys when locked together.

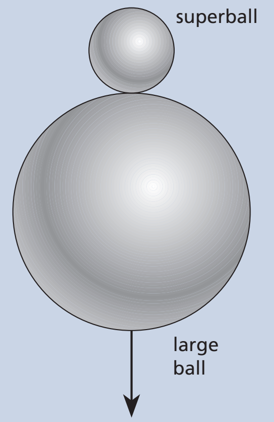
Solution

$$m_1\boldsymbol{u}_1 + m_2\boldsymbol{u}_2 = m_1\boldsymbol{v}_1 + m_2\boldsymbol{v}_2$$
$$m_1\boldsymbol{u}_1 + m_2\boldsymbol{u}_2 = (m_1 + m_2)\boldsymbol{v} \quad (\text{as } \boldsymbol{v}_1 = \boldsymbol{v}_2)$$
$$60 \times 4 + 25 \times 0 = 85 \times \boldsymbol{v}$$
$$\boldsymbol{v} = 2.8 \text{ m s}^{-1}$$

NOVEL CHALLENGE

A superball is placed on top of a tennis ball and they are dropped together.

Predict what happened — wow, what a funny rebound — and why?



Practical use of coupled collisions

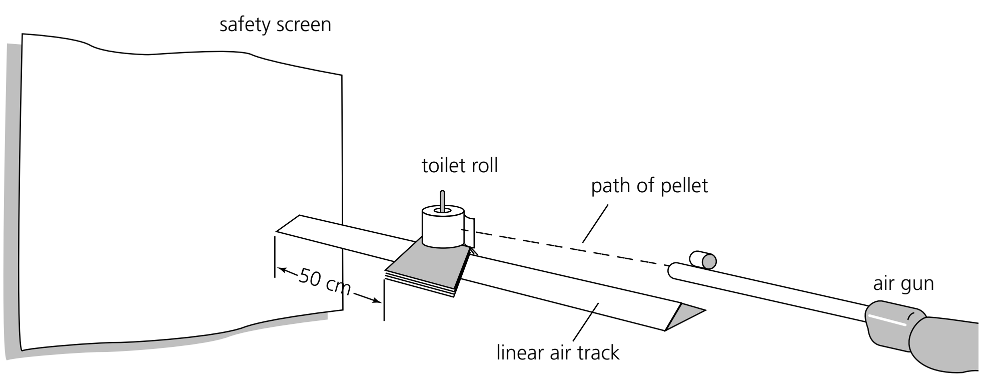


Figure 8.13
Measuring the velocity of an air-rifle pellet in the laboratory.

One way of measuring bullet speeds is to make use of a coupled collision. If an air-rifle pellet is shot into a soft absorbent target (such as a toilet roll) that is attached to a linear air-track glider, the glider moves away under the impact of the pellet. By measuring the time it takes the glider to move say 50 cm, the velocity of the glider can be determined. These data can be used to calculate the velocity of the pellet.

Example

When a 0.45 g air-rifle pellet is fired into a target attached to a glider on a linear air track, the glider moves 50 cm in 3.8 seconds. Calculate the velocity of the pellet. The glider and target have a combined mass of 643 g.

NOVEL CHALLENGE

A fly crashes into the front windscreen of a train and reverses its direction. Therefore, at one instant its velocity is zero but as it is squashed onto the window, the window's velocity must also be zero for a short time.
How could a fly stop a speeding locomotive?

Solution

- Mass of pellet = 0.45 g = 0.000 45 kg.
- Velocity of glider = $\frac{s}{t} = \frac{0.5}{3.8} = 0.13$ m s−1.

$$\boldsymbol{p}_{\mathrm{i}} = \boldsymbol{p}_{\mathrm{f}}$$

$$m_{\text{pellet}} \times \boldsymbol{u}_{\text{pellet}} = m_{\text{target}} \times \boldsymbol{v}_{\text{target}}$$

$$\boldsymbol{u}_{\text{pellet}} = \frac{m_{\text{target}} \times \boldsymbol{v}_{\text{target}}}{m_{\text{pellet}}} = \frac{0.643 \times 0.13}{0.000\,45}$$

$$= 186 \text{ m s}^{-1}$$

Note: the final mass of the glider and target should include the mass of the embedded pellet but as it is negligible it can be ignored in this case. Of course, if the mass of the embedded object was large then it would have to be included.

In the next chapter a device called a ballistic pendulum, used for measuring the speed of high-speed bullets, will be described.

Questions

9 An object of mass 5 kg moving with a velocity of 10 m s^{-1} strikes another of mass 3 kg at rest. The two masses continue in motion together. Find their common velocity.

10 An archer fires an arrow of mass 96 g with a velocity of 120 m s^{-1} at a target of mass 1500 g hanging from a long piece of string from a tall tree. If the arrow becomes embedded in the target, with what velocity does the target move?

11 Two carts, one of mass 0.6 kg and the other of mass 0.8 kg, are moving north along a smooth horizontal surface with speeds of 4 m s^{-1} and 2 m s^{-1} respectively, as shown in Figure 8.14. After the collision, the 0.6 kg mass continues to travel north but with a speed of 1.2 m s^{-1}. What is the speed of the 0.8 kg mass?

NOVEL CHALLENGE

An egg timer is placed on an electronic balance and its mass recorded when all the sand is at the bottom. It is inverted.
As the sand falls, will it weigh the same or less?

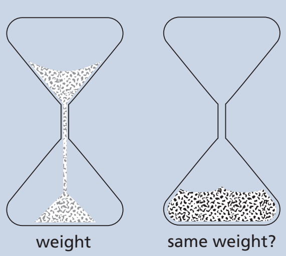
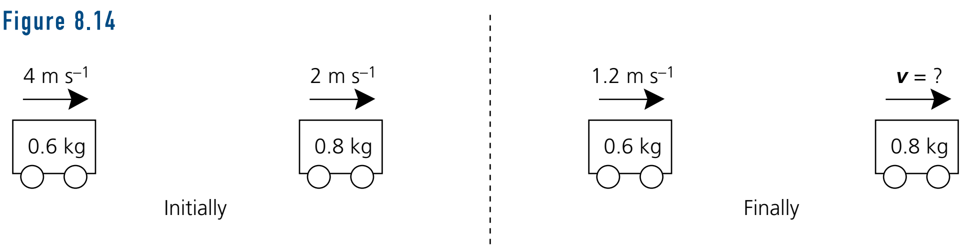


Figure 8.14



12 A 0.41 g air-rifle pellet is fired into a target made up of a 170 g toilet roll attached to a glider of 350 g. The target slides along a linear air track a distance of 50 cm in 2.8 s (refer back to Figure 8.13).
(a) Calculate the velocity of the pellet.
(b) What additional information would you need to calculate the recoil speed of the air rifle?

REAL-LIFE EXAMPLES OF CONSERVATION OF MOMENTUM 8.6

Examples of the concept and uses of the law of conservation of momentum abound in everyday life but often they need pointing out to become obvious.

Sports

Both collisions and explosions are features of many sports.

- Explosions: firing an arrow, throwing a ball or jumping into the air.
- Collisions: hitting a ball, karate chopping a brick or punching a bag.

Sporting explosions

In many sports the athlete is striving to deliver the maximum momentum to the ball or other projectile such as a discus or javelin. Sports physicists use special platform balances that measure the force being exerted on the ground. Figure 8.15 shows the force–time graph of a shotputter. When the force is in the direction of the ball it is called positive; when it is in the opposite direction it is called negative. Obviously it is the negative force that provides the propulsion force.

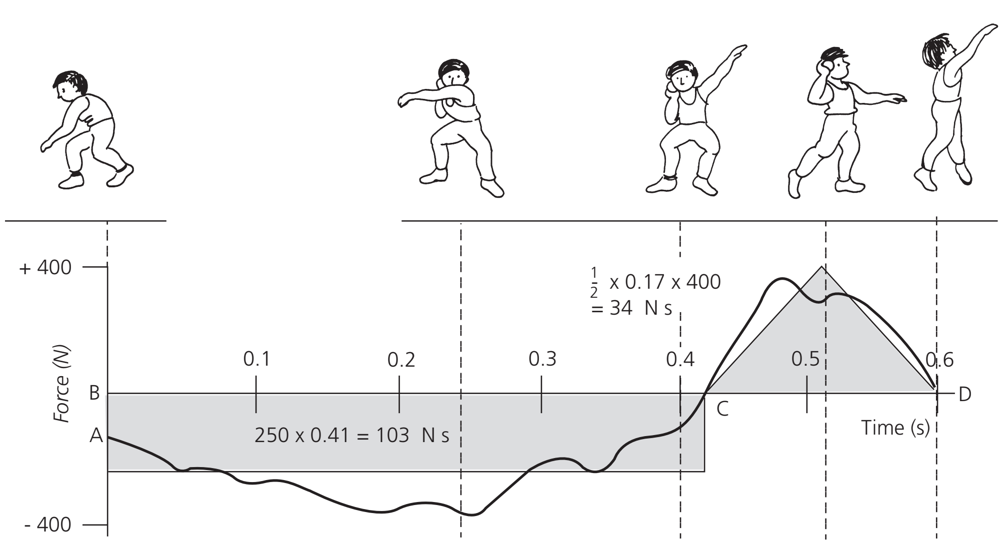


Figure 8.15
Changes in force during a shotput throw. Note how the downward (negative) force changes to an upward force as the ball leaves the hand.

The total impulse is the total area under the curve. The first 0.4 seconds have a negative impulse of −103 N s, whereas the final 0.2 second period is +34 N s. The total impulse is −69 N s.

$$\text{Total impulse } (= \Delta \boldsymbol{p}) = -69 \text{ N s}$$
$$\Delta \boldsymbol{p} = m\Delta \boldsymbol{v}$$

For a 4 kg shot the velocity would be 17.3 m s^{-1}.

Sporting collisions

In some sports, the player has a racquet or bat to strike the ball and momentum is transferred to the ball. In badminton and squash, players flick their wrists to increase the momentum of the light head of the racquet by making it move very fast. In tennis, where the ball's mass is much greater than the shuttle in badminton, this technique is not effective. Players must keep their wrists stiff as they swing at the ball so that the racquet acts as an extension of their body and the effective striking mass is that of the racquet, arm and shoulder.

Table 8.1 shows typical velocities of balls hit in various sports.

Table 8.1 TYPICAL VELOCITIES OF BALLS HIT FROM REST IN A VARIETY OF SPORTS*

BALL	BALLS MASS (kg)	BALL'S VELOCITY ($m\ s^{-1}$)		STRIKER'S VELOCITY ($m\ s^{-1}$)		IMPACT TIME (s)
		BEFORE	AFTER	BEFORE	AFTER	
Baseball	0.15	0	39	31	27	1.35×10^{-3}
Football (punt)	0.42	0	28	18	12	8.0×10^{-3}
Golf ball (drive)	0.047	0	69	51	35	1.25×10^{-3}
Squash ball (serve)	0.032	0	49	44	34	3.0×10^{-3}
Tennis ball (serve)	0.057	0	51	38	33	4.0×10^{-3}

* Also shown is the impact time of the strike and the velocity of the striking mass before and after impact.

In each case the effective striking mass can be calculated and you can see how much of the player's body is added to the mass of the racquet.

Example

The actual mass of the tennis racquet used in Table 8.1 was 0.40 kg. Calculate its effective mass.

Solution

$$\boldsymbol{p}_{\text{initial}} = \boldsymbol{p}_{\text{final}}$$

$$m_{\text{ball}} \times \boldsymbol{u}_{\text{ball}} + m_{\text{racquet}} \times \boldsymbol{u}_{\text{racquet}} = m_{\text{ball}} \times \boldsymbol{v}_{\text{ball}} + m_{\text{racquet}} \times \boldsymbol{v}_{\text{racquet}}$$

$$0.057 \times 0 + m_{\text{racquet}} \times 38 = 0.057 \times 51 + m_{\text{racquet}} \times 33$$

$$5 \times m_{\text{racquet}} = 2.91$$

$$m_{\text{racquet}} = 0.58 \text{ kg (the racquet's mass was increased from 0.4 kg to 0.58 kg)}$$

Baseball

The 1993 World Championship was determined by an otherwise perfect swing of the bat but it was just 1 mm too high and a less than perfect shot resulted in a run out. There's not much room for error.

What baseball players are looking for is both high bat speed and good control. The problem is, the higher the bat speed the less control the player has over the accuracy of the hit. In major league games, the ball strikes the bat after coming from the pitcher's mound 17 m away in 0.45 s. It collides with a bat just 7 cm wide, which is being swung at 100 km/h so there's not much time for decision-making. The ball is squashed to half its diameter and leaves the bat after 0.001 s contact time. You'd wonder how anyone could have control over the placement of the ball. But they do, although the difference between a foul and a hit over second base is only 0.01 s in timing. What a game!

The role of momentum in a good hit is crucial. Players want to give their bats high momentum and they can do this by increasing the mass of the bat or by swinging it faster. Legendary baseball champion 'Babe' Ruth used heavy bats, often as heavy as 52 ounces (1.5 kg). Today's players use bats of about 850 g, but Ruth had exceptional strength and could whip his bat around at high speed. However, changing from a bat of six times a ball's mass to one of seven times its mass adds little to the transfer of momentum to the ball. What it does is slow down the swing considerably.

But how can bat speed be increased? Watch the lead-off hitters, the small players who must get on base so the power hitters can drive them in. Lead-off hitters need to be able to punch the ball to the opposite field or find a hole in the in-field. They need excellent control and good bat speed. They 'choke-up' on the bat — sliding their hands up higher on the handle, making it easier and faster to swing.

But a short bat is not long enough to reach those fast balls on the outside of the plate. A better solution is to use a lighter bat and, over the past few decades, bat masses have decreased to the current 800–900 g. But as the wood is thinner, the risk of breaking is also increased, so aluminium and composite plastics (graphite, fibre glass) are used in most games except major league, where aluminium is too fast and dangerous.

Squash gets fast too but because the ball heats up during the game. A hot ball has a greater change in momentum than a cold one. The physics of that is interesting to contemplate.

Questions

13 **(a)** Calculate the force imparted to a 145 g ball during a hit as described above. Assume the rebound speed of the ball from the bat is the same as the impact speed of the ball.

(b) Assuming a player can supply the same momentum to bats of different mass, calculate the speed of an 850 g bat if he can swing a 1500 g bat at 30 m s^{-1}.

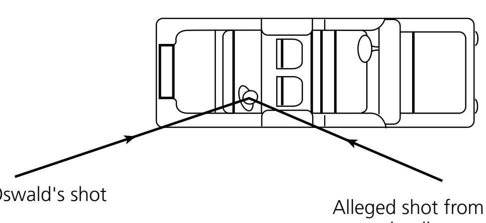
Forensic science

In the course of police investigations into crimes, physicists often play a vital role. Much of the scientific evidence in a forensic investigation (Latin *forum* = 'of the court') is biological or chemical in nature but when car accidents or guns are involved, the physicists who are experts in kinematics or ballistics are called in.

One famous case concerns the assassination of US President John F. Kennedy in 1963. JFK was shot in the head and neck by high powered rifle bullets. Movie film of the event shows that his head tilted forward as he was struck in the back of the neck and then his head moved rapidly back as he suffered a head wound. Assassination buffs have split into two groups, depending on whether they believe in a single 'lone nut' gunman or a conspiracy between two gunmen. The 'lone nutters' believe all wounds were caused by a lone gunman (Lee Harvey Oswald) firing from the sixth floor of a building behind the President's car. The conspiracy theorists believe that Oswald was responsible for the neck wound but another gunman firing from the grassy knoll to the front right of the car was responsible for the fatal head wound. No video of the assassination exists, the only clear film of the events being made by Abraham Zapruder on a hand-held Super-8 movie camera from a distance of about 60 m.

People who claim that JFK was shot from the front say that, because his head moved backward, a second gunman fired from the front (from the 'grassy knoll') (see Figure 8.16). Nobel-Prize-winning physicist Luis Alvarez contradicts this. He has shown that when an object such as a taped-up watermelon (simulating a head) is shot, the melon generally moves towards the gun as chunks are blown out the other side. Dubbed the 'jet effect', Dr Alvarez showed that the matter blown out of the melon carried with it more momentum than was brought in by the bullet. This is similar to the motion of a rocket as jet fuel is ejected and is a good example of conservation of momentum. Other physicists further argued that it was the shot by Lee Harvey Oswald to the back of the neck that caused JFK's arms to fly up under his chin and his body to jerk backward in a nervous reaction known as the 'Thorburn Position'. Either way, physicists agreed that the head shot came from behind JFK and have dismissed the conspiracy theory. Oswald used a \$12.50 Italian Carcarno hunting rifle that fired high velocity (670 m s^{-1}) full-metal-jacket bullets, each with a mass of 10.37 g. By the time they reached the President, the bullets had lost momentum and were travelling at 545 m s^{-1}. Imagine such a bullet striking a melon and remaining embedded in it while simultaneously blowing a jet of melon out the other side. The equation becomes:

Figure 8.16
The Presidential limousine: the Kennedys in the rear and the Connallys in the centre.



momentum of bullet = momentum of remains of melon and bullet + momentum of jet

$$m_b v_b = m_r v_r + m_j v_j$$

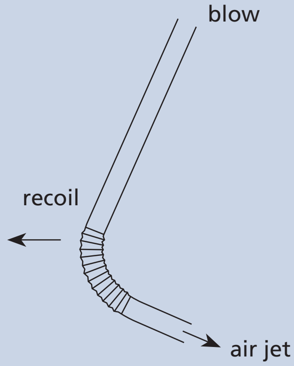
If the momentum of the bullet ($0.010\,37 \times 545 = 5.65$ kg m s^{-1}) was less than the momentum of the jet, then the momentum of the remains of the melon with the embedded bullet would have to be negative, and hence the velocity would also have to be negative. This means the remains of the melon would have moved toward the gunman. The 'lone nutter' theory is supported.

Questions

14 A 10.0 g Carcarno bullet is fired with a muzzle velocity of 545 m s^{-1} at a 3.0 kg watermelon and remains embedded in it.
(a) Calculate the motion of the melon if there is no jet exiting the other side.
(b) Calculate the motion of the melon if a 300 g jet of melon exits the rear of the melon at a speed of 35 m s^{-1}.

NOVEL CHALLENGE

When you blow through a bent drinking straw it recoils away from the jet of air. *But when you suck air in does the reverse happen? I think not! Explain that one in terms of momentum. Try it and see.*



SR Activity 8.2 KENNEDY ON THE INTERNET

1 If you have access to the Internet, try reading the newsgroup alt.assassination.jfk, which deals with the Kennedy investigation. You'll meet lots of cranks but also some physicists who will discuss ballistics and forensic science.

2 If you can't get to sleep, try one of the JFK Web pages. There are dozens of them — some favour the conspiracy theory, the others favour the lone gunman theory. Try www.jfklancer.com for a start. Failing that, try joining a chat room. The chat times are listed on the Web pages. You never know, someone might try to sell you a Carcarno.

SR Activity 8.3 HIRE A VIDEO

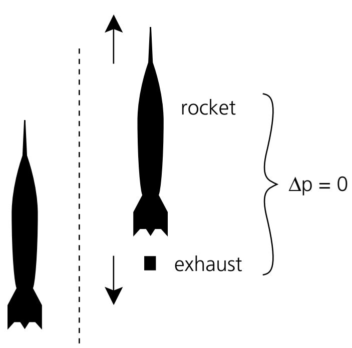
Oliver Stone's movie *JFK* starring Kevin Costner is out on video. Stone takes a different line from the one above but still examines the evidence in a scientific way. If you can hire it, look for the discussion on the ballistics evidence. Make notes and compare it to the discussion opposite.

Propulsion of rockets

A rocket moves forward because burning gases are ejected at high speed behind it. If an engine supplies a constant force (thrust), the acceleration of the rocket will increase because the total mass of the rocket decreases as fuel and oxygen are burnt. Have you noticed how much faster an inflated balloon goes at the end of its journey than when you first let it go?

Many people think that rockets only work if they have something to push against. But they work in space where there is no air. The momentum of the exhaust gases is equal in magnitude but opposite in direction to the gain in momentum of the rocket.

Figure 8.17



Example

The German V-2 rockets used to bomb London in the Second World War had a mass of 12 000 kg and produced thrust from exhaust gases that were ejected at the rate of 1500 kg every second and at a speed of 170 m s^{-1}. Calculate **(a)** the initial forward force on the rocket (thrust); **(b)** the net force if the rocket was fired vertically; **(c)** the initial acceleration.

Solution

Consider the rocket to be made up of two exploding parts: the rocket itself and the exhaust gases (Figure 8.17). Initially both components are at rest and the momentum of each is zero. When fired, the momentum of each is equal and opposite.

(a)

$$\begin{aligned} \boldsymbol{p}_{\text{rocket}} &= \boldsymbol{p}_{\text{exhaust}} \\ \boldsymbol{p}_{\text{rocket}} &= m_e \times \boldsymbol{v}_e \\ &= 1500 \times 170 = 255\,000 \text{ kg m s}^{-1} \end{aligned}$$

The interaction time for this 'explosion' is 1 s, so the impulse ($\boldsymbol{F}t$) equals this change in momentum from zero to 10 000 kg m s^{-1}.

$$\boldsymbol{F} = \frac{255\ 000}{1\ \text{second}} = 255\ 000\ \text{N (or kg m s}^{-2}\text{)}$$

(b) The net force is the result of the thrust (upward) and the weight (downward):

$$\boldsymbol{F}_{\text{net}} = \boldsymbol{F}_{\text{upthrust}} - \boldsymbol{F}_{\text{w}} = 255\ 000 - 120\ 000 = 135\ 000\ \text{N (upward)}$$

(c) Initial acceleration: $\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$, hence $\boldsymbol{a} = \boldsymbol{F}/m$ = 135 000/12 000 = 11.25 m s^{-2}.

As these rockets burnt fuel their mass decreased, and hence their acceleration increased. After 65 s all fuel had been burnt and the rockets were moving at 2 km s^{-1}.

Such rockets were particularly dangerous because there was no warning sound. The whine of the rocket engines came *after* the sound of the explosion on landing because they travelled faster than the speed of sound.

8.7 CONSERVATION OF MOMENTUM IN TWO DIMENSIONS

Rarely is the world as simple as portrayed in the previous discussion. We live in a three-dimensional world and interactions occur in three dimensions. You have been introduced to momentum in one dimension so that the principles can be seen. Now it is time to venture into the two-dimensional world — *Flatland*. Interactions occurring in the 3-D world are beyond the scope of this book. Wait until first-year university physics for that.

It doesn't matter whether it is in one, two or three dimensions — momentum is always conserved. That is, total momentum before the collision equals total momentum after the collision.

Explosions in two dimensions

An object at rest has zero momentum. If it explodes into several pieces, the pieces will still have a zero total momentum. If it is moving when it explodes or separates, then the fragments will have a total momentum equal to the momentum before the explosion.

Example

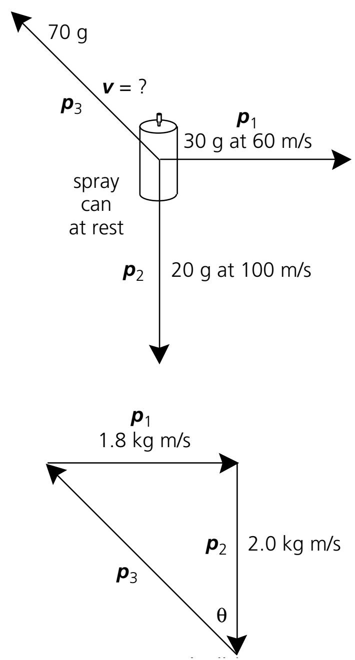
An empty spray can of mass 120 g rests on top of a fire and explodes into three fragments. One 30 g fragment travels east at 60 m s^{-1} and another 20 g fragment goes south at 100 m s^{-1}. (See Figure 8.18.) Calculate the velocity of the third piece.

Solution

The law of conservation of momentum states that the final momentum will be equal to the initial momentum, which in this case is zero. Hence, the sum of the three momentum vectors after the explosion will also be zero. That is, the three vectors will form a closed triangle when added head to tail. All we need do is draw the two known vectors and fill in the remaining gap ($\boldsymbol{p}_3$) to see the missing vector.

Using Pythagoras' theorem, $\boldsymbol{p}_3$ equals 2.7 kg m s^{-1}. As the mass is 70 g, the velocity must be 38.4 m s^{-1} in a direction θ of N42°W. This angle is sometimes expressed as '318° True'. Can you see why?

Figure 8.18
Vector diagram showing the momentum of each of the three fragments of the exploding spray can in the example to the left. When placed head-to-tail, the vector arrows $\boldsymbol{p}_1$, $\boldsymbol{p}_2$ and $\boldsymbol{p}_3$ must add to zero.



Questions

15 A bomb, initially at rest, explodes into three fragments as shown in Figure 8.19. Calculate the mass of the third fragment.

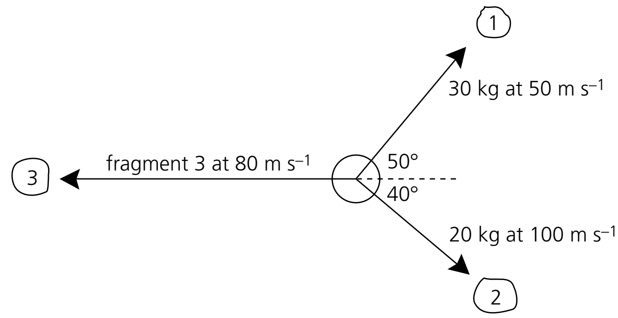


Figure 8.19
For question 15.

16 A radioactive nucleus of mass 5×10^{-26} kg is at rest and emits two neutrons, each of mass 1.6×10^{-27} kg, at right angles to each other. If both have speeds of 360 m s^{-1}, calculate the recoil speed of the nucleus.

Collisions in two dimensions

As with collisions in one dimension, collisions in two dimensions can be of the rebound type or the objects can stay coupled together. For example:

- rebound: billiard balls or cars colliding at an angle
- coupled: cars colliding off-centre and becoming tangled.

Rebound collisions

In a two-dimensional collision the objects approach and rebound obliquely. This means that their paths follow different lines but in the same plane. If you follow these four steps you have a good way of solving problems:

1. Construct a vector diagram showing the total momentum before the collision.
2. Construct another vector diagram showing the total momentum after the collision.
3. Equate these two vectors since $\boldsymbol{p}_f = \boldsymbol{p}_i$.
4. Calculate the unknown quantity by vector analysis.

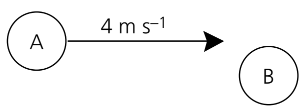


Figure 8.20
Initial motion of ball A in Example 1.

Example 1

A ball A of mass 1.0 kg is moving east at 4 m s^{-1} when it collides with a stationary ball B of mass 2 kg (Figure 8.20). Ball A heads north at 4 m s^{-1} after the collision. Determine the final velocity of ball B.

Solution

Step 1 Initial momentum

$$\boldsymbol{p}_i = \boldsymbol{p}_A + \boldsymbol{p}_B = 4 \text{ kg m s}^{-1} \text{ E} + 0 \text{ kg m s}^{-1} = 4 \text{ kg m s}^{-1} \text{ E (Figure 8.21)}$$

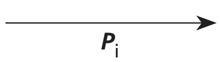


Figure 8.21
Total initial momentum of balls A and B in Example 1.

Step 2 Final momentum

Add $\boldsymbol{p}'_A + \boldsymbol{p}'_B$

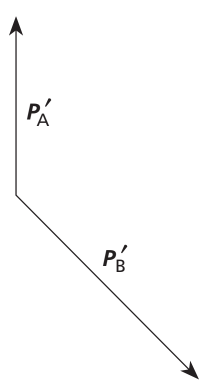


Figure 8.22
The final momentum of the balls.

Note: the symbol $\boldsymbol{p}'$ (pronounced p-prime) is used to indicate the *final* momentum.

$$\boldsymbol{p}_f = \boldsymbol{p}'_A + \boldsymbol{p}'_B = m_A\boldsymbol{v}_A + m_B\boldsymbol{v}_B = 1\text{ kg} \times 4\text{ m s}^{-1} + 2\text{ kg} \times \boldsymbol{v}_B \text{ (vectorily)}$$

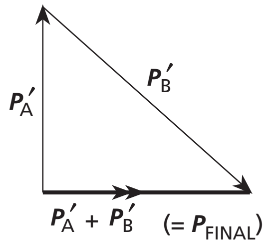


Figure 8.23 The final momentum of ball A and ball B is found by placing the momentum vectors head to tail.

Step 3 Equate $\boldsymbol{p}_f$ and $\boldsymbol{p}_i$

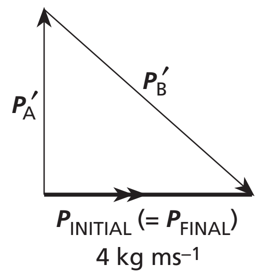


Figure 8.24 The sum of the final momentum of ball A ($\boldsymbol{p}'_A$) and of ball B ($\boldsymbol{p}'_B$) has to be equal to their intial momentum.

Step 4 Vector analysis

Using Pythagoras' theorem: $\boldsymbol{p}'_B = 5.6\text{ kg m s}^{-1}$.
As the mass of B is 2 kg, $\boldsymbol{v}_B = 2.8\text{ m s}^{-1}$.
Angle θ = S45°E (or SE or 135° True).

Example 2

A 2 kg ball (A) is travelling at 10 m s^{-1} east when it strikes a stationary 2 kg ball (B) a glancing blow. The two balls move away at right angles to each other with ball A travelling 30° to the north of its original path. Calculate the velocity of balls A and B after the collision.

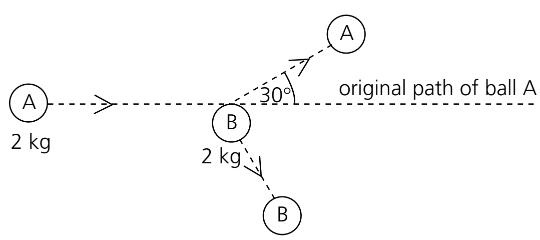


Figure 8.25 Motion of balls in Example 2.

Solution

$$\boldsymbol{p}_i = m_A\boldsymbol{u}_A = 2 \times 10 = 20\text{ kg m s}^{-1}$$
$$\boldsymbol{p}_f = \boldsymbol{p}'_A + \boldsymbol{p}'_B = m_A\boldsymbol{v}_A + m_B\boldsymbol{v}_B = 2\boldsymbol{v}_A + 2\boldsymbol{v}_B$$

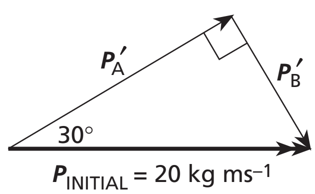


Figure 8.26 The final momentum equals the vector sum of the initial momentums.

- $\sin 30° = p'_B/20 \quad p'_B = 10$ kg m s^{-1} $\quad v_B = 5$ m s^{-1}.
- $\cos 30° = p'_A/20 \quad p'_A = 20 \times 0.866 = 17.3$ kg m s^{-1} $\quad v_A = 8.65$ m s^{-1}.

Note: as the above example has shown, when two objects of the same mass collide and bounce off, the angle between their paths after collision is a right angle. When the masses are different, the angle may not be a right angle and the solution to the problem is more difficult — the cosine rule is often used. If you try this experiment in class, you might find that the angle is just slightly *less* than 90° because of friction effects.

Coupled collisions

Example

Two skaters, Alfred (A) and Barbara (B), collide and hold each other together after impact. Alfred, whose mass is 83 kg, is originally moving east with a speed of 6.2 m s^{-1}. Barbara, whose mass is 55 kg, is originally moving north with a speed of 7.8 m s^{-1}. What is their velocity after the impact?

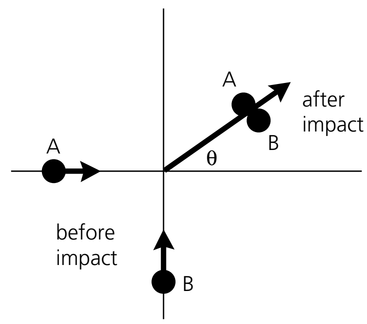


Figure 8.27 The two skaters A and B collide and move off together.

Solution

$$\boldsymbol{p}_A + \boldsymbol{p}_B = \boldsymbol{p}'_{(A+B)}$$
$$m_A\boldsymbol{v}_A + m_B\boldsymbol{v}_B = (m_A + m_B)\boldsymbol{v}$$
$$83 \times 6.2 + 55 \times 7.8 = 138\boldsymbol{v} \text{ (vector addition; do not solve for } \boldsymbol{v} \text{ using algebra)}$$

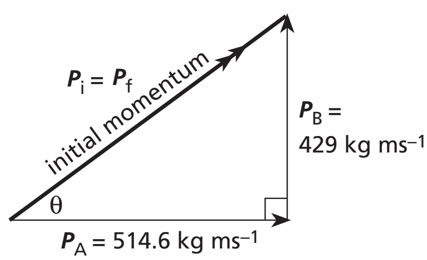


Figure 8.28 Final momentum (thick arrow) is the vector sum (head to tail) of the two initial momentums.

The initial momentum is shown by the hypotenuse. This is also the final momentum because of the law of conservation of momentum.

$$p_{\text{final}} = 670 \text{ kg m s}^{-1}$$
$$\text{hence velocity is } \frac{670}{83 + 55} = 4.9 \text{ m s}^{-1} \text{ at angle } \theta = 40°$$

Questions

17 A stationary 6.0 kg bomb suddenly explodes and the fragments fly off in the directions shown in Figure 8.29. Determine the final velocity of the 1.0 kg piece.

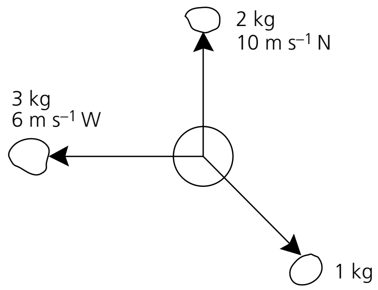


Figure 8.29
For question 17.

18 A ball A with a mass of 1.0 kg and moving at 4 m s^{-1} strikes a glancing blow on a second ball B of mass 2.0 kg that is initially at rest. After the collision, ball A is moving at right angles to its original direction at a speed of 3 m s^{-1} as illustrated in Figure 8.30. Calculate the velocity of B.

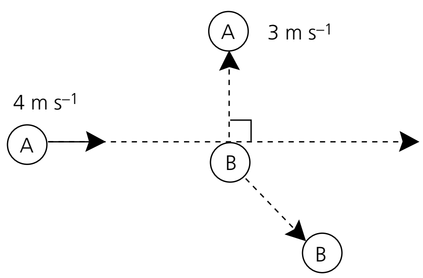
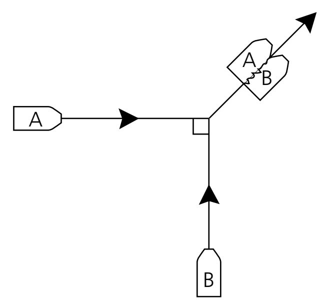


Figure 8.30
For question 18.

Figure 8.31
For question 19.



19 A barge with a mass of 1.5×10^5 kg is proceeding east along a river at 6.2 m s^{-1} in heavy fog when it collides with a barge heading north. The second barge has a mass of 2.78×10^5 kg and was moving at 4.3 m s^{-1}. Upon impact the barges remain coupled together (Figure 8.31). Find their resultant velocity.

8.8 TORQUE: MOVING IN CIRCLES

Not everything that has a force applied to it will move from one place to another. Sometimes it just goes in a circle. When you spin a bicycle wheel, the applied force goes into making the wheel rotate. When you tighten up the wheel nuts when changing a car tyre with a spanner, the force just makes the nuts turn around.

The turning effect is known as the **torque**, the symbol of which is the Greek letter 'tau' (τ). The magnitude of the torque depends on:

- the size of the force applied
- the direction of the force applied
- the distance between the point at which it acts and the pivot.

For example, when you open a door you are applying a torque to twist it about its hinges. You pull on the knob at right angles to the door. If you pulled sideways, the force would be wasted or reduced. As well, the door knob is located as far as possible from the pivot point to make your pull as effective as possible.

If $\boldsymbol{F}$ and r are perpendicular:

$$\text{Torque } \tau = \boldsymbol{F}r$$

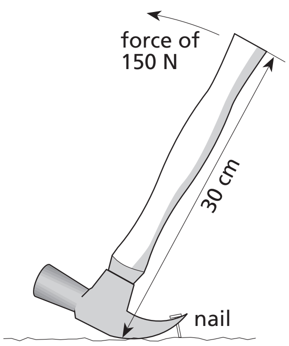
If $\boldsymbol{F}$ and r are at an angle θ:

$$\text{Torque } \tau = \boldsymbol{F}r \sin \theta$$

NOVEL CHALLENGE

The torque to remove a lid off a jar of food has been set at 2 N m by food manufacturers. This torque is such that all 20- to 40-year-olds and 97% of 50- to 94-year-olds can remove it. Estimate the force necessary to manage this, assuming the radius of a typical lid is 3.5 cm. Why does it help to put a tight lid under a hot water tap?

Figure 8.32
Calculating the torque applied to a hammer.



Example

A force of 150 N is applied at right angles to the end of a hammer handle 30.0 cm long to pull a nail from some wood. (See figure 8.32.) What torque is applied to the hammer?

Solution

Torque applied by the hand: $\tau = \boldsymbol{F}r = 150 \times 0.300 = 45$ N m.

Engine torque

If you hear people talking about four-wheel drives, the subject of torque eventually comes up. Diesel engines have a big reputation for providing a lot of torque at low engine speeds compared with their petrol-engined counterparts. Table 8.2 compares the engine performance of the turbo diesel and petrol Toyota Landcruiser 4WDs.

Table 8.2 ENGINE PERFORMANCES

	PETROL	DIESEL
Capacity (L)	4.5 L	4.2 L
Power (kW)	158 kW @ 4600 rpm	115 kW @ 3600 rpm
Torque (N m)	373 N m @ 3200 rpm	357 N m @ 1800 rpm

Although the petrol engine produces more torque, the diesel engine produces it at a much lower engine speed. This gives it tremendous advantage in climbing sandhills and getting out of bogs. Also, the torque produced by either engine is not constant over the range of engine speeds but peaks at the value shown in the table.

NOVEL CHALLENGE

When removing a cork out of a champagne bottle, it is easier if you hold the cork and rotate the base of the bottle rather than holding the base and rotating the cork as most people do. Why is this easier? It seems to defy logic, doesn't it?

NEI Activity 8.4 DIESEL VS PETROL

Motor enthusiasts seem to either love diesels or hate them.

1 If you know someone with an interest in cars and trucks see if you can get him or her to help you make a comparison of the advantages and disadvantages of diesel versus petrol engines. Use the following criteria as a guide: engine life (wear and tear), cost of engine, fuel price, fuel economy, acceleration ability, availability of fuel, water in cylinders, air pollution.

2 Why do farmers prefer diesels? Is it because of the low-speed torque?

3 What is the difference between a supercharger and a turbocharger?

Questions

20 Calculate the torque on a wheel nut produced by a force of 90 N at right angles on the end of a spanner 40 cm away from the pivot point (the wheel nut).

ANGULAR MOMENTUM 8.9

Now that you've seen how rotation occurs, it's time to look at the laws involved and some other things that rotate. Have you seen the spinning chair at the Sciencentre in Brisbane? Or the spinning chairs at Questacon or just about every other science expo around? They have something in common with a car engine, a springboard diver, a frisbee and the incredible shrinking stars.

Bodies that spin have momentum — **angular momentum**. It is different from linear momentum in that it is the spinning, not the movement from place to place, that is important. Just as you need a force to get a bicycle to move, you have to apply a torque to a bicycle wheel to make it spin. An external torque can change an object's angular momentum.

So far in this chapter you have seen that linear momentum is equal to the product of mass and linear velocity ($\boldsymbol{p} = m\boldsymbol{v}$). Similarly, angular momentum ($\boldsymbol{L}$) is the product of inertia ($\boldsymbol{I}$), which is related to mass, and angular velocity (ω). Hence: $\boldsymbol{L} = \boldsymbol{I}\omega$.

NOVEL CHALLENGE

The Earth gains 100 thousand tonnes each day as interstellar dust settles on this beautiful planet of ours. Calculate how much longer each day will be because of this. You may need to do this long-hand as most calculators won't show an answer unless you know some tricks.

Inertia

For rigid bodies such as a bicycle wheel or a rolling ball, physicists have developed formulas that enable us to calculate their **rotational inertia**. This is different from mass because for a rotating object not all the mass is travelling at the same speed — the outside goes faster than the inside. How the mass is distributed in that object will determine how difficult it is to start or stop the object rotating. Some simple objects are shown in Figure 8.33.

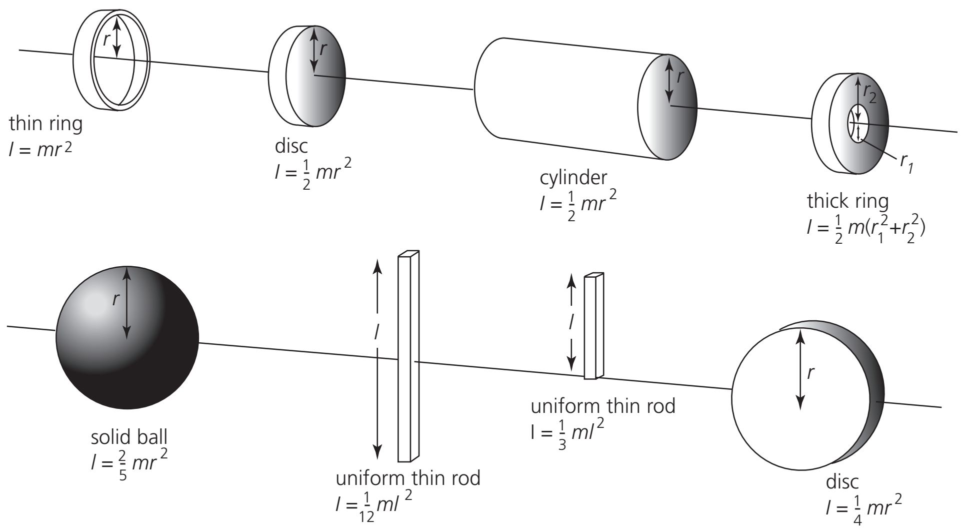


Figure 8.33
Rotational inertia equations for objects rotating about the indicated axes.

Example

Calculate the rotational inertia of a 20 kg snowball of diameter 1.5 m.

Solution

For a solid sphere:

$$\boldsymbol{I} = \tfrac{2}{5}mr^2$$
$$\boldsymbol{I} = \tfrac{2}{5} \times 20 \times 1.5^2$$
$$\boldsymbol{I} = 18 \text{ kg m}^2$$

A rolling snowball will therefore have both rotational momentum *and* linear momentum. To stop it moving you have to stop its translational motion *and* its rotational motion. Not an easy task.

NOVEL CHALLENGE

If everyone faced the same way on Earth and took a step at the same time would the Earth's rotation change? What data would you need to calculate this mathematically?

Angular momentum

The angular momentum $\boldsymbol{L}$ of a rigid body of rotational inertia $\boldsymbol{I}$ rotating at an angular speed ω about an axis is given by $\boldsymbol{L} = \boldsymbol{I}\omega$. The angular speed needs to be expressed in radians per second (rad s^{-1}) as was shown in Chapter 6.

Example

Calculate the angular momentum of the snowball in the previous example if it is rolling at 3 revolutions per second.

Solution

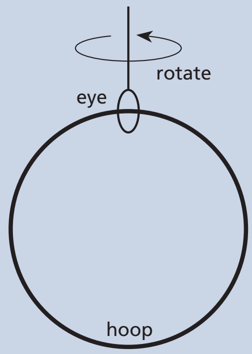
- 1 revolution equals 2π radians, hence 3 rev/s = 6π rad s^{-1}.

$$\begin{aligned}\boldsymbol{L} &= \boldsymbol{I}\omega \\ &= 18 \text{ kg m}^2 \times 6\pi \text{ rad s}^{-1} \\ &= 340 \text{ kg m}^2 \text{ s}^{-1}\end{aligned}$$

CONSERVATION OF ANGULAR MOMENTUM 8.10

NOVEL CHALLENGE

A piece of wire has an eye at one end with a small wire hoop through it. When it is rotated in a drill the hoop lifts up. *Now why is this?*



Just as linear momentum is conserved, so too is angular momentum.

If no net torque acts on a system, the angular momentum $\boldsymbol{L}$ of that system remains constant no matter what changes take place within that system.

$$\boldsymbol{L} = \boldsymbol{I}\omega = \text{a constant}$$
$$\boldsymbol{I}_i\omega_i = \boldsymbol{I}_f\omega_f$$

In the simplest case, if a solid body is spinning at a particular rate then it will continue to spin at that rate unless an outside torque (twisting force) acts on it. More interestingly though, if the distribution of mass changes within the body, then inertia changes and so the angular speed will have to change to keep the angular momentum constant.

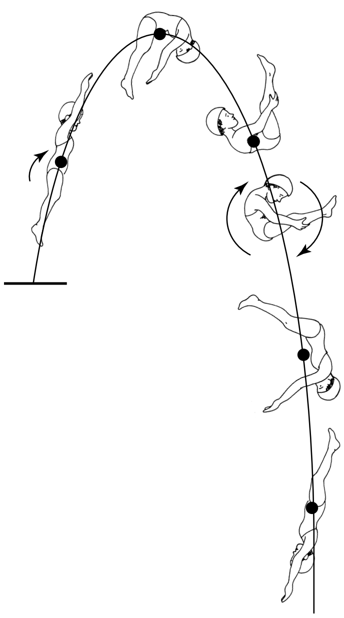
Imagine a student seated on a stool that can rotate freely. The student, who has been set into rotation at a slow initial angular speed ω_i, holds two dumbells in his outstretched hands. The student now pulls his arms in close to his body. This reduces his rotational inerta from its initial value $\boldsymbol{I}_i$ to a smaller value $\boldsymbol{I}_f$ as his mass is closer to the rotational axis (the radius r is smaller). His rate of rotation increases markedly, from ω_i to ω_f. If he wants to slow down all he has to do is extend his arms once more.

EVERYDAY EXAMPLES OF ANGULAR MOMENTUM 8.11

The springboard diver

Figure 8.34 shows a diver doing a forward one-and-a-half somersault dive. As you would expect, her centre of mass follows a parabolic path. By pulling her arms and legs into the closed tuck position she reduces her rotational inertia and hence increases her angular speed. Pulling out of the tuck position into the open layout position slows her rotation rate.

Figure 8.34
The diver's angular momentum is constant throughout the dive. Her centre of mass follows a parabolic path.



The incredible shrinking star

When a star runs out of nuclear fuel, its temperature decreases and its diameter gets smaller; in fact, it may go from the size of our Sun to just a few kilometres. The star becomes a *neutron star*, so called because the core of the star has been compressed to just an incredibly dense neutron gas. Because stars rotate, the effect of this decrease in radius is an increase in rotational speed. Our Sun rotates once per month; a neutron star may rotate at 800 revolutions per second.

Bullets

When bullets are projected up the barrel of a gun they are guided by spiral grooves inside the barrel. These grooves are called the 'rifling' and give the bullet a high rotational speed by the time it leaves.

One of the most popular firearms among Queensland farmers and sporting shooters is the .257 Weatherby Mark V rifle. It has a barrel 66 cm long and a 'twist' of 30 cm. This means that as the bullet moves up the rifle barrel, it does one complete turn for every 30 cm of barrel length. Seeing that it exits the muzzle with a velocity of 850 m s^{-1}, it means that the bullet is spinning at about 2850 rev/s. You can show this to be correct by dividing the 850 m by 0.30 m to see how many revolutions it does in 1 s. This spinning is designed to keep the bullet travelling point-first so as to reduce air resistance. Without this rotational stabilisation, the bullet would begin to tumble after a short distance and lose its velocity rapidly. It would become useless.

A spinning bullet can be thought of as a cylinder rotating about its long axis. The .257 bullet mentioned above has a diameter of .257 inches (6.5 mm) and a popular type has a mass of 100 grains (6.47 g). For such a bullet to rotate about any other axis, it can be shown that inertia would be greater (by a factor of about 10) and for angular momentum to be conserved, its spin rate would have to be reduced. This is not likely without external forces being applied. So it travels point-first.

Example

Compare **(a)** the linear momentum and **(b)** the angular momentum of a 45 grain .223 Armalite bullet on exiting the muzzle. A .223 bullet has a diameter of .223 inches (5.56 mm) and 45 grains is equal to 2.9 g. The Armalite has a muzzle velocity of 1030 m s^{-1} and a twist of 25 cm, which produces a rotation rate of 4120 rev/s.

Solution

- Linear momentum: $\boldsymbol{p} = m\boldsymbol{v}$
  $= 2.9 \times 10–3 \text{ kg} \times 1030 \text{ m s}^{-1} = 3.0 \text{ kg m s}^{-1}$.
- Angular momentum:
  rotational inertia $\boldsymbol{I} = \frac{1}{2}mr^2$
  $= \frac{1}{2} \times 2.9 \times 10^{-3} \text{ kg} \times (2.78 \times 10^{-3} \text{ m})2 = 1.12 \times 10^{-8} \text{ kg m}^2$
  rotational speed $= 4120 \text{ rev/s} \times 2\pi \text{ rad/rev} = 25\,887 \text{ rad s}^{-1}$
  angular momentum $\boldsymbol{L} = \boldsymbol{I}\omega = 1.12 \times 10^{-8} \text{ kg m}^2 \times 25\,887 \text{ rads}^{-1} = 2.9 \times 10^{-4} \text{ kg m}^2 \text{ s}^{-1}$.

The linear momentum is 10 000 times greater than the angular momentum but each has its own job to do and both are precisely engineered to produce the high impact and good stabilisation effects.

Ballistics experts define 'twist rate' as the number of turns a bullet does per linear metre. Although the rotational speed of the bullet remains constant for most of its journey (due to conservation of angular momentum), the linear speed decreases. Hence the bullet does a lot more spins in a slow metre than it does in a fast metre — so the twist rate increases. It is just a strange way of expressing rotational speed but even keen shooters won't believe you when you tell them that a bullet's twist rate increases as it travels towards the target. You could even win money in a bet. (See Table 8.3.)

NOVEL CHALLENGE

If you spin around a chocolate-covered almond while it is lying flat, an amazing thing happens (usually) — it stands on its end.

Explain this phenomenon in terms of rotational inertia. Before you try it, predict if the fat end or the pointy end will be on the top. What would happen with a Smartie or an M&M?

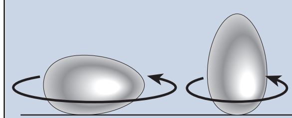


Table 8.3 SOME BALLISTIC DATA*

RANGE (m)	VELOCITY (m s^{-1})	ROTATIONAL SPEED (rev/s)	TWIST RATE (rev/s)	LINEAR MOMENTUM (kg m s^{-1})	ANGULAR MOMENTUM ($\times 10^{-8}$ kg m^2)
0	855	3050	3.6	8.3	7
100	790	3050	3.9	7.7	7
200	730	3050	4.2	7.1	7
300	680	3050	4.5	6.6	7
400	620	3050	4.9	6.0	7
500	570	3050	5.4	5.5	7

* The data are for a Winchester bullet fired from a Remington .308 rifle with a 28 cm twist. The bullet has a diameter of 7.62 mm, a mass of 9.7 g and a muzzle velocity of 855 m s^{-1}.

Activity 8.5 BOOK THROWING

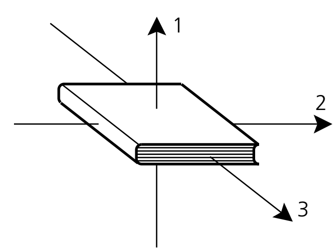
EI

Put a rubber band around a hardback book and try throwing it in the air with a rotation about one of its axes as shown in Figure 8.35.

It should be fairly easy to achieve stable rotation about two of the axes, but the third one is very hard. Which of the axes labelled in Figure 8.35 is this difficult one?

The explanation is that rotation about axes that produce maximum or minimum inertia is relatively simple as they are stable against small deviations (wobbles). Intermediate inertia is easier to get to wobble.

Figure 8.35
The three rotational axes of a book.



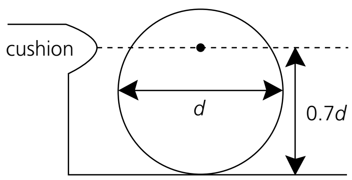


Figure 8.36(a)
The cushion is set at 0.7 times the ball's diameter. This is the same as two-fifths of the radius above the centre of the ball.

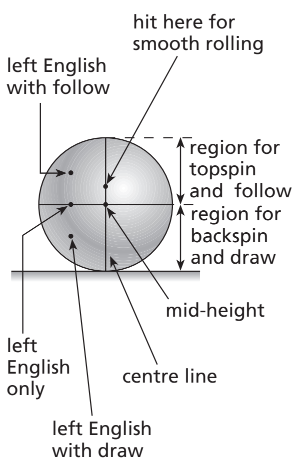


Figure 8.36(b)
Strategic points to hit the cue ball.

Billiards and pool

These games have the feel of physics. Balls collide with each other and bounce off the cushion. But actually the physics of these games is a bit more subtle than that. A skilled player can impart backspin and topspin and other important rotational motions they call 'left and right English'.

When the cue (stick) hits a ball, both linear and rotational motion is imparted. When you strike a billiard ball at mid-height, it will skid away from you and then begin to roll until it collides with another ball or the cushion. Rolling friction is very low. But if it is struck *above* the mid-point it will acquire top spin: the top of the ball moves away from you faster than it otherwise would. Striking the ball *below* the centre results in backspin. You are thus able to control three features of the ball's motion: its linear velocity (by how hard you strike it); the direction of the spin; and how fast it spins.

A spinning ball experiences considerable friction, unlike a rolling ball, which experiences almost none. The direction of the friction is toward you for backspin and so the spin is eliminated quickly; the ball slows down until all spin is lost and then it just continues to roll. With topspin, friction is away from you and slippage tends to speed up the ball. The slippage gradually slows until it exactly matches the forward motion and the ball just rolls. There is a special point on the ball two-fifths of the radius above the centre point, which, when struck, produces a roll with no slippage at all. That is the reason the cushion on the inside edge of the table has a bump at this height — to bounce the ball back without causing it to slip (Figure 8.36**(a)**).

However, players are more interested in collisions with other balls. When the cue ball strikes another ball it transfers its momentum. In a head-on collision, the transfer is complete, leaving the cue ball with no linear motion. In a glancing collision, the cue ball loses only part of its momentum and continues to travel almost at right angles to the motion of the struck ball. In any collision, virtually none of the angular momentum is transferred because of the small amount of friction between the balls. So after a head-on collision, the linear motion stops but the rotation continues. The cue ball will then move back toward you if it has backspin (called 'draw') or away from you if it has topspin (called 'follow') after the collision. If the ball is struck on the left side, it will acquire a clockwise spin (called 'left English') and if struck on the right side will acquire 'right English'. These rotations also affect the result of the collision. See Figure 8.36**(b)**.

Angular momentum (and linear momentum) is conserved in all collisions. A study of momentum transfer is the job of a physicist. To take advantage of the physics is the job of the player.

Activity 8.6 BILLIARD PHYSICS

1 If you can get access to a billiard table, see if you can achieve the following (take notes): topspin, smooth rolling, backspin, left English, right English. Note the motion in each case and see if it agrees with the text above. Try hitting the cue ball directly at mid-height.

2 Try hitting the ball with each of the above motions head-on into a stationary ball. Look for topspin and follow, backspin and draw. What happens with left English and follow, left English and draw?

3 Does a glancing blow produce a separation angle of 90° as stated in the above text? What difference does spin have on this angle, if any?

4 What effect on angular momentum transfer does putting chalk on the balls have? This is illegal but it's in the interests of science. Don't do it in a real game — it may give you an unfair advantage.

The falling cat

Don't try this! When a cat falls out of a window upside-down it turns over and lands on its feet. How is this possible? If it starts with no angular momentum, how can the cat acquire it without violating the law of conservation of angular momentum? The answer is that the cat bends itself into a V-shape and by stretching out its front legs while curling up its back legs can change its rotational inertia and turn half its body. It then curls up its front legs and stretches its back legs to change its inertia again and completes the rotation. Pretty clever for a dumb animal! Explore the physics of it for yourself but not with a cat.

NOVEL CHALLENGE

A large ball bearing is placed on a sheet of paper on a desk and the paper is pulled quickly from under the ball.
Does the ball stay in the same place relative to the desk, or what? Please explain!

Questions

21 The 7.62 mm (diameter) bullet has become the NATO standard cartridge for the armed forces. Calculate the angular momentum of a 9.7 g bullet fired from a Russian SKS rifle with a muzzle velocity of 671 m s^{-1} and a rotation rate of 1266 rev/s.

22 Calculate the angular momentum of a smoothly rolling billiard ball of mass 100 g and diameter 8 cm rotating at 10 rev/s.

NEI Activity 8.7 SOME TRICKY QUESTIONS

Here are a few tricky questions on momentum. Before you look at the answers below, try discussing them in class.

1. Why is it hard to stand a bicycle upright but if you give it a push, it will roll along without falling over?
2. Why shouldn't you put your foot on the brakes while you're driving a car through a corner? Racing car drivers only accelerate as they come out of a curve, not while they are in it. Why?
3. A ski turn requires a sinking of the whole body followed by a powerful upthrust and a rotation of the upper part of the body. The lower part of the body rotates the opposite way. Why is this?
4. If you spin a hard-boiled egg and stop it with your finger it stays stopped. A fresh egg will start to spin again. Why?
5. Imagine an egg timer that uses falling sand. If you weigh it while some of the sand is falling in mid-air will it weigh less than when all the sand is at the bottom? After all, some of the sand is in the air and not being supported by the balance.

Answers to Activity 8.7

Answer 1 The spinning wheels acquire angular momentum and for the bike to fall over there has to be a change in this momentum. As well, the bike acquires linear momentum and this also has to be altered.

Answer 2 Sudden braking in a turn throws extra weight on to the front wheels and less on the back wheels resulting in less friction in the rear tyres. This makes it more likely for the car to spin out. Conversely, accelerating puts extra weight on the rear tyres, increasing the friction as drivers come out of a turn.

Answer 3 To conserve angular momentum, when the top part of your body twists one way, the lower part twists in the opposite direction.

Answer 4 The contents of a fresh egg continue to spin when the shell is stopped. When the shell is released the contents make the shell spin again.

Answer 5 They will weigh the same. The loss in weight because some of the sand is in mid-air is compensated for by the impact of the sand when it strikes the bottom. These sand grains transfer their momentum and hence extra force to the balance.

Figure 8.37
For question 27.

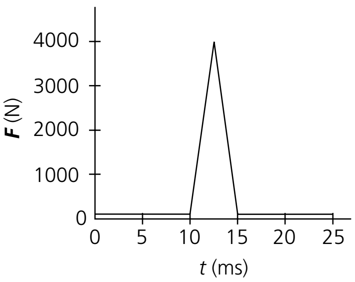


Figure 8.38
For question 28.

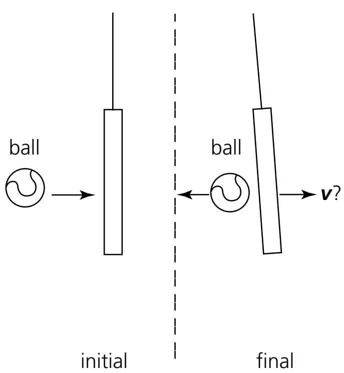
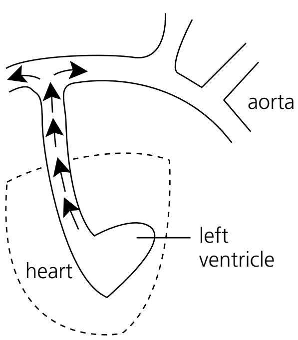


Figure 8.39
Blood from the heart comes up the aorta from the left ventricle. The aorta branches at a 'T'-junction.



Practice questions

The relative difficulty of these questions is indicated by the number of stars beside each question number: * = low; ** = medium; *** = high.

Review — applying principles and problem solving

***23** Masses of 20 kg and 35 kg are on the ends of a 1.4 m long bar. Determine the centre of mass of the system.

***24** What is the momentum of: **(a)** a cricket ball of mass 160 g moving at 12.5 m s^{-1} east; **(b)** a billiard ball of mass 200 g moving at 8.5 m s^{-1} N35°E; **(c)** a 100 kg footballer moving with a velocity of 8 m s^{-1} north?

***25** An alpha particle of mass 7×10^{-27} kg is accelerated from 5×10^{3} m s^{-1} to 2×10^{4} m s^{-1}. Calculate the change in momentum.

***26** A car of mass 2200 kg accelerates from rest at 3 m s^{-2} for 10 s. Determine the impulse imparted to the car.

****27** When a ball of mass 180 g is struck by a bat moving in the opposite direction, the force acting on the ball is as shown in the graph (Figure 8.37). Determine **(a)** the impulse; **(b)** the final velocity of the ball if it was initially moving at 10.0 m s^{-1} south.

****28** A ball of mass 50 g moving horizontally at a speed of 40 cm s^{-1} strikes a suspended plate of mass 1000 g and rebounds from it with a speed of 25 cm s^{-1} as illustrated in Figure 8.38. Find the speed with which the plate begins to move.

****29** Two masses of 4 kg and 3 kg respectively are travelling east along a frictionless surface with respective speeds of 12 m s^{-1} and 5 m s^{-1}. If the 3 kg mass continues to move east with a speed of 9 m s^{-1} after the collision, calculate the speed of the 4 kg mass.

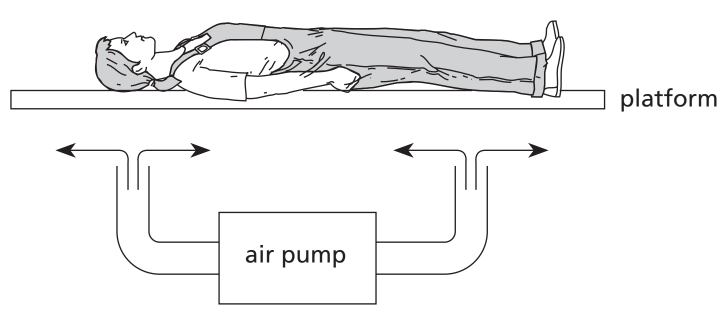
****30** A railway truck of mass 4000 kg moving with a speed of 3.6 m s^{-1} collides with a stationary truck of mass 2400 kg. The two trucks become coupled together. What is their common speed?

****31** What is the angular momentum of the Earth associated with rotation about its own axis? The mass of the Earth is 5.98×10^{24} kg and its radius is 6.37×10^{6} m.

****32** A ball A of mass 6 kg is moving east at 3.5 m s^{-1} when it collides with a stationary ball B of mass 8 kg. Ball A heads north at 5 m s^{-1} after the collision. Determine the final velocity of ball B.

****33** The **ballistocardiograph** (BCG) is an important device in medicine and is designed to employ simple physics to analyse the effectiveness of operations on a patient's heart. With each heartbeat, about 70 g of blood is ejected from the left ventricle of the heart into the aorta (Figure 8.39). The speed of the blood is about 30 cm s^{-1}. Hence the blood from each heartbeat has momentum. The body recoils with each heartbeat due to conservation of momentum. This can be registered on a very sensitive balance attached to the platform on which the body rests. (See Figure 8.40).

Figure 8.40
The air table for the ballistocardiograph.



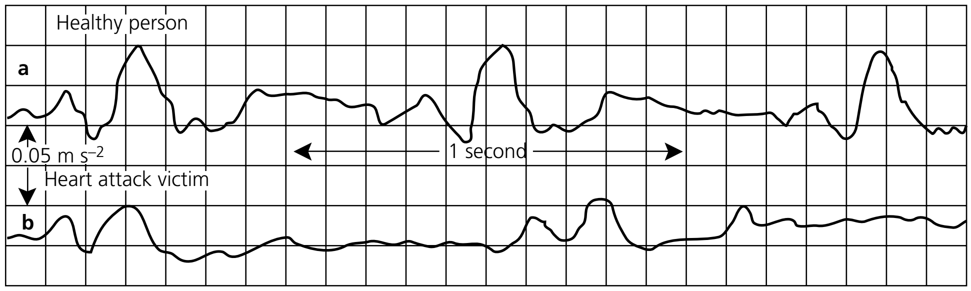


Figure 8.41
A ballistocardiogram of a healthy person and a heart attack victim.

The graph produced shows the acceleration of the body during the different stages of the heartbeat. In Figure 8.41 a ballistocardiogram is shown for both a healthy person and one who has suffered a heart attack. Accelerations can be read to an accuracy of 10^{-5} m s^{-2}.

(a) When the body recoils, the table moves. How has friction been taken into account?

(b) When a pulse of blood travels from ventricle to aorta, which way would the body move?

(c) On the graph, the acceleration goes negative after the main part of the heartbeat. Why is this?

(d) An acceleration of about 0.06 m s^{-2} is considered healthy. What is the value for the heart attack victim?

(e) Extract the information needed to calculate the momentum of a blood pulse and calculate it.

Extension — complex, challenging and novel

*****34** A radioactive Thorium (Th) nucleus decays by emitting an electron and a neutrino at right angles to each other in a horizontal plane. The momentum of the electron is 6×10^{-21} kg m s^{-1} and that of the neutrino 2×10^{-20} kg m s^{-1}. If the mass of the resulting protactinium (Pa) nucleus is 3.905×10^{-25} kg, calculate **(a)** the total momentum of the three particles immediately after the decay; **(b)** the recoil momentum of the nucleus; **(c)** the recoil speed of the nucleus.

*****35** William Tell fires an arrow into a 100 g apple. The arrow has a mass of 100 g and travels at 50 m s^{-1} horizontally. As the arrow hits it, the apple splits into two pieces. One piece weighing 50 g flies vertically up at 20 m s^{-1} while the other piece gets stuck on the arrow and continues on. Calculate the velocity of the arrow and second piece of apple together.

*****36** A body of mass 400 g is moving along a smooth surface at a velocity of 10 m s^{-1} east. It strikes a body of mass 650 g, initially at rest, and then the 400 g body moves at a velocity of 8 m s^{-1} in a direction E35°N. What is the velocity of the 650 g object?

*****37** A rocket of mass 25 000 kg is cruising through space with a constant speed of 1×10^{3} m s^{-1} when exhaust gases are expelled for 10.0 seconds at a rate of 500 kg s^{-1} with a speed of 5×10^{4} m s^{-1}. Calculate the new speed of the rocket.

✔ TEST YOUR UNDERSTANDING

(Answer true or false)

- Objects moving in straight lines cannot have angular momentum.
- Momentum is not a vector.
- Momentum is the same as a force.