CHAPTER 09

Work and Energy

RUNNING OUT OF ENERGY 9.1

If you leave a torch turned on, its batteries will run out of energy. But has the energy gone forever? Where did it go? These are fundamental questions when it comes to energy. As you probably learnt in earlier science studies, energy is not lost — it just gets transferred from one place to another. This is called the **law of conservation of energy**. The universe seems to have a finite amount of energy that is continually being rearranged.

Think about these questions:

- If you stand still, you are using up energy, but where does it go?
- Why can't a ship extract heat from sea water to power its engines?
- Will there really be an energy crisis soon? Are we running out of energy?

NOVEL CHALLENGE

Why do you lean forward when you get up out of a chair?

NOVEL CHALLENGE

When bodies interact, the energy of one may increase at the expense of another. But we can't intercept the energy and bottle it. So comment on this assertion: 'Energy is not a thing; it is a property of a body.'

NEI Activity 9.1 ENERGY AT HOME

To help you become more familiar with the energy, work and power terms, find out the following:

1. *Electric kettle* If you have an electric kettle, look underneath for its rate of energy consumption, which will be expressed in watts (W). For example, the Kambrook Flash has 2000 W stamped on it. Did anyone in the class get below 1600 W or over 2400 W? Why couldn't the manufacturers make a 20 000 W kettle? It would boil water in a flash!
2. *Food energy* Most foods have their energy content written on the label. A 'Popper' apple juice carton states that the energy is 206 kJ per 100 g. But it also expresses it another way. Look at a food container from your cupboard and note the two ways that energy content is expressed.
3. *Engine power* If your family owns a car, truck or motorbike and you can find the owner's manual, find out the power output of the engine. For example, a GXL Turbo Land Cruiser has a power output of 118 kW. But following this number is a further specification to do with the power. What is it?

ENERGY AT WORK 9.2

A simple definition of energy is that **energy is the capacity to do work**. The word 'energy' stems from the Greek *en* meaning 'in', and *ergos* meaning 'work'. But this doesn't really give us a good understanding of the idea of energy and work. Physicists didn't develop a good understanding of these concepts until 100 years after Newton's death. Today these ideas are considered fundamental to the processes of nature.

Energy transfers

The above definition indicates that energy can be converted into useful work; for example, when the electrical energy in a car's battery is used to start the engine. The reverse is also true — work can be converted into stored energy. For example, we can do work to pump water from a lake to a high reservoir. That stored water has higher energy because of its height and can later be used to drive electric generators and produce electrical energy. When energy is transferred *to* an object we say work is done *on* the object; when energy is transferred *away from* an object we say that work is done *by* the object:

- Energy transferred *to* an object (work done *on* the object): e.g. water pumped up to a reservoir.
- Energy transferred *away from* an object (work done *by* the object): e.g. water flows back down.

Energy losses in transfer

When a torch is turned on, some of the energy stored in the chemical bonds is transferred to the electric charge that flows through the bulb. Some of this energy is transferred into light and some as heat energy to the glass bulb and air.

Energy transfers never achieve 100% **efficiency**, that is, some of the energy is transferred to places you don't intend it to go to. For instance, the energy from the torch that goes to heating up the glass and air is wasted — it is a loss in the sense that it didn't get turned into light. But it is not really lost; energy never is. It just goes to the wrong place. **Efficiency is a measure of the useful energy output compared with the energy input**.

$$\%\text{ efficiency} = \frac{\text{energy out}}{\text{energy in}} \times 100\%$$

Some energy transfers are listed in Table 9.1.

NOVEL CHALLENGE

In 1916, a Dr Taylor observed a man carrying 40 kg 'pigs' of iron 11 m up a 2.4 m high incline to a train carriage. He carried 1156 pigs in 10 hours. The man's mass was 65 kg and he rested for 15% of the time.
What was his average power output for the $8\frac{1}{2}$ hours?
On a later occasion and without a rest he could only carry 305 pigs in the 10 hours. By what factor was his power output increased when he had proper rest? Suggest why cyclists use a sprint–coast–sprint sequence.

Table 9.1 ENERGY TRANSFERS AND LOSSES

DEVICE	USEFUL ENERGY TRANSFER (ENERGY IS CONVERTED TO USEFUL WORK)	% OF TOTAL ENERGY TRANSFERRED THAT IS USEFUL (% EFFICIENCY)	NON-USEFUL ENERGY TRANSFERS (ENERGY IS NOT CONVERTED TO USEFUL WORK)
Petrol engine	chemical → mechanical	25	heat, sound
Electric light	electrical → light	5	heat
Fluorescent light	electrical → light	20	non-visible radiation
Solar cell	light → electrical	21	heat; re-emission of light
Battery	chemical → electrical	85	heat
Electric motor	electrical → mechanical	90	heat

INVESTIGATING

What energy transfer occurs for humans? What percentage of the input energy is transferred to non-useful purposes?

Forms of energy

The jumble of terms like light, heat, electricity, sound, mechanical and chemical doesn't provide a systematic way of organising the different forms of energy. Before we can go any further, we need a way of classifying energy.

Bodies that are moving have **kinetic energy** (E_K, *KE*), e.g. a flying bird, a shooting star, a moving locomotive, a speeding bullet.

Bodies that can do work because of their position have **potential energy** (E_P, *PE* or *U*), e.g. water in a reservoir, a compressed spring, a stretched rubber band.

Kinetic and potential energy are said to be forms of **mechanical energy**.

Where does this leave chemical, heat and electrical energy? Because they are to do with the random vibrations or motions of electrons, atoms and molecules within an object, they are said to be forms of **internal energy** (E_i). This chapter deals only with mechanical energy. Heat, sound, electricity and nuclear energy are dealt with in later chapters. Chemical energy is mainly left to the other physical science — chemistry.

WORK 9.3

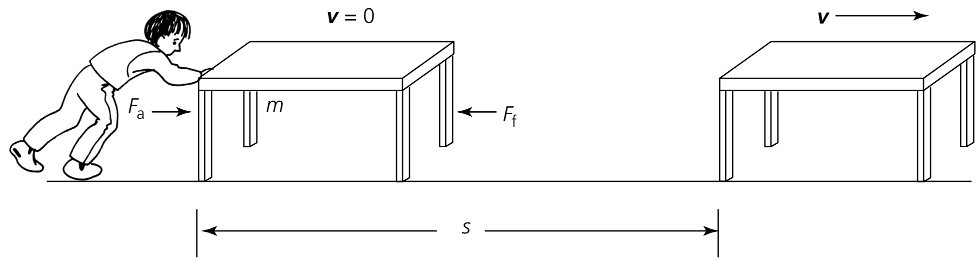


Figure 9.1
The desk with mass *m* is moved from rest a distance *s* across the floor by an applied force ***Fa*** against the frictional force ***F***$_f$. It acquires a velocity v.

NOVEL CHALLENGE

Could you shift a destroyer (a 20 000 tonne ship) moored in a dock with the ropes slack? Let's assume you can apply a force of 500 N. We say 'Yes'; but how long do you think it would take to push it 2 m away from the dock: 400 seconds, 400 hours, or forget it?

If you tried to push a desk across the floor and it didn't move, you might say you did a lot of work on the desk. But to a physicist, if it didn't move then no work was done. If you did move it, then the work done would depend on how hard you pushed and the distance it moved. The word 'work' is often used very loosely; for example, have you done your home*work* tonight? Physicists define work very carefully; **work is defined as the product of the force and the distance moved in the direction of an applied force**. It is a scalar quantity, and yet is the product of two vector quantities.

$$\text{Work} = \text{force} \times \text{displacement} \quad \text{or} \quad W = \mathbf{F}\mathbf{s}$$

Since force is measured in newtons and displacement is measured in metres, work has the units **newton metre** or N m. The newton metre is called the **joule (J)** in honour of James Joule (1818–89), an English physicist who studied heat and electrical energy.

Example

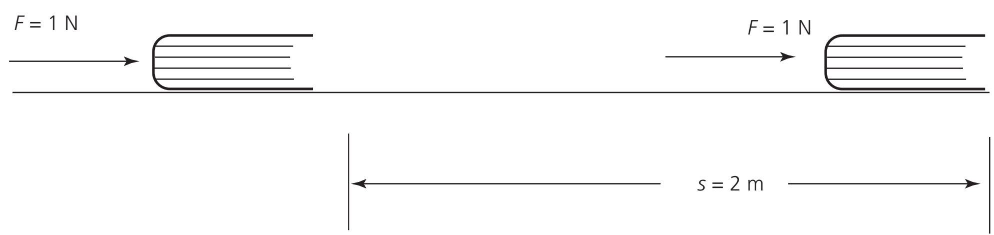


Figure 9.2
Work is done when a force is used to push a book along a desk.

Calculate the work done when a horizontal force of 1 N is used to push a book a distance of 2 m along a desk (Figure 9.2).

Solution

$$W = \mathbf{F}\mathbf{s} = 1\ \text{N} \times 2\ \text{m} = 2\ \text{J}$$

Doing no work in class

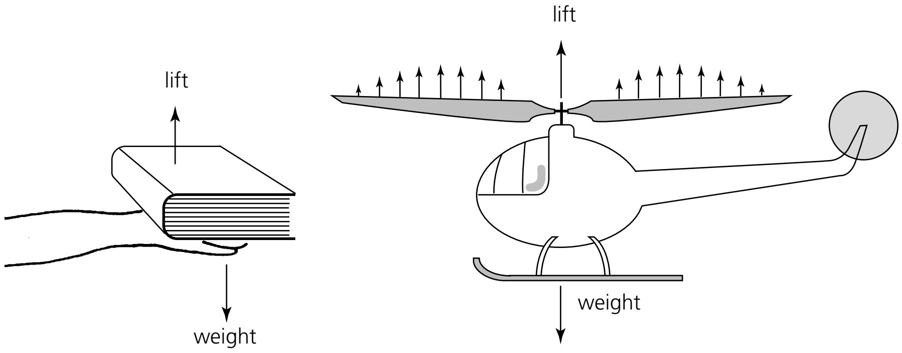


Figure 9.3
No work is done on the book or the helicopter if there is no movement.

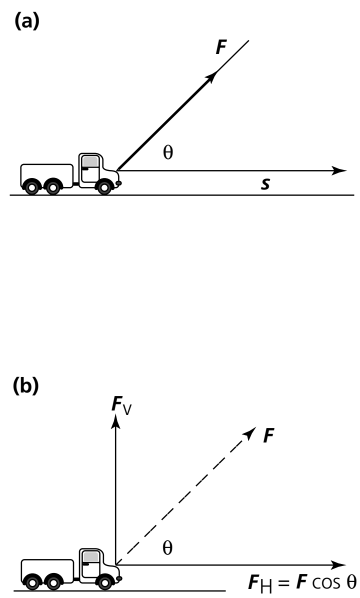
Our definition of work leads to the surprising conclusion that, in a scientific sense, you are not doing any work on a book if you hold it in your outstretched arm for a long period of time. Sure, you get tired — but no work is done *on the book*. You will feel tired because your muscles are using energy and burning up fuel. As the muscle fibres relax and contract just keeping your arm still you are using energy. But it is not being transferred to the book. No work is done on the book. But there is a change in the internal energy of your body as microscopic molecular internal motions and reactions go on. The same is true for a helicopter hovering in a stationary position above the ground. These motions never result in any measurable displacement and therefore never do any work in the sense that the word is used in physics.

Forces at an angle

When you push a lawnmower or a shopping trolley, the force from your arms is at an angle to the direction of the motion. The same is true if a child pulls a toy by a string at an angle θ along the floor as shown in Figure 9.4 **(a)**. In this case the equation for work done is a little different.

If a force (***F***) is used to pull the toy along a horizontal floor, the useful part of the force is the component in the direction of motion. In Chapter 4 you would have seen that this force ($\boldsymbol{F}_{\text{horizontal}}$ or $\boldsymbol{F}_{\text{H}}$) is equal to ***F*** cos θ. This is shown in Figure 9.4 **(b)**. Some of the force is 'wasted' and tends to lift the toy off the floor. It is not converted to useful work pulling the toy along the floor.

Figure 9.4
Pulling a toy truck with a cord that makes an angle θ with respect to the direction of the truck's motion: $W = \boldsymbol{F}\boldsymbol{s} \cos \theta$.



Example

A force of 5.0 N is applied to a string attached to a toy truck that makes an angle of 40.0° to the floor. How much work is done in dragging the truck a distance of 4.0 m across the floor?

Solution

The horizontal component ($\boldsymbol{F}_{\text{H}}$) = ***F*** cos 40° = 5.0 × 0.766 = 3.83 N.

$$W = \boldsymbol{F}\boldsymbol{s} = 3.83 \times 4.0 = 15 \text{ J}$$

Alternatively:

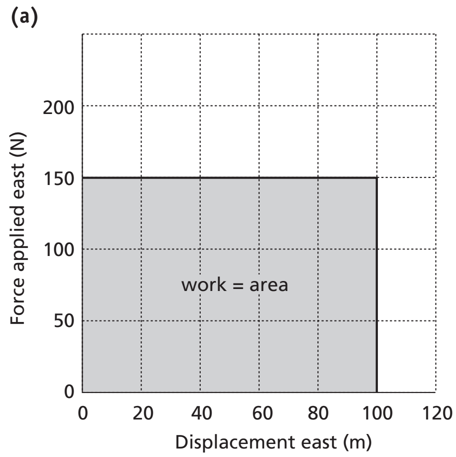
$$W = \boldsymbol{F}\boldsymbol{s} \cos \theta = 5.0 \times 4.0 \times \cos 40° = 15 \text{ J}$$

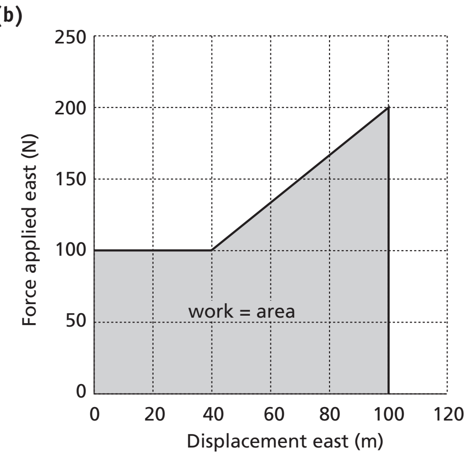
Notice that when $\theta = 0°$, the force is in the direction of motion and the formula returns to $W = \boldsymbol{F}\boldsymbol{s}$ because the cosine of 0° (cos 0°) = 1.

Graphs

Force and displacement can be expressed graphically. In Figure 9.5 **(a)**, a constant force of 150 N is applied to an object and shifts it a distance of 100 m. The work done is simply the product of $\boldsymbol{F} \times \boldsymbol{s}$ and so is the shaded area under the line (150 000 J). If the force does not remain constant but varies as in Figure 9.5 **(b)**, the work is still determined by calculating the shaded area (13 000 J).

Figure 9.5
(a) Area = 150 × 100 = 15 000 J.
(b) Area = (100 × 100) + (60 × 100) ÷ 2 = 13 000 J.





Lifting things

When calculating the work done in lifting an object vertically, the force applied will be equal to the object's weight ($\boldsymbol{F}_w = m\boldsymbol{g}$). This is assuming that it is lifted at constant speed. If the speed is varied then Newton's second law formula ($\boldsymbol{F} = m\boldsymbol{a}$) would have to be applied.

Example

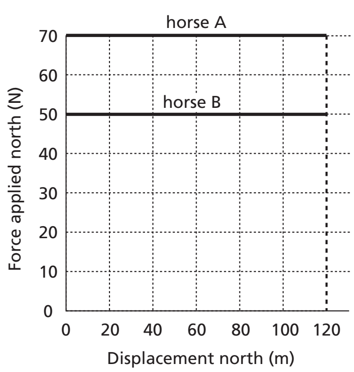
Calculate the work done in lifting a 15 kg schoolbag at constant speed from the floor to a port rack 1.8 m off the ground.

Solution

$$\boldsymbol{F}_w = m\boldsymbol{g} = 15 \times 10 = 150 \text{ N}$$
$$W = \boldsymbol{Fs} = 150 \times 1.8 = 270 \text{ J}$$

A *Mars bar* provides you with 1 235 000 J (1235 kJ) of energy. This is equivalent to lifting your bag 4500 times to burn off the energy. It seems hardly worth the effort! But luckily, in lifting a bag, your body uses up a lot more energy than just the amount needed to overcome gravitational forces. Just as well — wouldn't you get really fat!

Figure 9.6
For question 3.



Questions

1 Calculate the amount of work done in:
(a) pulling a bag of dog food 3.5 m along a table by applying a 25 N horizontal force; **(b)** lifting a 20 kg bag of dog food at constant speed on to a table 85 cm off the ground; **(c)** pumping 200 kg of water at a constant flow rate into a tank 25 m high.

2 A 200 kg piano is lowered by a rope out of a third-floor window.
(a) Calculate how much work is done in lowering the piano a distance of 9 m to the ground at constant speed.
(b) Was work done *on* or *by* the piano in this process?

3 A team of two horses is pulling a loaded cart in a northerly direction along a horizontal road at constant speed. A force–displacement graph is shown in Figure 9.6.

(a) Calculate the work done by each horse.
(b) What is the total work done on the cart?

4 Figure 9.7 shows different forces acting on different objects. Calculate the work done in each case.

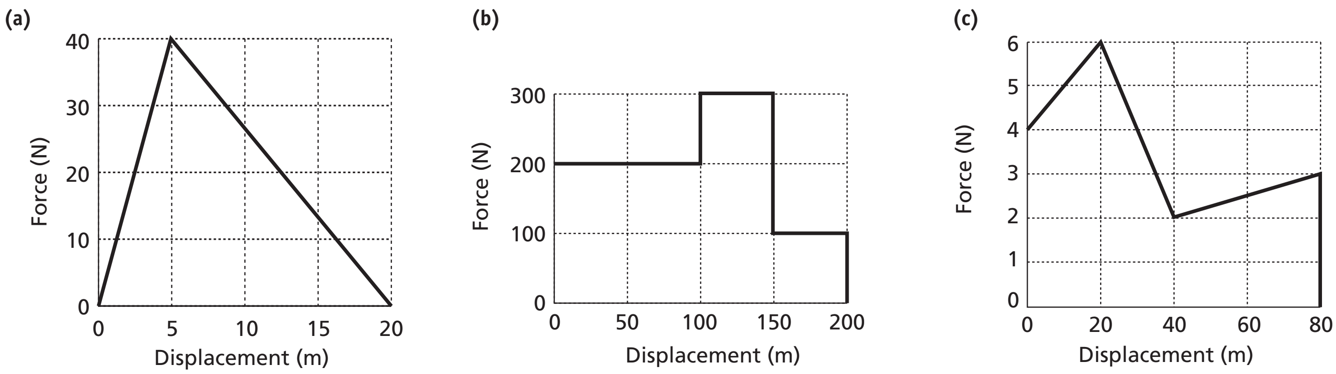


Figure 9.7
For question 4.

9.4 KINETIC ENERGY

A bowling ball resting on the floor has no energy of motion. One that is rolling along a bowling alley does have energy of motion. Energy due to the motion of an object is called **kinetic energy** (Greek *kinema* = 'motion').

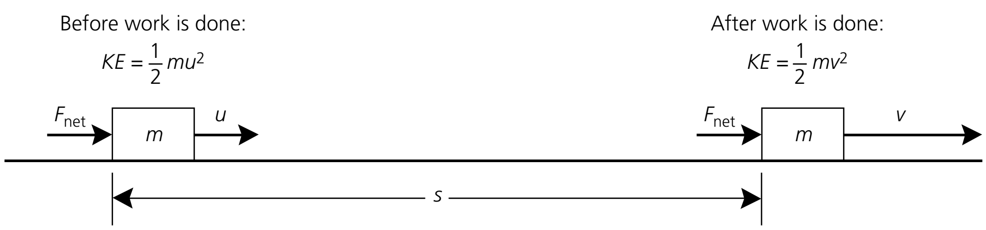


Figure 9.8
Hockey puck on an air table.

To determine an equation for kinetic energy, we will use concepts already developed. Imagine a hockey puck moving on a frictionless surface such as an air table. If an unbalanced force $\boldsymbol{F}_{net}$ is applied to it for a period of time t, the force produces accelerated motion and the object goes from an initial velocity $\boldsymbol{u}$ to a final velocity $\boldsymbol{v}$:

$$\boldsymbol{v}^2 = \boldsymbol{u}^2 + 2\boldsymbol{a}\boldsymbol{s}$$
$$\boldsymbol{a} = \frac{\boldsymbol{v}^2 - \boldsymbol{u}^2}{2\boldsymbol{s}}$$

By letting $F = m\boldsymbol{a}$, the work done in accelerating the puck is given by:

$$W = \boldsymbol{F}\boldsymbol{s} = m\boldsymbol{a}\boldsymbol{s} = m\left(\frac{\boldsymbol{v}^2 - \boldsymbol{u}^2}{2\boldsymbol{s}}\right) \times \boldsymbol{s} = \tfrac{1}{2}m\boldsymbol{v}^2 - \tfrac{1}{2}m\boldsymbol{u}^2$$

This represents a change in the quantity that we call kinetic energy.

Work done equals change in kinetic energy $W = \Delta E_K$

Hence: if an object starts from rest, its final kinetic energy is given by:

$$E_K = \tfrac{1}{2}m\boldsymbol{v}^2$$

This is properly referred to as its **translational** kinetic energy because rolling or rotating objects also have **rotational** kinetic energy. This is not dealt with here.

You or your teacher may prefer to use the symbol *KE* for kinetic energy — it's a matter of choice. Note that kinetic energy is a scalar quantity and hence does not require direction.

Example 1

Calculate the translational kinetic energy of a 6.0 kg bowling ball rolling at 5.0 m s^{-1}.

Solution

$$E_K = \tfrac{1}{2}m\boldsymbol{v}^2 = \tfrac{1}{2} \times 6 \times 5^2 = 75 \text{ J}$$

Example 2

A 520 kg rocket sled at rest is propelled along the ice by an engine developing a constant thrust of 12 000 N. Assuming all of the work goes into motion, calculate its velocity after 40 m.

Solution

- Work done by engine: $W = \boldsymbol{Fs} = 480\ 000$ J.
- Work is converted to kinetic energy, hence $E_K = 480\ 000$ J.

$$E_K = \tfrac{1}{2}m\boldsymbol{v}^2 \quad \text{or} \quad 480\ 000 = \tfrac{1}{2} \times 520\ \boldsymbol{v}^2$$

$$\boldsymbol{v} = \sqrt{\frac{2 \times 480\ 000}{520}} = 43 \text{ m s}^{-1}$$

The J-manoeuvre

In aircraft dogfights, it can be difficult to out-manoeuvre planes of similar ability. A technique developed by the US Airforce can tip the balance. It is called the *J-manoeuvre*. When an exhaust nozzle that can turn sideways, up or down is added to the rear of their F-16 fighters, they can radically alter the performance of the planes. The rotating exhaust is similar to that used in the British Harrier Jump Jets (remember Schwarzenegger in the movie *True Lies*!), but is far more flexible. With it, pilots can bring a 2200 km h^{-1} F-16 to a halt in a few seconds.

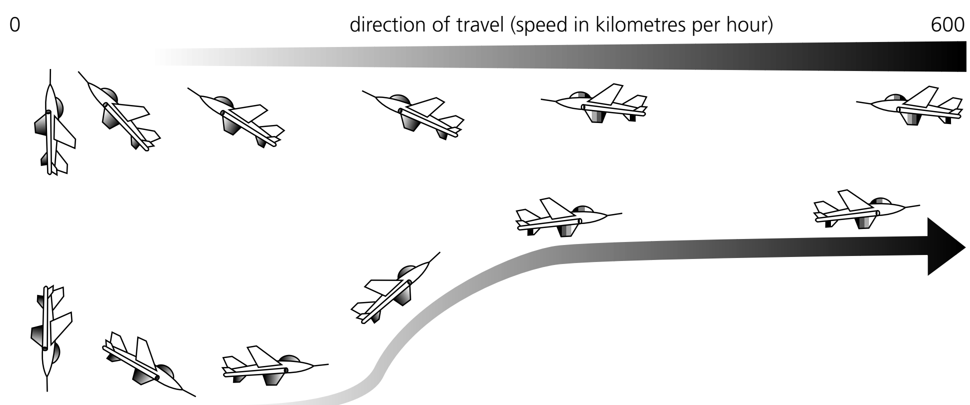


Figure 9.9
The J-manoeuvre: the aerial equivalent of a handbrake turn.

Conventional jet engines thrust the plane forward and the pilot steers using aerodynamic controls — ailerons, elevator and rudder — which alter the air stream and cause the plane to change directions. With the J-manoeuvre, the pilot pulls the nose of the plane through 90° vertically while the plane continues to travel horizontally 'belly-first' (Figure 9.9). This brings the plane to a halt without climbing (the normal way to slow down). The pilot then turns the plane through 180° sideways and points its nose downward and as it picks up speed, the pilot brings it back to level flight. It is much the same as a motorcycle 'wheelie'. So, a 13 tonne jet fighter can stop in about 5 seconds and then reverse directions. This is a 'loss' of 2.4 billion joules of kinetic energy or about 500 megawatts of power being shed without change in potential energy (altitude) — a clever trick. The pilot needs a really strong seatbelt harness system; and a strong stomach.

NOVEL CHALLENGE

In November 1998, forensic police officer Gerard Dutton of the Tasmania Police in Hobart investigated an incident where a man was killed by a piece of fencing wire 27 mm long, 2.4 mm diameter and 0.89 g mass that was hurled up by a roadside slasher mower. The death was curious because international data on 'incapacitation energy' developed by the US Army said that a person would be killed only if the energy of a projectile was between 40 and 236 joules. The fencing wire fragment was thrown from the edge of a rotating blade of radius 54 cm travelling at 2000 revolutions per minute. Was the projectile's energy within the incapacitation energy range?

9.5 ENERGY CHANGES AND COLLISIONS

In the chapter on momentum we saw that in *all* collisions, momentum is conserved. We will now look at conservation of kinetic energy in collisions.

Collisions can be either **elastic** or **inelastic**. The word elastic comes from the Greek *elastikós* meaning 'to drive' or 'propel'.

Elastic collisions

An elastic collision is one in which kinetic energy is conserved.

Conserve is from the Latin for 'to preserve' or 'keep the same'. Jams are often called conserves because they preserve the fruit. In elastic collisions the total amount of kinetic energy before the collision is the same as the total kinetic energy after the collision. Collisions between gas molecules are perfectly elastic. If they weren't, the gas would lose energy and the pressure in a spray can would decrease while it was only sitting on a shelf. Clearly this does not happen. Collisions between steel ball bearings is also approximately elastic.

For an elastic collision (Figure 9.10):

$$E_K(\text{initial}) = E_K(\text{final})$$
$$\tfrac{1}{2}m\boldsymbol{u}_a^2 + \tfrac{1}{2}m\boldsymbol{u}_b^2 = \tfrac{1}{2}m\boldsymbol{v}_a^2 + \tfrac{1}{2}m\boldsymbol{v}_b^2$$

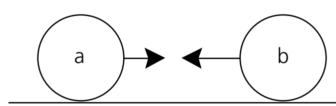


Figure 9.10
Balls colliding head-on.

Example 1

A 3 kg steel ball moving east at 4 m s^{-1} collides with a stationary 1 kg ball. After the collision, the 3 kg mass moves east at 2 m s^{-1} and the 1 kg mass moves east at 6 m s^{-1} (Figure 9.11). Is this collision elastic?

Solution

$$\begin{aligned} E_K(\text{initial}) &= \tfrac{1}{2}m\boldsymbol{u}_a^2 + \tfrac{1}{2}m\boldsymbol{u}_b^2 \\ &= \tfrac{1}{2}\ 3 \times 4^2 + \tfrac{1}{2}\ 1 \times 0^2 \\ &= 24 \text{ J} \\ E_K(\text{final}) &= \tfrac{1}{2}m\boldsymbol{v}_a^2 + \tfrac{1}{2}m\boldsymbol{v}_b^2 \\ &= \tfrac{1}{2}\ 3 \times 2^2 + \tfrac{1}{2}\ 1 \times 6^2 \\ &= 24 \text{ J} \end{aligned}$$

The collision is elastic.

Note: check for yourself that momentum is conserved.

Figure 9.11
Collision of balls in Example 1.

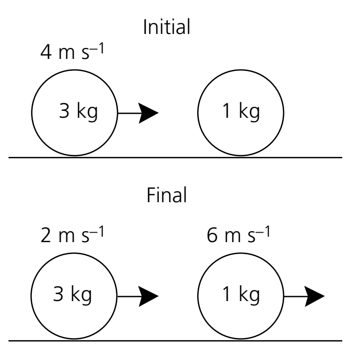
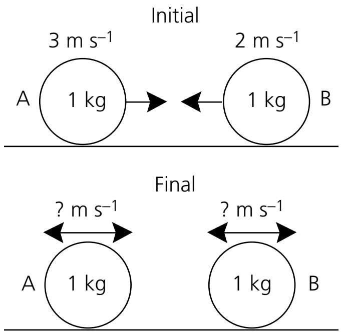


Figure 9.12
Collision of balls in Example 2.



Example 2

An object A of mass 1.0 kg moving at 3.0 m s^{-1} east collides elastically head-on with an object B of mass 1.0 kg moving at 2 m s^{-1} west. Determine the final velocity of each object. (See Figure 9.12.)

Solution

The solution involves some *complex reasoning* as conservation of both momentum *and* kinetic energy is required.

Let east be the positive direction.

1 *Conservation of momentum:*

$$
\begin{aligned}
m_A\boldsymbol{u}_A + m_B\boldsymbol{u}_B &= m_A\boldsymbol{v}_A + m_B\boldsymbol{v}_B \\
1 \times 3 + 1 \times {}^-2 &= 1 \times \boldsymbol{v}_A + 1 \times \boldsymbol{v}_B \\
3 - 2 &= \boldsymbol{v}_A + \boldsymbol{v}_B \\
1 &= \boldsymbol{v}_A + \boldsymbol{v}_B \\
\boldsymbol{v}_A &= 1 - \boldsymbol{v}_B
\end{aligned}
$$

2 *Conservation of kinetic energy:*

$$\tfrac{1}{2}m\boldsymbol{u}_A^{\,2} + \tfrac{1}{2}m\boldsymbol{u}_B^{\,2} = \tfrac{1}{2}m\boldsymbol{v}_A^{\,2} + \tfrac{1}{2}m\boldsymbol{v}_B^{\,2}$$

$$\tfrac{1}{2} \times 1 \times 3^2 + \tfrac{1}{2} \times 1 \times -2^2 = \tfrac{1}{2} \times 1 \times \boldsymbol{v}_A^{\,2} + \tfrac{1}{2} \times 1 \times \boldsymbol{v}_B^{\,2}$$

Multiply both sides by 2 to cancel out the $\tfrac{1}{2}$

$$
\begin{aligned}
9 + 4 &= \boldsymbol{v}_A^{\,2} + \boldsymbol{v}_B^{\,2} \\
13 &= \boldsymbol{v}_A^{\,2} + \boldsymbol{v}_B^{\,2} \\
\boldsymbol{v}_A^{\,2} &= 13 - \boldsymbol{v}_B^{\,2}
\end{aligned}
$$

3 *Solving both equations simultaneously and eliminating the term* $\boldsymbol{v}_A$:

$$\boldsymbol{v}_A^{\,2} = (1 - \boldsymbol{v}_B)^2 = 1 - 2\boldsymbol{v}_B + \boldsymbol{v}_B^{\,2} \text{ and } \boldsymbol{v}_A^{\,2} = 13 - \boldsymbol{v}_B^{\,2}$$

hence: $$1 - 2\boldsymbol{v}_B + \boldsymbol{v}_B^{\,2} = 13 - \boldsymbol{v}_B^{\,2}$$

$$2\boldsymbol{v}_B^{\,2} - 2\boldsymbol{v}_B - 12 = 0$$

using the quadratic formula or factorising into $(\boldsymbol{v}_B - 3)(\boldsymbol{v}_B + 2) = 0$, gives two solutions for $\boldsymbol{v}_B$. They are $\boldsymbol{v}_B = 3$ m s^{-1} and -2 m s^{-1}. The second of these solutions is the case where the velocities are the same as the initial. In other words, no collision took place; they were on parallel but separate tracks.

The first solution is where a collision took place. In this case $\boldsymbol{v}_B = 3$ m s^{-1} and substituting this into the equation $\boldsymbol{v}_A = \boldsymbol{v}_B - 1$ gives a value for $\boldsymbol{v}_A$ of -2 m s^{-1} (west).

This is a long procedure and students find many pitfalls. But if you are careful and meticulous and practise many more examples, success should be yours.

Equal masses colliding elastically

Note that in the above example, the objects swapped their velocities. Objects A and B had speeds of 3 and -2 m s^{-1} respectively; after collision, these had become -2 and 3 m s^{-1} respectively. This is true of linear elastic collisions between objects of **equal mass**. It is sometimes called the 'pool player's result'.

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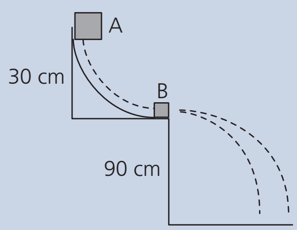
Activity 9.2 THE POOL HALL REVISITED

1. Next time you are playing pool or billiards, hit a ball into a stationary one and see if they swap velocities. The cue ball should stop moving and the struck ball should take off. As explained in previous chapters, if you impart spin to the ball other effects come into play. For this experiment you should hit the ball about seven-tenths of the diameter from the base. This produces no spin. Can you explain why it won't spin?

NOVEL CHALLENGE

Blocks A and B are of mass 2*m* and *m* respectively. Block A is allowed to slide down the curved incline until it hits block B in an elastic collision. Which block will travel the farthest out? How far away from each other will they strike the ground? We say 1.6 m; how about you? We think you'll need conservation of energy *and* momentum to do this one.



INVESTIGATING

To counter the high bounce of the WACA (Perth) cricket ground, many players use bats that have the centre of percussion (the 'meat') higher up the bat than normal. When in Brisbane they can revert to a bat with lower centre of percussion and in India where the wickets are really flat, it's even lower. *How many centimetres variation are there in the bats?*

2 Try it again but have one ball moving slowly and the other fast. Do they swap velocities?

3 If you can't get to a pool table, try it with ball bearings rolling from opposite directions along a grooved and curved ruler or try it with Newton's cradle (Newton's balls). (See Figure 9.13**(b)**.)

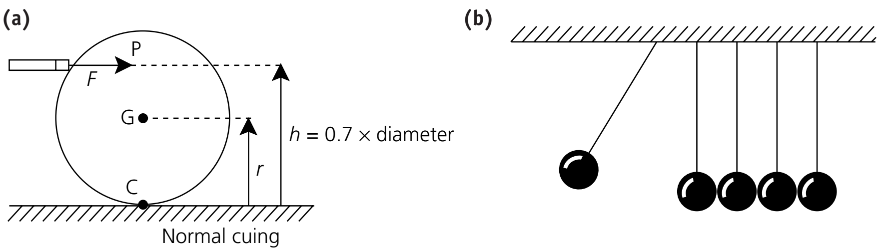


Figure 9.13
(a) The force *F*, applied along the line through point P, causes translation and also rotation about the centre of mass G;
(b) Newton's balls.

Inelastic collisions

An inelastic collision is one in which the total kinetic energy is **not** conserved.

Kinetic energy is lost. The 'missing' kinetic energy is transferred to other types of energy such as thermal (heat) energy and sound. Most collisions in the real world are of this sort. When two objects cling or lock together after impact (i.e. they become coupled), the collision is inelastic.
Examples include:
- cars colliding
- bullet hitting a target
- meteorite striking the earth
- tennis ball being struck by a racquet.

When different objects are dropped on to a concrete floor, they bounce to different heights. A perfectly elastic collision would see the ball returning to its original height. A superball is about 90% elastic, a golf ball about 60% and a lump of putty is perfectly *inelastic*.

EI Activity 9.3 ANYONE FOR TENNIS?

What part of the tennis racquet should you use to hit a tennis ball? Well, it all depends on whether you are serving or returning. In this activity you can check and extend the results of Rod Cross, a physicist from the University of Sydney who clamped a tennis racquet to a bench and measured the bounce of a tennis ball from different power points of the racquet head. He found that a point 5 cm from the top end was a dead spot giving no bounce at all. This is good for getting maximum power into the serve as all the kinetic energy of the racquet goes into the ball. At the other end (5 cm in), there is another power spot, which has maximum bounce. This is good for returning a fast ball, but no good for a serve. In the middle is the centre of percussion — no 'ringing' of the hands and gives medium bounce.

1 Clamp a racquet to a bench and repeat his tests. Graph the bounce (in cm) for a given drop height versus the distance from the top of the racquet.

2 Does the shape of the graph change when the drop height (i.e. the speed) changes? Of what significance is this?

3 How does the bounce height vary across the racquet? Have you ever seen a three-dimensional graph? How could you show the variation in bounce height across the racquet as well as from the top-to-bottom of the racquet on the one graph?

4 Obtain a videotape of a power server and see where the player hits the ball for a serve. Does he or she really use the dead spot?

Figure 9.14

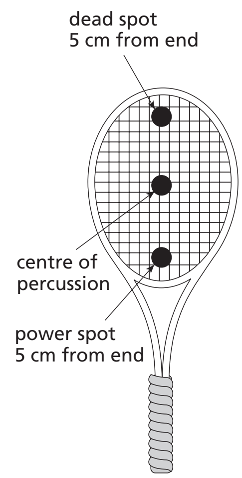
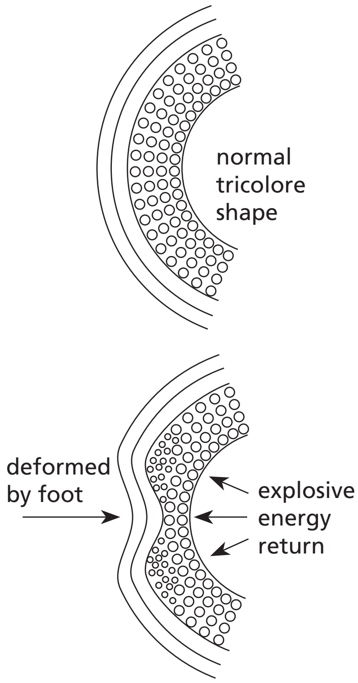


Figure 9.15



World Cup players face a whole new ball game

In the mid-1990s, the governing body of soccer (FIFA) decided to change the nature of the regulation soccer ball to add more excitement and goals to matches. Adidas produced a new ball that had 5% extra 'zing', much to the consternation of soccer players, who began over-hitting the ball. The Questra ball has a special polyurethane coating that reduces friction and, on the inside, has a rubbery layer to get 'more bang from the boot'. It achieves this by reducing deformation of the ball and allowing more of the kinetic energy to be transferred from the foot to the ball. In the FIFA bounce test, a ball is dropped from 2.0 m and the time is measured for it to reach its maximum height after bouncing. The Questra ball takes 5% less time than the traditional World Cup ball — the Italian Etrusco Unico.

For the World Cup 'France-98', the Adidas Tricolore was used. It has a mass of 450 g and circumference of 70 cm. The outer layer was a printed polyester film, the second layer a polyurethane matrix of individually closed gas-filled microballoons (see Figure 9.15). The third layer was a poly-cotton mixed fibre backing and the fourth layer was the latex rubber balloon of about 70 kPa pressure.

NOVEL CHALLENGE

The US baseball manufacturer Rawlings said they hadn't changed the manufacture of balls since 1931; however, it seems odd that balls from the 1970s bounced 157 cm when dropped from 462 cm (15 foot), whereas balls from the 1990s bounced 208 cm. What is intriguing is that in the 1970s players hit 61 home runs per season, whereas in the 1990 they hit 68 home runs. What is the interpretation?

EI Activity 9.4 BOUNCING BALLS

1. You are going to drop different balls on to a concrete floor from a height of 1.8 m and measure the height to which they bounce. Design a means of measuring the bounce height. Express their bounce height as a percentage of the drop height. This is called **restitution**. Some secondhand data are listed on the next page.
2. What characteristics do the higher bouncing balls have in common and how do they differ from the less bouncy balls?
3. Try inflating a volleyball to different pressures and comparing pressure versus bounce height. Is it linear? If not, why not?
4. Where does the energy go?

EI Activity 9.5 DATA-LOGGING BALLS

You can perform the activity illustrated in Figure 9.16 using a TI graphing calculator and a TI 'Ranger'. Other manufacturers (e.g. Casio) have similar equipment.

1. Hold the ball at least 50 cm away from the detector. When you are ready to start collecting data, press ENTER on the TI calculator to start the motion graph.
2. When the motion detector starts clicking, release the ball from rest and allow it to bounce up and down directly below the detector.
3. Analyse your graph to determine the maximum height.

You could also perform this experiment using the CBL and a Bounce program downloaded from the TI web page.

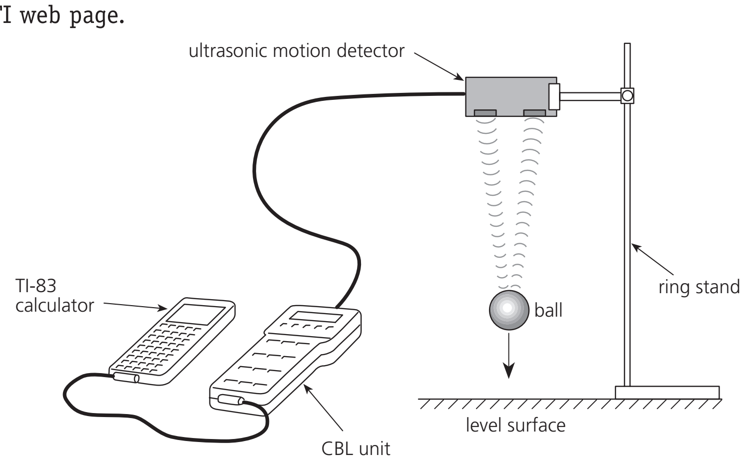


Figure 9.16
A motion detector and data-logger being used to measure bounce heights.

Activity 9.6 GETTING WARMER

In this activity you are to compare the bounce heights of a squash ball at different temperatures. Use a 2 m drop height to get a reasonable rebound.

1. Take three squash balls, put one in ice water (0°C), leave one at room temperature (measure), and put one in boiling water (100°C).
2. Compare the bounce height as before and account for your results.

In the 1965 baseball series, the Detroit Tigers accused the Chicago White Sox of illegally refrigerating the ball. The Tigers only scored 17 runs in 5 games using the cold balls whereas the White Sox accused the Tigers of cooking the balls to score 59 runs in the previous 5 games (including 19 home runs).

Although there is always some bounce associated with collisions, in this section we will deal with collisions that are almost totally inelastic for the sake of simplicity.

Example

A body of mass 6 kg travelling east at 4 m s^{-1} strikes a 2 kg mass at rest. After the collision they remain coupled and the mass moves east at 3 m s^{-1}. Is the collision elastic or inelastic?

Solution

$$\begin{aligned} E_K(\text{initial}) &= \tfrac{1}{2}m\boldsymbol{u}_A^{\,2} + \tfrac{1}{2}m\boldsymbol{u}_B^{\,2} \\ &= \tfrac{1}{2}\,6 \times 4^2 + \tfrac{1}{2}\,2 \times 0^2 \\ &= 48\text{ J} \\ E_K(\text{final}) &= \tfrac{1}{2}m\boldsymbol{v}^2_{A+B} \\ &= \tfrac{1}{2}(6+2) \times 3^2 \\ &= 36\text{ J} \end{aligned}$$

As kinetic energy is lost, the collision is not elastic.

Questions

5 A 2 kg steel ball A travelling west at 5 m s^{-1} collides elastically head-on with a stationary ball B also of mass 2 kg. Without doing any calculations, state the velocities (including directions) of the two balls after collision.

6 A 0.20 kg ball A moving with a speed of 1.75 m s^{-1} approaches a stationary second ball B of mass 0.15 kg, head-on. After the collision, ball B travels at 2.0 m s^{-1} in the same direction. **(a)** Calculate the speed of ball A after the collision. **(b)** Was the collision elastic?

7 A 12 000 kg railway truck travelling along a straight track at a speed of 12 m s^{-1} collides with an identical stationary truck. If the trucks lock together as a result of the collision, calculate **(a)** their common speed; **(b)** the loss of kinetic energy.

9.6 POTENTIAL ENERGY

When you lift something off the floor and put it on a desk, you are applying a force to the object and displacing it — you are doing work on it. If work is done then it gains energy. If it is moved at constant speed then it is not gaining kinetic energy but gaining energy of position. This is called **gravitational potential energy** (Latin *potens* = 'capable'). It has the symbol E_P, *PE* or *U*. When you need to distinguish it from other forms of potential energy it is often written as GPE. In this book we will use E_P in formulas and PE or GPE as an abbreviation.

OUR RESULTS FOR ACTIVITY 9.6

If you need second-hand data or just want to compare, here are some typical results. The drop height was 100 cm.

Tennis ball	48%
Golf ball	60%
Table tennis	26%
Superball	83%

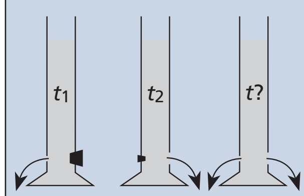
Note: the official standards for a basketball is a rebound height of 125.5 cm to 137 cm from a drop height of 182.9 cm; volleyball — 152.4 to 165 cm from 254 cm. Go and check the PE department's ball.

PHYSICS FACT

The 'restitution' (% energy returned after compression) values for animal bodies are as follows: kangaroo 40%, protein collagen 93%, resilin 97%. Resilin was discovered in 1960 by Danish scientist Dr Torkel Weis-Fogh. It works so well because it doesn't get hot. It is the natural polymer in flying insects; they use it for elastic storage in their wing hinges. This sounds like a good topic for an investigation.

NOVEL CHALLENGE

A plastic measuring cylinder has two stoppered holes near the base, one twice the diameter of the other. The cylinder is filled with water and the small stopper removed. It takes t_1 seconds to empty. When repeated with just the big stopper removed it takes t_2 seconds. *How long will it take with both stoppers removed? Do it algebraically first before you wreck a good measuring cylinder.*



NOVEL CHALLENGE

I live on a hill, and if I let my 2000 kg car roll down the incline, by the time it has travelled 120 m to the bottom it is going at 25 km h^{-1}. The start is 5 m higher than the finish. What average frictional force must be acting?

The work done is a measure of the change in gravitational potential energy. If the object is lifted at constant speed then the force applied equals its weight ($\boldsymbol{F}_w$) and the vertical distance is given the symbol for height h:

$$W = \boldsymbol{F}\boldsymbol{s} = mgh$$

Hence a 5 kg ball raised 20 m will have work done on it or a change in potential energy equivalent to 5 × 10 × 20 J = 1000 J. We assume that an object on the ground has zero GPE so the GPE of the ball is 1000 J. When the ground is the zero reference we can say that **objects raised gain GPE and objects falling lose GPE**.

Example 1

Calculate the GPE of a 20 kg box of groceries lifted 0.75 m to a bench top.

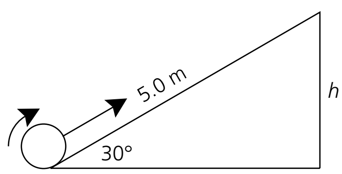
Solution

$$E_P = mgh = 20 \times 10 \times 0.75 = 150 \text{ J}$$

Example 2

A 35 kg beer keg is rolled up a 5 m long plank, which makes a 30° incline to the ground. What is the GPE of the keg at the top (Figure 9.17)?

Figure 9.17
Beer keg being rolled up incline.



Solution

A 30° incline with an hypotenuse of 5 m has a vertical height given by: 5.0 sin 30° = 2.5 m.

$$E_P = mgh = 35 \times 10 \times 2.5 = 875 \text{ J}$$

In summary, gravitational potential energy is defined as the energy associated with the state of separation between bodies that attract each other via the gravitational force. Mathematically, $E_P = mgh$. This of course will only be true over distances where the gravitational force remains constant. When you get too far away from the Earth's surface this relationship does not hold.

Questions

8 A brick of mass 2.5 kg is lifted to a height of 2.5 m above the ground by a bricklayer. Calculate **(a)** the GPE acquired by the brick; **(b)** the work done by the bricklayer in lifting it.

9 Assume that 1 kJ of work is done in lifting a 30 kg steel ball from the ground to the top of a tower. How high is the tower?

10 A 75 kg skier travels down a 40° slope a distance of 100 m. What is his change in potential energy?

NOVEL CHALLENGE

Have you ever tried to compare airconditioner specifications? It's most confusing. We know that 1 horsepower (HP) is equivalent to 0.746 kW, but we saw an airconditioner at school that said '2 HP (4.6 kW)'. What the …! The problem is that HP is the output of the motor, whereas kW is the output of the cooling system after the losses due to conversion of electrical energy to heat energy have been taken into account. The formula is: kW × motor efficiency = HP × 0.746. So what is the percentage efficiency of the school airconditioner?

POWER 9.7

A horse generates about 1 horsepower; a Corolla engine develops about 57 kilowatts of power. Which is the stronger? A horsepower (hp) is the old measure of **power** output where 1 horsepower equals 746 watts, so the car is more powerful. The word power comes from Anglo-Norman *poer* = 'ability to do things'. **Power is a measure of the rate of energy output** — it has the units **joules per second** (J s^{-1}). One J s^{-1} is called 1 **watt** (W) in honour of James Watt (1736–1819), a Scottish physicist who was the inventor of the first practical steam engine. One of his engines, the 26 hp Boulton and Watt, was restored in 1971 and is now in operation pumping water on the Kennet and Avon Canal in the UK.

$$P = \frac{W}{t} = \frac{\Delta E}{t}$$

Example 1

What is the power output of a cyclist who transforms 2.7×10^4 J of energy in 3.0 minutes?

Solution

$$P = \frac{W}{t}$$
$$= \frac{2.7 \times 10^4 \text{ J}}{180 \text{ s}} = 150 \text{ W}$$

NOVEL CHALLENGE

In 1894, bored British house-husband J. C. Ware cycled on a stationary bike for 16 hours and used 42 kJ. What was his average power output?

Example 2

A 52 kg student runs up a flight of stairs of vertical height 3.0 m in 4.7 s. Calculate the power output.

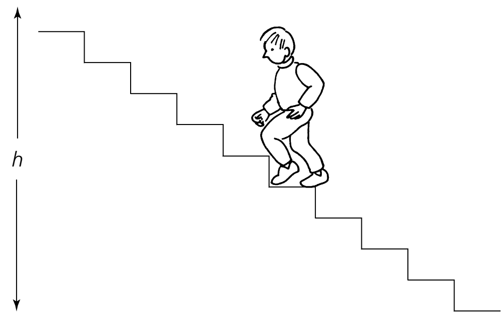
Solution

$$P = \frac{W}{t}$$
$$= \frac{mgh}{t}$$
$$= \frac{52 \times 10 \times 3}{4.7}$$
$$= 330 \text{ W (about half a horsepower)}$$

EI Activity 9.7 YOUR PERSONAL POWER OUTPUT

1. Measure the vertical height of a flight of stairs. If you have access to a building with several storeys this is even better.
2. Use a stopwatch to time yourself running up the flight of stairs.
3. Measure your mass on some bathroom scales and calculate your power output.
4. If you use a multi-storey building, how does your power output over the first flight compare with the output over the last flight?

Figure 9.18



Typical results for Year 11 boys and girls is about 500 W. The Grand Rialto Stair Trek is a race held annually to see who can run up the steps of one of the tallest buildings in the world. The record is held by a 66 kg man who can run up the 122 steps (vertical height = 247 m) in 6 min 55 s. Show that his power output is 374 watts, or about a half-horsepower. The best result is for a 60 kg woman who did it in 7 min 58 s (304 W).

Power and velocity

The original definition of power was for the unit *horsepower*. It was defined as the rate of energy needed to lift a 550 pound weight at a speed of 1 foot per second upward. As weight is a measure of force, it implies that power is the product of force and velocity ($P = \boldsymbol{Fv}$).

This can be shown mathematically:

$$P = \frac{W}{t} = \frac{\boldsymbol{Fs}}{t}$$

$$\text{As } \frac{\boldsymbol{s}}{t} = \boldsymbol{v}, \text{ then } P = \boldsymbol{Fv}$$

NOVEL CHALLENGE

Postulate 1: knowledge = power
Postulate 2: time = money
As every physicist knows:

$$\frac{\text{work}}{\text{time}} = \text{power}$$

Since knowledge = power, and time = money, we have:

$$\frac{\text{work}}{\text{money}} = \text{knowledge}$$

Solving for money, we get:

$$\frac{\text{work}}{\text{knowledge}} = \text{money}$$

Thus, as knowledge approaches zero, money approaches infinity regardless of the amount of work done.
Evaluate the conclusion:
'The less you know, the more you make'.

Example

An upward force of 6 kN is required to raise a mine cage vertically at a speed of 2.5 m s^{-1}. Calculate the power output of the motor lifting the mine cage.

Solution

$$P = \boldsymbol{Fv} = 6 \times 10^3 \times 2.5 = 15\ 000\ \text{W}$$

Activity 9.8 POWER PACKED

SR

EI

1 *Car power* Use the *Guinness Book of Records* to find out the most powerful production car ever made. It seems a long time ago, doesn't it? Obtain a brochure or magazine with your most desirable car in it and note its power output. How many of them would be needed to equal the power of the world's most powerful car?

2 *Microwave power* Put 1 L of tap water in an icecream container, measure its temperature and place it in a microwave oven. Run for 1 minute on 'high'. Stir it, take the temperature again and calculate the power output by using the formula: $P = 70 \times \Delta T$ watts. Take note of the oven's power rating, usually stamped on a tag on the back. Calculate the efficiency of transferring electrical energy to heat energy in the water. Compare notes with others in your class. Which model is the most efficient? Where do the losses occur?

Questions

11 The world's most powerful windmill is the turbine built in Orkney (UK). Its blades are 60 m long and it was turned on in a hurricane in 1987. It now produces enough electricity for 2000 houses at an average of 1.5 kW each. Calculate the power output of the turbine.

12 Calculate the power involved in **(a)** lifting 100 kg on to a 1.2 m high bench in 2 s; **(b)** raising a 2.7 t Land Cruiser up 1.9 m on a hydraulic hoist in 15 s.

13 A 2200 kg Ford Falcon accelerates from 10 m s^{-1} to 15 m s^{-1} in 20 s. Calculate **(a)** the initial and final kinetic energy; **(b)** the work done; **(c)** the power output (in kW) assuming all engine energy goes to changing the kinetic energy of the car.

14 The engine of a jet aeroplane develops 2.5 MW of power when in level flight and travelling at a constant speed of 14 m s^{-1}. Calculate the force being developed by the engine (to overcome air resistance and generate lift).

15 James Watt defined the horsepower (hp) by measuring what weight of coal a work horse could raise up a mineshaft at a standard 1 foot per second. He found that it weighed 550 pounds. Show that a 550 pound weight being lifted vertically at 1 foot per second equals 746 W (1 hp), given: 1 foot = 0.3048 m, 1 pound = 0.4536 kg. Use $\boldsymbol{g} = 9.81$ m s^{-2}.

CONSERVATION OF MECHANICAL ENERGY 9.8

Mechanical energy was earlier described as including both kinetic and potential energy. The total mechanical energy of a system can be defined as the sum of kinetic and potential energy. Within an isolated system this mechanical energy is conserved.

For example, when a ball is thrown upward in the air, it starts with a high kinetic energy and then at the top of its travel it has none. The kinetic energy has been transferred to gravitational potential energy. As it falls back to earth, it gains kinetic energy at the expense of potential energy. The word potential comes from the Latin *potens* meaning 'able'. In this sense, the object with potential energy is able to do work.

Consider a ball dropped from a high cliff (Figure 9.19). If the ball had a mass of 1 kg and the cliff was 100 m high, we can calculate the KE and GPE at any stage. The sum of the two has to equal the potential energy at the start. Table 9.2 sets this out:

Figure 9.19

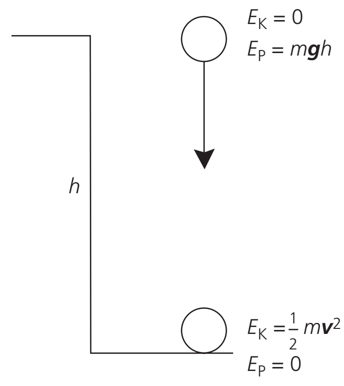


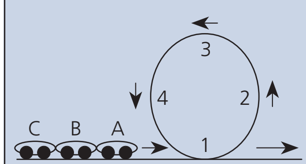
Table 9.2 CONSERVATION OF MECHANICAL ENERGY

HEIGHT (m)	POTENTIAL ENERGY mgh (J)	KINETIC ENERGY $\frac{1}{2}mv^2$ (J)	TOTAL PE + KE (J)	VELOCITY (m s^{-1})
100	1000	0	1000	0
75	750	250	1000	22
50	500	500	1000	32
25	250	750	1000	39
0	0	1000	1000	45

The velocities can be confirmed by using your vertical motion formulas from Chapter 2.

NOVEL CHALLENGE

A roller coaster is made up of three carriages A, B and C, which enter a loop as shown. Which position — 1, 2, 3 or 4 — will give the greatest speed for **(i)** carriage A, **(ii)** carriage B, **(iii)** carriage C?



Applications of energy transfers

The ability of mechanical energy to be converted into useful work has been known for thousands of years. Some examples of transfers are discussed below.

Amusement parks

If you want examples of physics principles and laws being put to use then there is no better place for an excursion than an amusement park. Newton's laws, conservation of momentum, centripetal forces, rotation and weightlessness are all there.

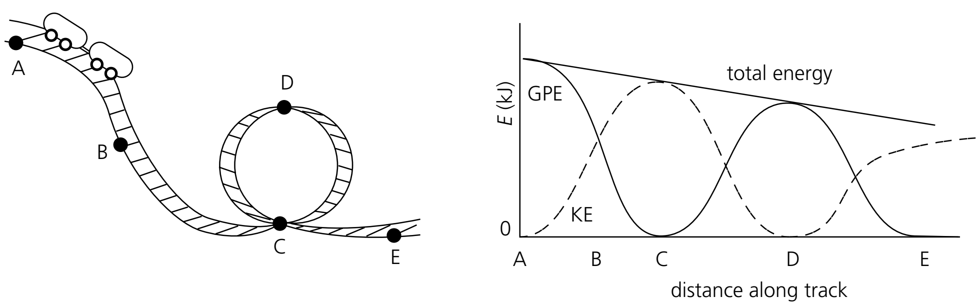


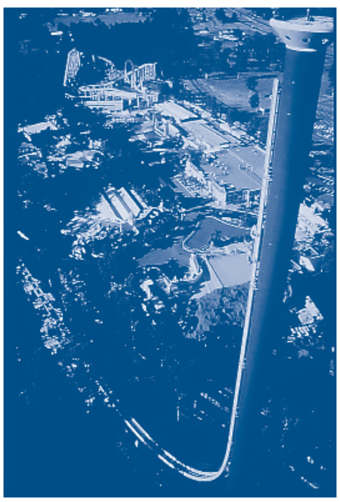
Figure 9.20
Energy changes during a roller coaster ride.

The roller coaster is a good example of conservation of mechanical energy. Electrical energy is used to propel the carriages to great heights, giving them high gravitational potential energy. At the point of release, the kinetic energy is almost zero but as they roll down the tracks, GPE is converted to KE and the carriages accelerate. At the next hill, some of the KE is converted back to GPE but some is transferred to thermal energy (by friction) and sound and can never rise to the previous height.

If there was no transfer to the structure, then mechanical energy would be conserved and the sum of GPE and KE would be constant. But there always is some loss and the designers take this into account. On rainy days, however, the losses become quite small as frictional losses are reduced and high speeds result. Sometimes the speeds are too high and the roller coaster has to be closed down.

Photo 9.1
The feeling of weightlessness on the Tower of Terror ride at Dreamworld.

Photo 9.2
The tower at Dreamworld is used for the Tower of Terror ride and the Giant Drop.



Activity 9.9 A DAY AT THE FUN PARK

NEI

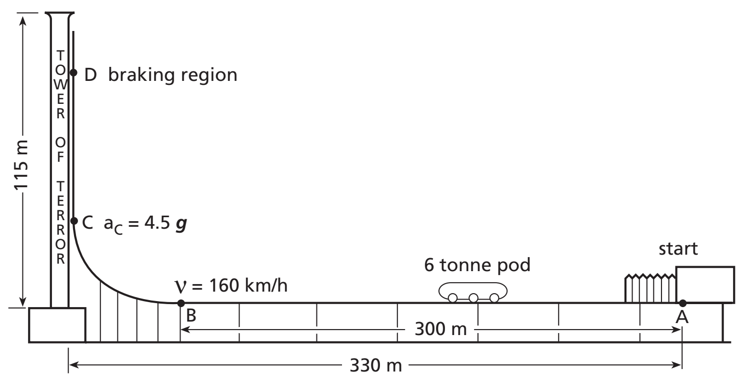
1 Next time you visit an amusement park, make a video of the roller coaster ride from the ground.
2 From the videotape, devise a way of measuring the speed of the carriage as it travels around the circuit.
3 Calculate the KE at various sections of the circuit (top of ramp, bottom of ramp, top of loop etc.). Plot these on a graph to show the changes. Estimate GPE at the same points and add these to your graph. What assumptions did you have to make?

Activity 9.10 DREAMWORLD'S 'TOWER OF TERROR'

SR

The 'Tower of Terror' is a 400 m track that stretches for 300 m horizontally before curving upward for 100 m (Figure 9.21). A 6 tonne pod with 16 people aboard (total mass about 7000 kg) is accelerated from rest (point A) to 160 km/h (at point B) along the horizontal section by electromagnets that draw 2.2 megawatts for 6 s. After this the pod goes unassisted into a vertical curve of radius 100 m, which gradually tightens to a curve of radius 50 m (point C) before travelling vertically for the last part of the trip (Photo 9.1). By this stage 12 s has elapsed (point D). In another 12 s the pod will be back to the start.

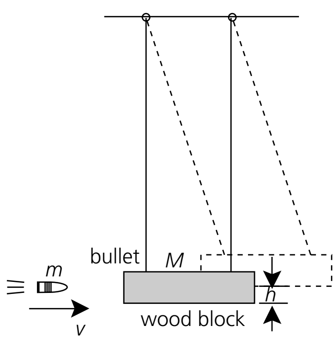
Figure 9.21
Tower of Terror dimensions.



1 If all of the kinetic energy is converted to GPE, to what height will the pod rise?
2 At the top of the curve (point C), the centripetal acceleration is 4.5 ***g***. Calculate the velocity at this point.
3 Why does the curve start with a radius of 100 m and then decrease to 50 m. Why not go straight into a 50 m curve?
4 Do the data suggest that the initial acceleration (from A to B) is uniform? Explain.
5 At what stage are you weightless? Explain.

The ballistic pendulum

Figure 9.22
A ballistic pendulum, formerly used to measure the speeds of rifle bullets.



A ballistic pendulum is a device that was used to measure the speeds of bullets before electronic timing was developed. The device consists of a large block of wood hanging by two long cords from the ceiling. When a bullet is fired into the wood, the bullet quickly comes to rest. The block with the embedded bullet swings upward to a maximum vertical height, which is measured. (Figure 9.22.)

We can let m be the mass of the bullet, and M the mass of the block. On collision, momentum must be conserved so if we assign $\boldsymbol{u}$ as the bullet's velocity and $\boldsymbol{v}$ as the velocity of the bullet + block after collision:

$$m\boldsymbol{u} = (m + M)\boldsymbol{v}$$

or

$$\boldsymbol{v} = \frac{m\boldsymbol{u}}{m + M}$$

Since the bullet and block stick together, the collision is perfectly inelastic. But mechanical energy is conserved and all of the kinetic energy is transferred to gravitational potential energy. So the kinetic energy of the block at the bottom of its arc must equal the potential energy at the top of its swing.

We can eliminate $\boldsymbol{v}$ from the equations:

$$\tfrac{1}{2}(m + M)\boldsymbol{v}^2 = (m + M)\boldsymbol{g}h$$

or

$$\boldsymbol{v}^2 = 2\boldsymbol{g}h;\ \boldsymbol{v} = \sqrt{2\boldsymbol{g}h}$$

Example

$$\frac{m\boldsymbol{u}}{m + M} = \sqrt{2\boldsymbol{g}h}$$

$$\boldsymbol{u} = \frac{m + M}{m}\sqrt{2\boldsymbol{g}h}$$

A bullet of mass m = 9.5 g was fired into a block of mass M = 5.4 kg and the block rose a vertical distance of 6.3 cm. Calculate the speed of the bullet.

Solution

$$\boldsymbol{u} = \frac{m + M}{m}\sqrt{2\boldsymbol{g}h}$$

$$= \frac{9.5 \times 10^{-3} + 5.4}{9.5 \times 10^{-3}}\sqrt{2 \times 9.8 \times 6.3 \times 10^{-2}}$$

$$= 630 \text{ m s}^{-1}$$

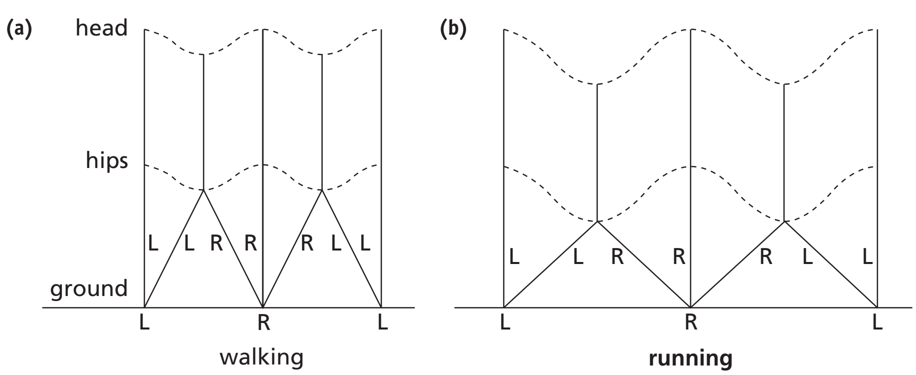
In practice, it is easier to tie a piece of cotton thread to the pendulum, and the sideways motion (the amplitude) is indicated by how far the piece of thread is dragged along the floor. By knowing how long the pendulum is, the vertical displacement h can be calculated.

Human energy conversions

Do you do any work to carry a bag while walking along a horizontal road? The force needed to carry the bag is vertical (the weight) but the displacement is horizontal (along the road) so no work is done. The bag has no additional potential energy or kinetic energy than it had to start with. But you have used energy and you would feel tired. Where has this energy gone?

A great deal of the energy has gone into internal energy transfers. Your body burns fuel to keep the muscles working. What is overlooked in this description is that the bag and your body move up and down as you walk. Your centre of gravity bobs like a bouncing ball rather than travelling smoothly like riding along on a bike.

As each foot strikes the ground your body drops down and then as you step forward your body rises again. This is shown in Figure 9.23.

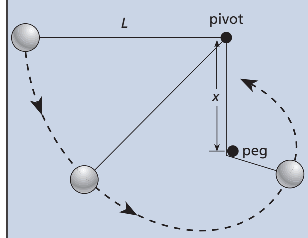


The up and down motion is a change in potential energy. A typical vertical displacement is 3 cm per step.

NOVEL CHALLENGE

A pendulum bob on a string is allowed to fall but the string strikes a peg. If the length of L is 2.5x, the bob has zero velocity when it gets to the suspension point P.

Can you prove this mathematically?



INVESTIGATING

If you ever go on the Tower of Terror, take a small piece of bark and release it in front of your face as you start to go up.

What do you notice on the way up and down? Surprised huh? Explain that if you can!

Figure 9.23
Note that the head and hips of a person walking move in a wave motion. They appear to move up and over the support leg and come down again as the foot touches the ground. In figure **(b)** the stride length is increased and the rise and fall of the wave increases.

Example

A 60 kg student is walking at 1 m s^{-1} with a stride of 50 cm. During each step his centre of mass rises and falls by 3 cm. Calculate **(a)** the work being done against gravity for each step; **(b)** the rate at which work is being done.

Solution

(a)

$$W = mgh$$
$$= 60 \times 10 \times 0.03$$
$$= 18 \text{ J (per step)}$$

(b)
- Each step is 50 cm so he makes two strides in one second.
- Each stride uses 18 J so the work done is 36 J in one second.

$$P = \frac{W}{t} = \frac{36}{1} = 36 \text{ J s}^{-1} = 36 \text{ W}$$

The up and down motion is the main use of energy in running and walking. Athletes try to minimise this vertical motion — just watch hurdlers in action and you will see how little the centre of gravity changes as they go over the hurdles.

Easy rollers — low friction tyres

While energy is expended when you walk or run because your body bobs up and down, you would expect very little energy loss with a rolling tyre. In this case there is no vertical motion of the centre of gravity. Unfortunately as a tyre travels along a road it undergoes two types of deformations — a macro-deformation and a micro-deformation. The first flattens the tyre tread against the road, creating a large 'footprint'. This is responsible for rolling resistance, which is unwanted, so the smaller the better. The second is a micro-deformation in which tiny irregularities in the road surface make imprints in the tread, and is responsible for the traction or friction between the surfaces. This is desirable so the bigger the better. A question you could ponder: do racing car 'slicks' maximise the micro-deformation and minimise the macro-deformation?

At 100 km h^{-1}, a tyre rotates about 20 times a second, with every part of the tyre flexing during each revolution. Most of the energy needed to cause flexing is transferred to heat within the tyre as the polymer chains slip over one another. If you feel a tyre after a trip, you'll note that the tyre is hot. Scientists are trying to develop polymer compounds that spring back into shape without generating heat. Michelin tyre scientists have found that by introducing silicon dioxide into the rubber, reductions of up to 40% in rolling resistance can be achieved. Such tyres are not in commercial production yet, but are expected to be much more expensive when they become available.

Stopping powerful locomotives

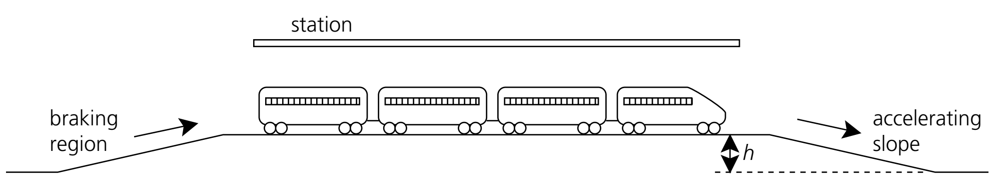


Figure 9.24
London Underground.

When deep sections of the London Underground railway were being constructed, the track at the stations was built at a higher level than the track between the stations (Figure 9.24). Trains approaching a station are decelerated by running up the slope, thus losing kinetic energy and gaining potential energy and at the same time reducing brake wear. Conversely, trains leaving the station are accelerated by running down the slope and saving on fuel. It all seems so logical you'd wonder why it was not thought of earlier. The system is ideal when trains stop at every station and is easy to construct when a new underground line is being built. It is not very practical on surface lines.

Underground trains can accelerate faster, too, not only if they have slope-assisted departures but because of the higher coefficient of friction. Being underground, the lines are protected from rain, dust and dirt so coefficients of friction of about 0.2 are typical, whereas on a good day above ground the coefficient is about 0.1 and can fall to 0.05 in wet weather. Acceleration is limited by the maximum frictional force that can be achieved (recall that the frictional force equals the coefficient of friction times the weight: $\boldsymbol{F}_f = \mu \boldsymbol{F}_N$). Trains used to have a sandbox located just in front of the drive wheels of the locomotive. On steep tracks sand was let out to increase the friction. Pretty ingenious, eh?

NOVEL CHALLENGE

Per kilogram, humans produce 10 000 times the energy produced by the Sun.
*But which produces the greatest **total** heat—the Sun or all the people on the Earth? Calculate the energy output of the Sun by estimating the population of the Earth, the average mass of a person and knowing that the average heat output of a person is 12 megajoules per day.*

NEI

Activity 9.11 SOME MORE GUINNESS

Using the *Guinness Book of Records*, find where in the world the steepest railway is. The one at Katoomba NSW relies on a cable to pull it up the incline, but where is the steepest that uses wheel friction only?

Questions

16 Using conservation of mechanical energy principles, calculate the impact speed of a 65 kg diver who dives off a platform 8.5 m above the water.

17 Champion weightlifter Leonid Taranenko lifted 266 kg in a snatch and jerk to a height of 2.4 m. Calculate **(a)** how much work he did lifting this; **(b)** the impact speed when he dropped the weight on to the mat from this height.

18 A 0.41 g air-rifle pellet was fired into a 112.3 g toilet roll target suspended from the ceiling by a piece of thread. This ballistic pendulum rose 1.5 cm vertically. Calculate the muzzle velocity of the pellet.

9.9 ELASTIC POTENTIAL ENERGY

When an archer pulls back on the string of a bow, work is done and energy is stored. When the string is released, most of the stored energy is transferred to the arrow as kinetic energy. The bow stores energy as **elastic potential energy** (EPE).

Elastic potential energy is the energy stored in a spring or other elastic body by virtue of its distortion, or change in shape.

Photo 9.3
Pole vaulter. These photos were taken at evenly spaced time intervals. They show that the elastic potential energy stored in the bent pole is returned to the vaulter, to increase his gravitational potential energy, and his kinetic energy so that he can clear the bar.

Examples include:
- rubber band on a speargun
- springs on a trampoline
- compressed gas in a nailgun
- flexible pole in a pole vault
- bungee rubber rope
- a polymer bumper bar on a car
- vehicle suspension springs or gas shock absorbers.

Springs

When a spring is stretched (action) there is a restoring force (reaction), which tries to restore the spring back to its original length. The two forces are equal but opposite in direction.

Figure 9.25
Hooke's law experiment.

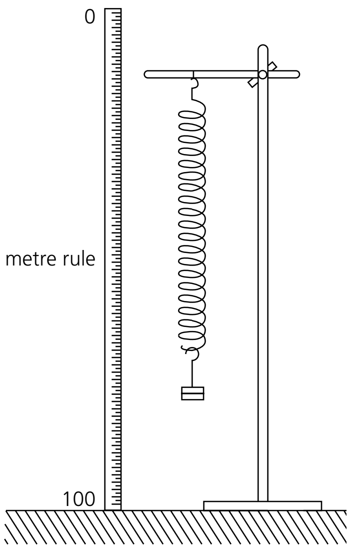
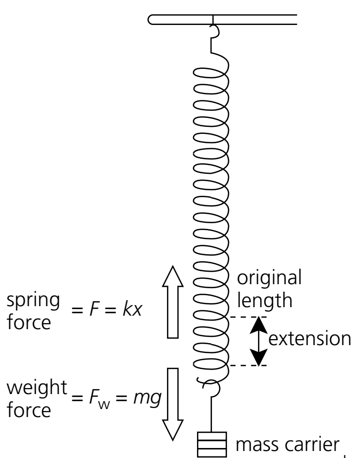
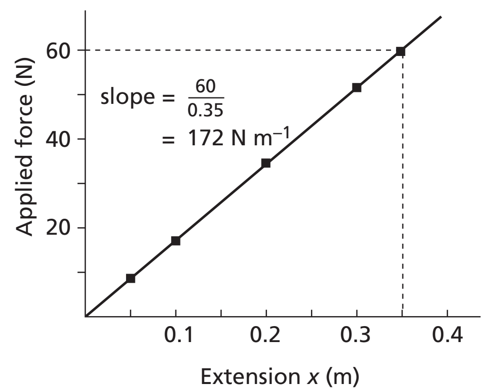


Figure 9.27
The forces acting on a stretched spring.



Imagine a spring hanging vertically with a ruler beside it as shown in Figure 9.25. As masses are added the spring stretches under the applied force (the weight of the masses). A graph of force applied versus the extension of the spring is shown in Figure 9.26. The slope of the line ($\boldsymbol{F}x$) is called the **spring constant** (k) and in the graph shown, its value is 172 N m^{-1}. In Chapter 4 this was discussed in detail — you may remember the formula $\boldsymbol{F} = -kx$. The stiffer the spring the greater the spring constant. This relationship is called **Hooke's law** after Robert Hooke, an English scientist of the late 1600s. He didn't publish his findings in a journal as scientists do today. He wrote it in a letter as 'CEIIINOSSSTTUV', which can be rearranged into the Latin *ut tensio sic vis* meaning 'the force is proportional to the displacement'. He wanted to be credited with the law's discovery so he secured it by the time-honoured device of the anagram. That way he could claim to be first without letting anyone know the details.

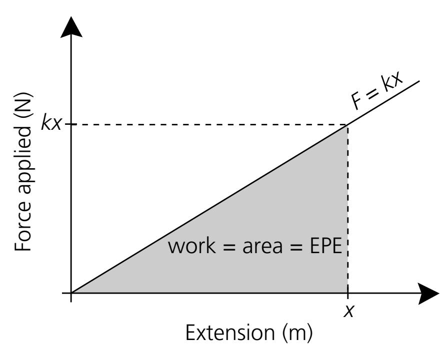
Figure 9.26



As the weight on a spring is increased, the spring stretches a little until it comes to equilibrium again. This is due to the fact that the spring exerts an equal but opposite force on the masses. This is shown in Figure 9.27. As the added force acts through a certain distance it implies that work is done. Where did the energy of this work go? It did not go into increased gravitational potential energy, nor did it go into increasing the kinetic energy of the spring. The energy went into stored elastic potential energy (EPE) of the spring.

In the graph shown in Figure 9.28, the area under the line is a measure of the work done, which equals EPE.

Figure 9.28



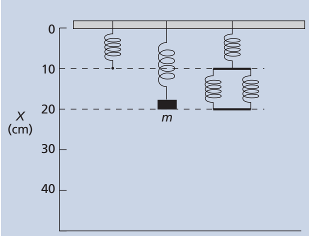
Area of triangle = $\frac{1}{2}$ base times height = $\frac{1}{2}x$ times $\boldsymbol{F}$ = $\frac{1}{2}x$ times kx.

Note: the symbols used here can be confusing. We will be using 'x' for displacement and '×' for multiplication ('times') to avoid confusion.

$$E_P \text{ or EPE} = \tfrac{1}{2}kx^2$$

NOVEL CHALLENGE

A spring has an unladen length of 10 cm. With a mass of 1 kg added it stretches to 20 cm. Three springs identical to the first one are arranged in the pattern shown in the diagram. *How far down will they stretch when 1 kg is added?*



Example 1

A spring with a spring constant of 250 N m^{-1} is stretched to a distance of 15 cm beyond its natural length. How much energy is stored in the spring?

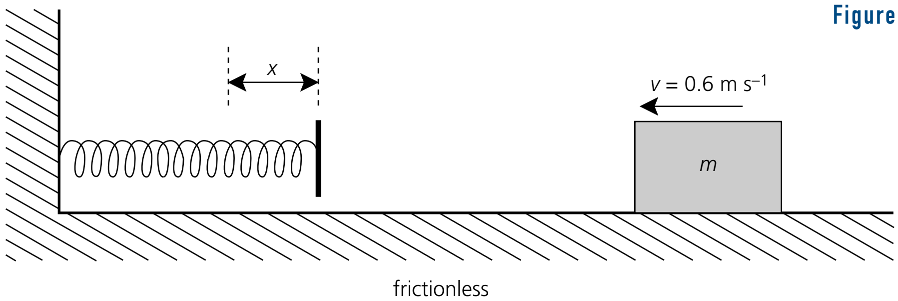
Solution

$$\text{EPE} = \tfrac{1}{2}kx^2 = \tfrac{1}{2}250 \times 0.15^2 = 2.8 \text{ J}$$

Example 2

A 200 g block of wood is travelling horizontally at 0.6 m s^{-1} and strikes a spring that has a spring constant of 15 N m^{-1} (Figure 9.29). Calculate the maximum compression of the spring.

Figure 9.29



Solution

Assume all of the KE is converted to EPE.

$$E_K = \tfrac{1}{2}m\boldsymbol{v}^2$$
$$E_P = \tfrac{1}{2}kx^2$$
Hence:
$$\tfrac{1}{2}m\boldsymbol{v}^2 = \tfrac{1}{2}kx^2$$
$$m\boldsymbol{v}^2 = kx^2$$
$$x = \sqrt{\frac{m\boldsymbol{v}^2}{k}} = 0.069 \text{ m}$$

INVESTIGATING

The spring constant is also called the elastic modulus. In maths, modulus is the number by which two given numbers can be divided and produce the same remainder. These meanings seem to be unrelated. Find out what the Latin *modus* means to explain this.

NOVEL CHALLENGE

A spring 20 cm long has a spring constant (modulus) of 86 N m^{-1}.
If the spring is cut into two 10 cm lengths, what will the modulus of each half be?

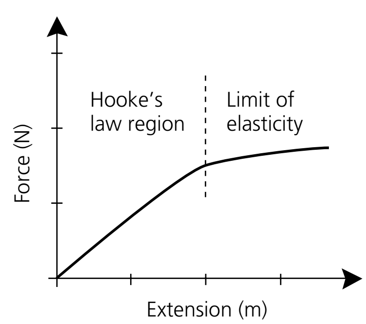
SR Activity 9.12 HOT LIPS

Hold a rubber band against your bottom lip. Stretch it tightly and note the temperature change. Let it go back to normal size and note how its temperature changes again. Can you explain this in terms of changes to the elastic potential energy and work being done?

Elastic limit

Many solids behave as if they are atoms or molecules linked together like a spring. When they are compressed or stretched by an external force they obey Hooke's law. However, every material has a limit to the amount of compressing, stretching or bending it can take. When Hooke's law is no longer obeyed, the object is said to have reached the **elastic limit** (Figure 9.30). A spring that has been stretched beyond the elastic limit will no longer return to its original length and its spring constant will change.

Figure 9.30
The limit of elasticity is eventually reached.



Questions

19 A catapult operates with the aid of a spring that has a spring constant of 265 N m^{-1}. Determine the amount of energy imparted to a rock if the spring is compressed a distance of 34 cm prior to release.

20 The graph in Figure 9.31 shows the extension of a spring when a force is applied to it. Using the graph, determine:
(a) the work done in stretching it to 20 cm; **(b)** the spring constant;
(c) the EPE stored in it when it is stretched 17 cm.

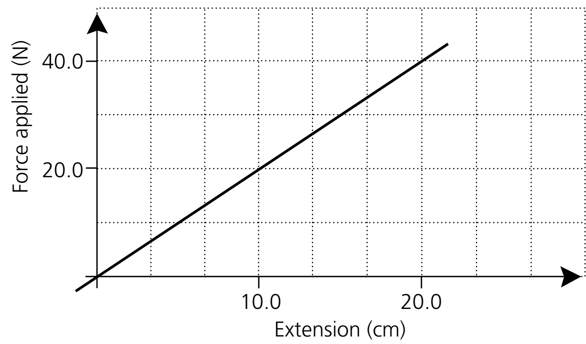


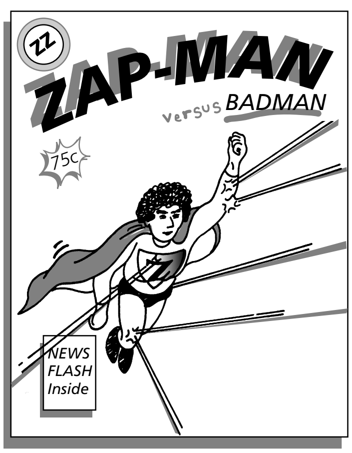
Figure 9.31
For question 20.

NOVEL CHALLENGE

A spring is 10 cm long. When a 0.6 kg mass is attached and allowed to fall the spring stretches by 0.76 cm.
Do you have enough information to calculate the spring constant?

21 A 336 t locomotive is moving at 5 km h^{-1} when it bumps into a carriage that has a spring bumper. If the spring has a spring constant of 4×10^8 N m^{-1} calculate the maximum compression of the spring.

Figure 9.32
It is well known that bullets and other missiles fired at Superman will simply bounce off his chest.



Human springs

In a previous example it was shown that walking and running use up energy because of the bobbing up and down of the body. But not all of the energy used to raise the body is lost when it falls. Some is stored as elastic potential energy in the muscles, tendons and bones of the foot and leg. Researchers have found that the Achilles tendon in a runner's leg stretches by about 5% on impact and can return more than 90% of the absorbed energy to the muscles of the calf. The foot, by virtue of the arched bones and tendons, can also act like a spring. Deformation of the arch on impact stores the energy elastically and can return about 80% of it. A typical runner loses about 100 J of kinetic and gravitational potential energy at each step but because the foot and leg return a large proportion of this energy back on rebound, the runner effectively only loses about 50 J — the rest is returned.

Professional running tracks can increase this rebound energy transfer. Most are made of a rubber compound that feels quite springy and when a typical runner depresses the track about 7 mm or so, about 90% of this energy is returned. Figures show that a 3% increase in speed can be attained.

FINISHING UP WITH SUPERMAN 9.10

This chapter concludes separate treatment of mechanics and kinematics. However, knowledge, processes and reasoning developed in these nine chapters will be used over and over throughout the rest of your course.

Let's look at Superman to see how our knowledge and reasoning can put some myths to rest.

1 *Energy of the leap* Superman can 'leap tall buildings at a single bound' because he grew up on Krypton where gravity was so strong that the inhabitants needed superstrength to stand up. One early leap by Superman was described as covering an eighth of a mile (200 m). Using projectile formulas we can show that he would have to have a launch velocity of 160 km h^{-1}. That is indeed 'faster than a train', to quote an early Superman description. But as Superman matured, the description became 'faster than a speeding bullet', which as you may recall is about 990 m s^{-1} (3500 km h^{-1}). This corresponds to about 50 million joules of kinetic energy for a man of normal weight. Superstrength doesn't exempt you from the law of conservation of energy, and to gain that energy from food requires more than 20 Big Macs just to do it once. The conclusion: Clark Kent must eat like a pig.

2 *Force and acceleration* How much force must go into Superman's legs to reach 3500 km h^{-1} and thus 'leap a tall building in a single bound'? If he is pushing off against the ground he has to reach this speed before his feet leave the ground, a distance of say 50 cm. This corresponds to an acceleration of about 10^6 m s^{-2} (100 000 '*g*'). This means that his legs have to exert a force of about 10^8 N, that is, 100 000 times his own weight. This is the same as the weight of about 10 000 t of lead. The thrust of the world's most powerful rocket motor (the Russian 'NI' rocket booster) is only 4620 t. Superman could easily stop a speeding locomotive.

3 *Flying* In recent stories, Superman has been shown changing directions in mid-air. With a take-off speed of 3500 km h^{-1}, he could reach an altitude of 50 km in one leap. But Superman can't turn off air resistance. For a body of Superman's size, his terminal velocity is about 200 km h^{-1}. This means even if he survived a take-off blast that started him off at 3500 km h^{-1}, he would slow down and complete his flight at 200 km h^{-1}. Some high-speed skiers do a little better than that; they achieve a speed of 250 km h^{-1} by streamlining their bodies with skintight suits and special aerodynamic headgear.

4 *Vacuum travel* As a boy, Superman carried his earthling father to the Moon without the benefit of a pressurised suit — just a helmet. Even if the father's clothing didn't burn up from air friction at lift-off, it wouldn't stop his dear dad's blood from boiling in the vacuum of space. And as Superman is not rocket-propelled, how can he change directions if he has no air to push against? Surely he doesn't violate the law of conservation of momentum! What provides the centripetal force that enables him to travel in a circle? One possibility for his rocket thrust is his superbreath. He might just blow his superbreath out in front of him and thereby be pushed backward. Some diapraghm he must have. Imagine its spring constant value!

5 *Light speed* Superman's orbital flights pose yet another problem. He is known to circle the Earth at 'seven times per second'. This corresponds to the speed of light. Using centripetal force formulas, it can be shown that to stay in a low earth orbit at this speed he would have to develop about a billion tonnes of thrust (10^{13} N). Chapter 30 (Relativity) has further problems examining travel at speeds close to that of light. You'll soon see that Superman knows little about senior physics. A 'very limited achievement' for him.

Maybe we should look at the Terminator, the Killer Tomatoes, Cinderella, Peter Pan, the Easter Bunny and the Tooth Fairy. No fantasy is safe from a physicist.

— Practice questions

The relative difficulty of these questions is indicated by the number of stars beside each question number: * = low; ** = medium; *** = high.

Review — applying principles and problem solving

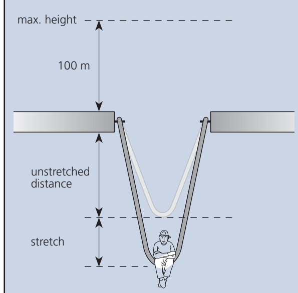
***22** A light bulb consumes 60 J of electrical energy per second but only converts this to 18 J of light energy.

(a) What is the efficiency of the bulb?

(b) Where does the remaining 42 J go — is it lost?

NOVEL CHALLENGE

A 'reverse bungee' is a long elastic cord hanging in a 'V' configuration (as shown in the diagram). It is stretched and held in position by an electromagnet before being released. It rises and goes 100 m above the top support. Prove that the elastic 'spring' constant equals 2×10^3 N m^{-1}.



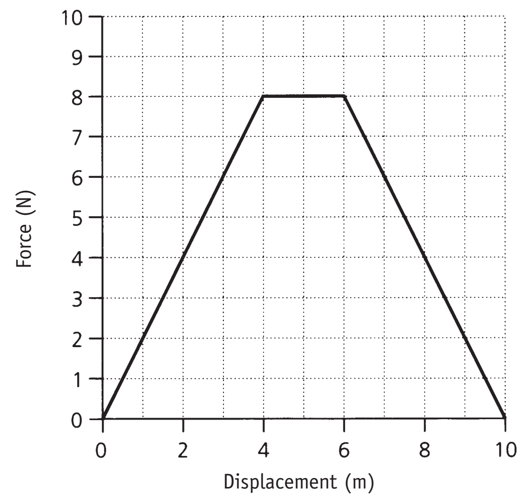
INVESTIGATION

This is a good extended experimental investigation. What factors influence the range of an arrow shot from a bow (angle of elevation, 'draw')? Experienced archers shoot feathers instead of plastic vanes (fletches) because they argue that feathers result in higher arrow velocities, greater stability, better guidance, higher accuracy and more forgiving flight. Here's their argument. First, feathers are faster because feathers weigh much less than plastic vanes. This means less mass to accelerate and less energy wasted. Feathers typically save 40 grains (2.6 g) over plastic. This is a lot of surplus mass — 30% of a typical 125 grain (8.1 g) steel head. Second, feathers produce less friction as they travel over the arrow rest or other bow parts. Less friction means higher speed. Third, the superior guidance of feathers prevents yawing and fishtailing of the arrows. Yawing and fishtailing add drag and slow arrow speed. What do you make of these arguments?

***23** A force of 90 N is applied to a string attached to a sled that makes an angle of 35.0° to the floor. How much work is done on the sled in dragging it across the lawn a distance of 10.0 m?

***24** A car is pulling a loaded trailer in a easterly direction along a horizontal road at constant speed. A force–displacement graph is shown in Figure 9.33. Calculate the work done by the car.

Figure 9.33
For question 24.



***25** Calculate the kinetic energy of a 60 g air hockey puck sliding at 8.0 m s^{-1}.

***26** What amount of energy is consumed by a 2000 W jug that took 1 minute 45 seconds to boil a cup of water?

***27** A body of mass 10 kg travelling east at 4 m s^{-1} strikes a 3 kg mass at rest. After the collision they remain coupled and the mass moves east at 3 m s^{-1}. Is the collision elastic or inelastic?

***28** Calculate the GPE of a 50 kg bag of cement lifted 1.4 m from the ground to a mixer bowl.

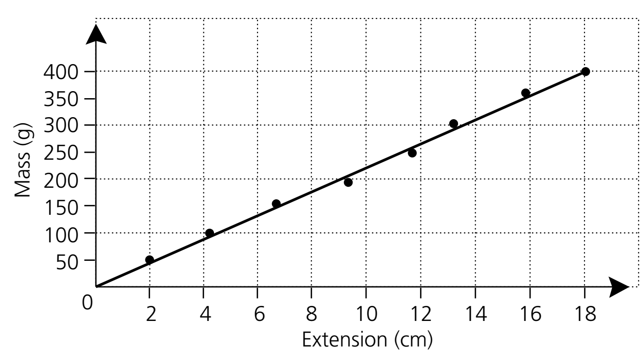
***29** A 65 kg student runs up a 12.0 m flight of stairs in 14 s. Calculate his power output.

***30** Using conservation of mechanical energy principles, calculate the impact speed of a 2 kg rock dropped off a cliff 8.5 m above the water.

***31** A spring with a spring constant of 150 N m^{-1} is stretched to a distance of 25 cm beyond its natural length. How much energy is stored in the spring?

***32** The graph in Figure 9.34 shows the extension of a spring when masses are added to it. Using the graph, determine:

Figure 9.34
For question 32.



(a) the work done in stretching it to 18 cm;
(b) the spring constant;
(c) the EPE stored in it when it is stretched to **(i)** 10 cm; **(ii)** 20 cm.

****33** In Figure 9.35, a sequence of drawings shows a high jumper in action. The changes in mechanical energy during the high jump (Fosbury flop) are shown in Figure 9.36.
(a) Match the numbered drawings with the three phases of the jump.
(b) The total energy at the peak height appears to be greater than the initial energy. If energy is conserved, how can there be energy created as the graph suggests?
(c) Shouldn't potential energy start at zero in the graph? After all, they do start from ground level!
(d) Extrapolate the graphs (on your own paper, not in this book) to show how the curves might look after the jumper clears the bar and lands flat on her back on the landing pad. Give an appropriate name to this phase.
(e) Would the GPE on landing be the same as at take-off? Explain.

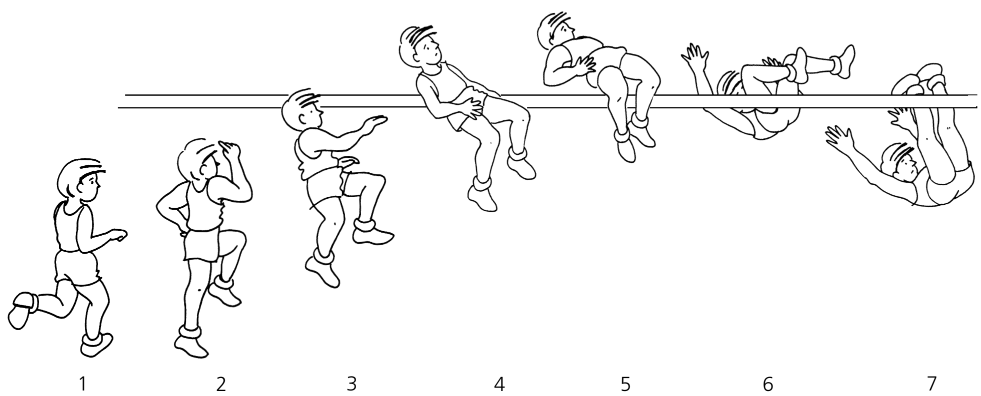


Figure 9.35
Stages of a Fosbury flop.

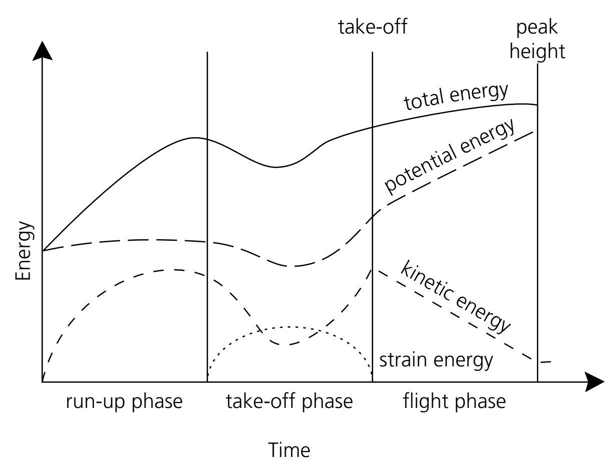


Figure 9.36
Energy changes in a Fosbury flop.

Extension — complex, challenging and novel

*****34** A 1.6 kg ball collides with a 2.4 kg ball as shown in Figure 9.37. After the collision the balls continue to travel, as shown in the diagram.
(a) What is the velocity ***v***?
(b) Is the collision elastic or inelastic? Show your proof.
(c) If the velocity of the 2.4 kg ball was in the opposite direction initially, could the velocity of the 1.6 kg ball after the collision be in the direction shown in the figure?

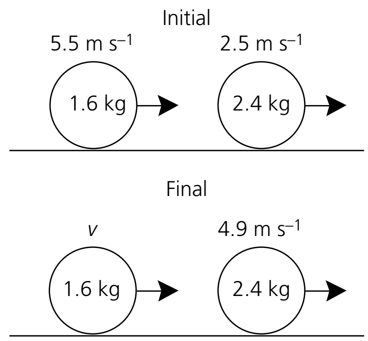


Figure 9.37
For question 34.

*****35** In a ballistic pendulum experiment, a bullet of mass 4.5 g was fired horizontally into a block of mass 3.4 kg suspended by a string 2.0 m long. The block and embedded bullet moved sideways a distance of 53.5 cm. Prove that the vertical displacement is 7.3 cm and then calculate the speed of the bullet.

*****36** Approximately 5.5×10^6 kg of water drops 50 m over Niagara Falls every second. If all of the water's potential energy could be converted to electricity, how much money could the owners get if it was sold at the industrial rate of 2 cents per megajoule?

*****37** A Toyota *Camry* uses about 11.5 L of petrol per 100 km.

(a) If petrol provides 31 MJ per litre, how far could you travel on 1 MJ of energy consumed?

(b) If you are driving at 60 km h^{-1}, at what rate would you be consuming energy (in watts)?

*****38** A 0.63 kg ball is thrown straight up into the air with an initial speed of 14 m s^{-1} and reaches a maximum height of 8.1 m before falling back down again. Assuming that the only forces acting on the ball are the ball's weight and air drag, calculate the work done on the ball during the ascent by the air drag.

Photo 9.4
Jimmy goes bungee jumping.

*****39** American advertising agency McCann decided to bungee jump a GMC Jimmy Sports Utility off a West Virginian bridge to show how well the truck was made. They used a 30 m rubber bungee cord used by the US army for supporting tanks during air drops. It had nine individual cords, each about 12 cm diameter. When released from the top of the bridge, 267 m above the river, the 1587 kg ute stretched the cord to six times its length before returning upward for a few more bounces. Neglecting air resistance, are you able to calculate the spring constant *k* of the bungee cord? If so, what is it? If not, what other information do you need?

*****40** A tough one. A body of mass 2.0 kg makes an elastic collision with another body at rest and continues to move in the same direction but with one-fourth of its original speed. What is the mass of the struck body?

*****41** A London Underground train consisting of a locomotive and 30 carriages with a total mass of 1586 tonnes travelling at 28 km h^{-1} is slowed by travelling up a slight incline to the horizontal platform. If it is raised vertically by 1.2 m over a distance of 300 m, calculate the speed of the train at the top of the incline, assuming no braking takes place.

*****42** The spring of a spring gun is compressed a distance d or 3.2 cm from a relaxed state, and a ball of mass m = 12 g is placed in the barrel. If the spring constant is 292 N m^{-1}, with what speed will the ball leave the barrel once the gun is fired? Assume no friction and a horizontal gun barrel.

*****43** The Greek historian Herodotus said it took 100 000 men 23 years to build the Great Pyramid at Giza but we don't believe him. The total GPE of the pyramid can be found by the formula GPE = $h^2\, d\, s^2\, \boldsymbol{g}/12$ where h = the height (146.7 m), d = density (2700 kg/m^3) and s = the side (230.4 m). But you have to add on the GPE of the stone blocks when they were raised the 19 m from the quarry to the base of the pyramid (GPE = $m\boldsymbol{g}h$). The mass can be found by multiplying the density by the volume ($V = s^2\, h/3$). If an Egyptian man can generate 160 kJ of energy per day, show that the average number of men over the 23 years was only 2845 and not the 100 000 Heroclotus claimed.

*****44** Have you ever been in a high-rise building that sways in the wind? Tall buildings oscillate with periods between 0.5 s and 10 s. To reduce the amplitude of the sway, engineers place 'tuned dynamic dampers' on the roof (see Figure 9.38). These are large blocks of concrete attached by springs to the side of the building, and can slide from side to side on a film of oil. A typical spring has a spring constant of 50 000 N m^{-1}. What mass of concrete would be needed to damp the oscillations of a building with the most sickening period of 5 seconds? Engineers use the formula:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

*****45** Imagine a block of wood held at the top of an inclined plane. At the bottom is a spring, attached to the plane. Write a question based on this and make up any data that may be required for the question. In fact, make up more data than is required. Write out a solution and ask a colleague (or even your teacher) to solve it.

TEST YOUR UNDERSTANDING

(Answer true or false)

- Energy gets used up or runs out.
- Something not moving can't have energy.
- A force acting on an object does work even if the object doesn't move.

Figure 9.38
For question 34.