CHAPTER 19

Lenses

19.1 INTRODUCTION

Did you know that the earliest eyeglasses were thick convex lenses, which reminded their makers of lentils, hence the term 'lens', from the Latin for 'lentil beans'?

Lenses play a major part in everyday life and in the scientific world. They are present in spectacles, contact lenses, microscopes, telescopes, overhead projectors, and photocopiers, to name a few. Simple lenses are used in science classes for mineral identification and for map reading in geography.

Lenses are transparent devices that refract light. Differently shaped lenses refract light differently. Once they were made of glass but now, to reduce the weight, many lenses are made of plastic. Magnetic lenses are used to focus charged particles in particle physics. Eyes of animals contain lenses made of organic material that can change shape to produce a range of depth of vision.

At the end of this chapter you may be able to answer questions such as these:

- What devices around me rely on lenses of various types?
- If lenses refract light why don't we see colours when using spectacles due to the fact that each colour of light is refracted differently?

19.2 SHAPE OF LENSES

Lenses are curved pieces of glass or plastic in many shapes, although they all serve a similar purpose — they refract or bend light in one way or another. Several shapes are shown in Figure 19.1. However, the ones most commonly used are **biconcave** (concave) or **biconvex** (convex) lenses.

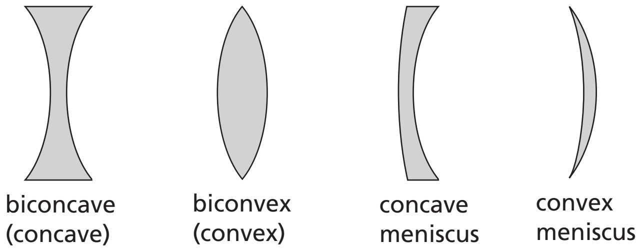
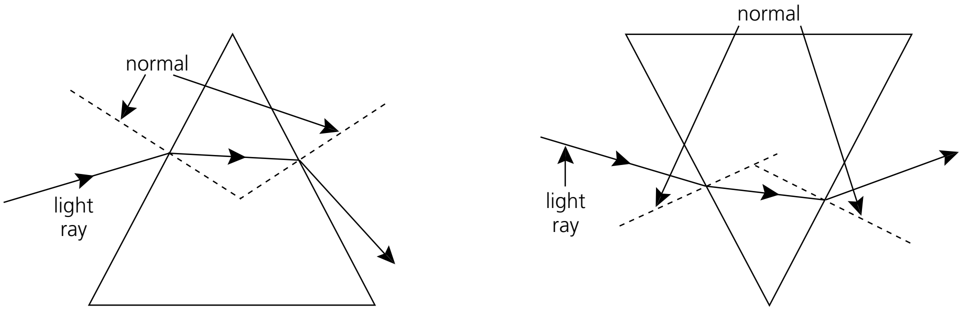


Figure 19.1
Some of the different shapes of lenses.

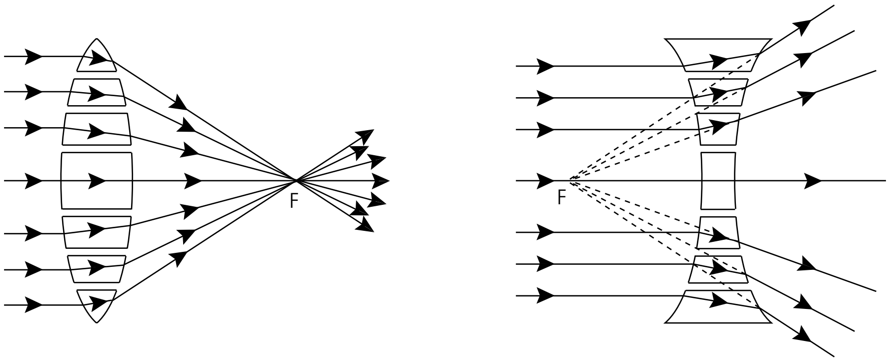
One common feature of all lenses is that light incident on the lens always bends toward the thickest part. Recall the refraction of light through a prism (Figure 19.2). Light is refracted toward the normal at the first surface and away from the normal at the second surface. The overall effect is that light always bends toward the thickest part of the prism.

Figure 19.2
Light through a prism always refracts toward the thickest part of the prism.



We can regard a lens as being a system of tiny prisms of slightly different shapes, so that light rays are refracted at both sides of the lens, which results in the bending of light rays toward the thickest part. This makes light rays incident on a convex lens bend toward the centre, thus focusing or converging the light. This type of lens is therefore also called a **converging lens**. Light incident on a concave lens spreads or diverges the light. This lens is also called a **diverging lens** (Figure 19.3).

Figure 19.3
Lenses can be considered to be made up of tiny prisms, thus light will always refract to the thickest part of the lens.



Other shaped lenses produce effects that depend on their shape. *Note:* refraction occurs at both sides of the lens but to simplify the drawing of ray diagrams we will draw a line through the centre of the lens and have all refraction occur at this line.

FEATURES OF LENSES

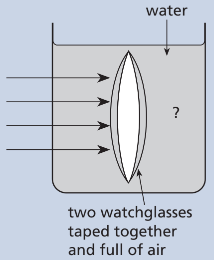
19.3

Similar terms to those associated with mirrors are used in describing lenses. Each face of the lens can be drawn using a compass. The centre of the circle that produces the curved surface is called the **centre of curvature**. Each face can have a different curvature. However, for our discussions we will use lenses whose sides have the same **radius of curvature**. The line joining the centre of curvature through the optical centre of the lens is called the **principal axis**. If rays of light parallel to the principal axis are incident on a convex lens the rays converge to a point on the other side of the lens — the **principal focus** (F). Since lenses can be used either way around they have two focal points, one on either side of the lens. If rays of light parallel to the principal axis are incident on a concave lens they diverge. However, if these rays are traced back they appear to come from a point on the same side of the lens as the light originates. This is a **virtual** focus. The distance from the focal point to the optical centre of the lens is the **focal length** (f). More powerful lenses have shorter focal lengths and are much thicker (Figures 19.4 and 19.5).

Lens makers use the unit of the **dioptre** (D) to define the **optical power** of a lens. This is equivalent to the reciprocal of the focal length (in metres). For example, the optical power of a 20 cm focal length convex lens is 1/0.20, which equals +5 D. For a concave lens this becomes 1/–0.20 = –5 D. The term power in this case refers to the ability of the lens to refract light. High power lenses will refract light to a greater degree than low power lenses.

NOVEL CHALLENGE

You have made an air lens by taping two watchglasses together. This does nothing in air but what will it do underwater? 'Myopic people see better underwater' — account for this.



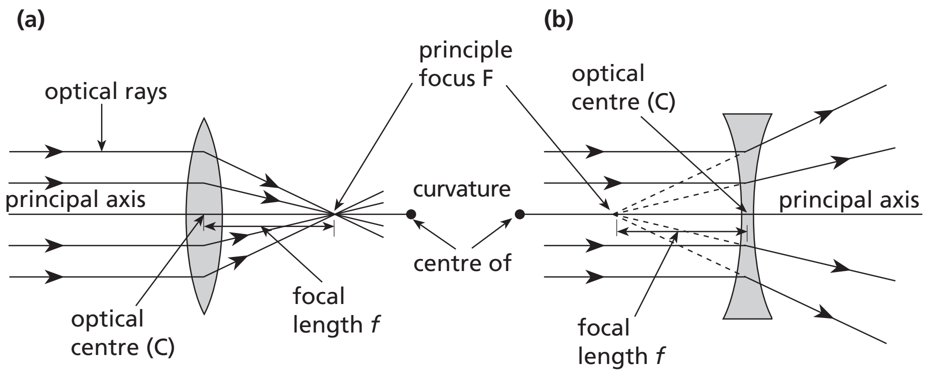
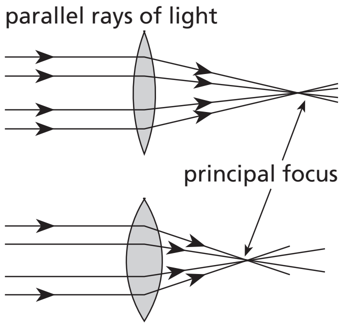


Figure 19.4
Features of convex **(a)** and concave **(b)** lenses.

19.4 RAY DIAGRAMS AND IMAGES

If you look through a magnifying glass (a convex lens) at an object you can see the object. In actual fact you are only seeing the image of the object. This can be easily verified if you look at an object a long distance away while holding the lens at arm's length. The image is upside-down.

Figure 19.5
Thick lenses have smaller focal lengths than thin lenses and are more powerful.



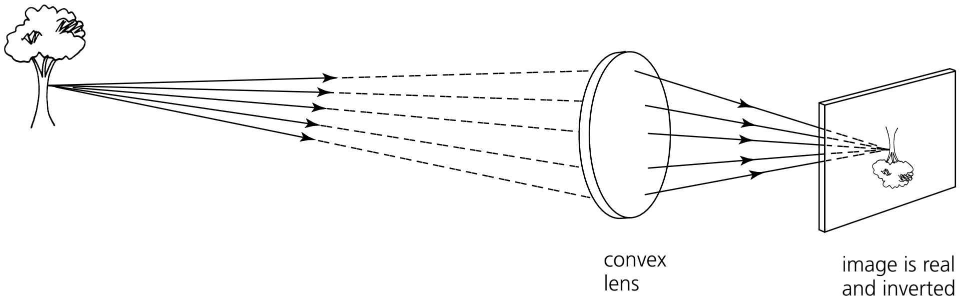


Figure 19.6
Images of distant objects are inverted, indicating that when you look through a lens you are observing the image and not the object itself.

Ray diagrams

Ray diagrams can be drawn to find the position and nature of an image. Again, any number of rays can be drawn to determine the position and nature of the image but many would require the use of protractors, many calculations, and measurements of the angles of refraction at both surfaces to produce an accurate ray diagram. However, three easily drawn rays are most commonly used to simplify the drawing of ray diagrams (Figure 19.7).

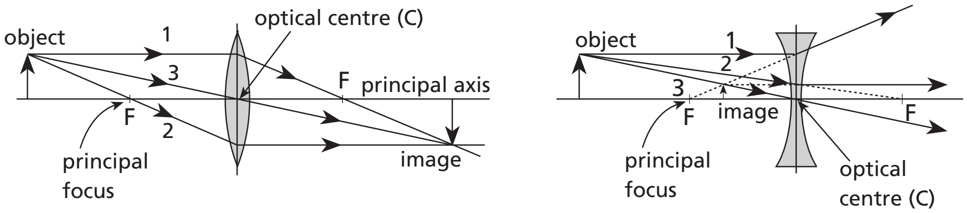


Figure 19.7
The three rays that are used to find the image produced by a lens: one parallel to the principle axis (1), one through the focal point (2), and one through the optical centre of the lens (3).

NOVEL CHALLENGE

A man wakes up and his digital clock reads 5:20. He then realises that there is a glass of water in front of the display acting like a convex lens, and he has arisen too early. What is the real time? Use a diagram to show this.

1. The ray parallel to the principal axis refracts through the principal focus for a convex lens, and appears to come from the focus for the concave lens.
2. A ray through the focus refracts parallel to the principal axis for the convex lens. For a concave lens the ray that is lined up with the focus on the opposite side refracts parallel to the principal axis.
3. The ray through the centre of the lens continues unchanged in direction.

Remember, to find the position of the image requires the use of two rays. The third can be used to double-check.

Images in convex lenses

Figure 19.8 shows a few examples of ray diagrams. The object has been placed at various positions with respect to the convex lens — greater than 2*f*, at 2*f*, between 2*f* and F, and between *F* and the lens (Figure 19.8).

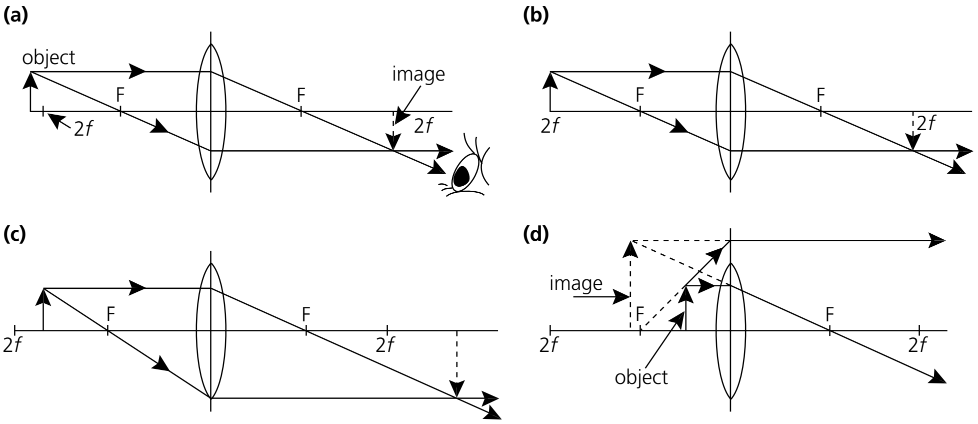


Figure 19.8
(a) When the object is outside 2*f* the image produced is: real, inverted, smaller, and between F and 2*f* on the opposite side of the lens. **(b)** When the object is at 2*f* the image produced is: real, inverted, the same size, and at 2*f* on the opposite side of the lens. **(c)** When the object is between 2*f* and F, the image produced is: real, inverted, larger, and outside 2*f* on the opposite side of the lens. **(d)** When the object is between F and the lens, the image produced is: virtual, upright, larger, and between F and the lens on the same side of the lens.

Notes:

- As an object moves toward F the image moves out and increases in size.
- When the object is between F and the lens a virtual image is formed.

Characteristics of the image

The terms used to describe the image are identical to those used for mirrors: real or virtual, upright or inverted, and smaller or larger.

In the previous examples:

In Figure 19.8**(a)**: the image is real (as the rays pass through the image), inverted, diminished, and between F and 2*f* on the opposite side of the lens.

In Figure 19.8**(b)**: the image is real, inverted, the same size, and at 2*f* on the opposite side of the lens.

In Figure 19.8**(c)**: the image is real, inverted, larger, and outside 2*f* on the opposite side of the lens.

In Figure 19.8**(d)**: the image is virtual (as the rays do not actually pass through the image), upright, larger, and between F and the lens on the same side of the lens.

NOVEL CHALLENGE

A concave mirror and a convex lens are placed in water. *Does their focal length change and if so, why?*

Images in concave lenses

The same three rays can be used to find the position and nature of an image in a concave lens. However, two important rules must be remembered:

- The rays always bend toward the thickest part of the lens.
- The focal point, the point where light rays parallel to the principal axis converge (or appear to come from), is on the same side of the lens as the object.

Figure 19.9 shows the use of rays to find the position and characteristics of the image in a concave lens.

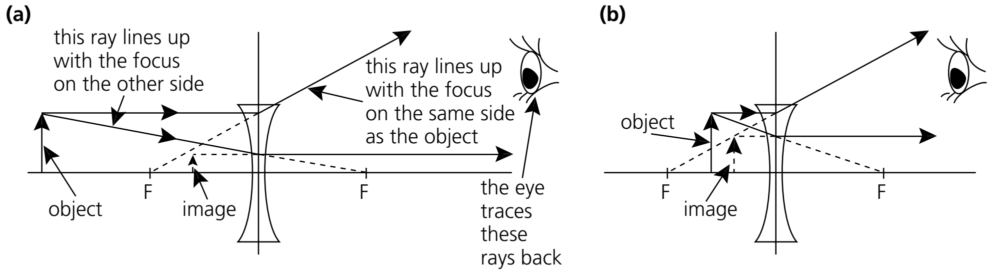


Figure 19.9
(a) When the object is at a distance from the concave lens, the image produced is virtual, upright, smaller, and between F and the lens on the same side of the lens.
(b) When the object is close to a concave lens, the image produced is virtual, upright, smaller, and between F and the lens, on the same side of the lens.

It will be noticed that whatever the distance of the object, even close to the lens, the image is virtual, upright, smaller and between the lens and F on the same side as the object. Concave lenses always produce these types of images no matter where the object is placed.

Questions

1 Complete Table 19.1 indicating the nature and position of the image as the object moves in from a long distance to a point where it is close to the convex lens.

Table 19.1 NATURE OF THE IMAGE

OBJECT POSITION	REAL/VIRTUAL	INVERTED/UPRIGHT	SMALLER/LARGER	POSITION OF IMAGE
Outside $2f$				
At $2f$				
Between $2f$ and F				
At f				
Between F and the lens				

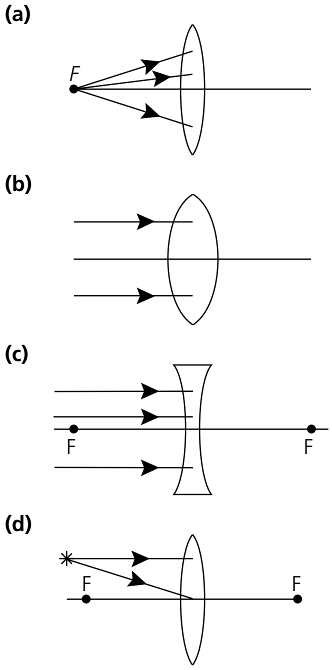
2 The power of a lens depends on its .... and therefore its ....

3 Complete the ray diagrams shown in Figure 19.10.

4 An object 5.0 cm high is placed 20 cm from a convex lens of focal length 15 cm. Draw a ray diagram to scale to find the position and characteristics of the images.

5 Use a ray diagram to find the position of the image of an object 25 cm in front of a concave lens of 10 cm focal length. (Use a scale of 1 cm = 5 cm.)

Figure 19.10
For question 3.



19.5 THE LENS FORMULA AND MAGNIFICATION

As for mirrors, the position and size of the image can be found mathematically. (See Figure 19.11.)

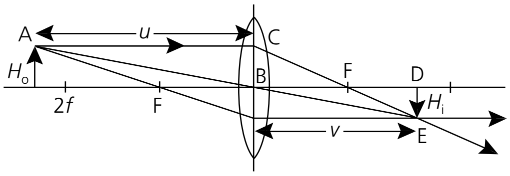


Figure 19.11
Using similar triangles the lens and magnification formulae can be derived.

NOVEL CHALLENGE

Plot a graph of v (y-axis) against M ($= v/u$) on the x-axis using suitable data. *Prove that the intercept on the y-axis equals the focal length. But what does the slope of the line equal? You'll be surprised and delighted.*

By using triangles as in Figure 19.11 the formula for the magnification can be derived as:

$$M = \frac{H_i}{H_o} = \frac{v}{u}$$

The **lens formula** $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ can also be derived using this figure,

where H_o is the object height; H_i is the image height; u is the object distance from the centre of the lens; v is the image distance from the centre of the lens; f is the focal length.
Note: When using the magnification formula, use the absolute values of v and u (don't worry about v being + or −).

Activity 19.1 A SIMPLE PROOF

Use similar triangles as in Figure 19.11 to derive the magnification and lens formulae.

Even though the formulae are derived using a convex lens they also apply to concave lenses but care needs to be taken in their use in solving problems. Again, since we have measurements on both sides of a lens, an order convention is required:

- All object distances (u) are positive.
- Since light passes through lenses, all image distances (v) on the opposite side of the lens (real images) are positive.
- Images formed on the same side of the lens as the object are virtual images, and their distances are negative.
- The focal length (f) of a convex lens is positive, and of a concave lens is negative.
- All measurements are made from the optical centre of the lens.

Example 1

A small 2.0 cm high candle is placed 50 cm in front of a 20 cm focal length convex lens. Find:

(a) the position of the image;
(b) the magnification;
(c) the height of the image;
(d) the position of the image of the candle if it is placed inside the focal point at a point 10 cm from the lens.

Solution

(a)

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} + \frac{1}{50} = \frac{1}{20}$$

$$\frac{1}{v} = \frac{1}{20} - \frac{1}{50}$$

$$= \frac{5}{100} - \frac{2}{100}$$

$$= \frac{3}{100}$$

$$\therefore\ v = 33.3\ \text{cm}$$

The image is at 33.3 cm distance on the other side of the lens.

(b)

$$\begin{aligned} M = \frac{H_i}{H_o} &= \frac{v}{u} \\ &= \frac{33.3}{50} \\ &= 0.67 \end{aligned}$$

(c)

$$\begin{aligned} H_i &= 0.67 \times H_o \\ &= 0.67 \times 2 \\ &= 1.33 \text{ cm} \end{aligned}$$

(d)

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\begin{aligned} \frac{1}{v} + \frac{1}{10} &= \frac{1}{20} \\ \frac{1}{v} &= \frac{1}{20} - \frac{1}{10} \\ &= \frac{1}{20} - \frac{2}{20} \\ &= -\frac{1}{10} \\ \therefore\ v &= -10 \text{ cm} \end{aligned}$$

The image is 10 cm away from the lens and is on the same side of the lens as the object.

Example 2

A small light 4.0 cm high is placed 25 cm in front of a concave lens of 10 cm focal length. Find **(a)** the position of the image; **(b)** the height of the image.

Solution

(a) (*Note:* since the focal length of a concave lens is negative, $f = -10$ cm.)

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\begin{aligned} \frac{1}{v} + \frac{1}{25} &= \frac{1}{-10} \\ \frac{1}{v} &= \frac{1}{-10} - \frac{1}{25} \\ &= -\frac{5}{50} - \frac{2}{50} \\ &= -\frac{7}{50} \\ \therefore\ v &= -7.1 \text{ cm} \end{aligned}$$

The image is at 7.1 cm distance on the same side of the lens as the object.

(b)

$$\frac{H_i}{H_o} = \frac{v}{u}$$

$$= \frac{7.1}{25} \times 4$$

$$= 1.1 \text{ cm}$$

Questions

6 A small candle (3.0 cm high) is placed 5.0 cm in front of a convex lens of 20 cm focal length.
  - **(a)** Draw a ray diagram to find the image. Describe its nature.
  - **(b)** Use the lens formula to find the location of the image.
  - **(c)** Use the magnification formula to find the height of the image.

7 A 5.0 cm high object is placed 30 cm in front of a concave lens of 10 cm focal length.
  - **(a)** Find the position of the image.
  - **(b)** Find the height of the image.
  - **(c)** Describe the image.

8 In each of the following cases draw a ray diagram to find the position and characteristics of the image.
  - **(a)** A 2.0 cm high object is placed 10 cm in front of a 20 cm focal length convex lens.
  - **(b)** A 5.0 cm object is placed 18 cm in front of a 6.0 cm focal length concave lens.
  - **(c)** A 10 cm high object is placed 50 cm in front of a 25 cm focal length convex lens.

9 A 2.0 cm high object is placed in front of a convex lens. A real image 4.0 cm high is produced on a screen 30 cm from the lens.
  - **(a)** Calculate the magnification. **(b)** Find the position of the object.

10 Students use a 8.0 cm high candle to produce images using a convex lens. At one particular object position a 2.0 cm high image is produced on a screen placed 10 cm from the lens.
  - **(a)** Calculate the magnification.
  - **(b)** Calculate the position of the object.
  - **(c)** What is the focal length of the lens?
  - **(d)** Describe the image produced.

11 An object 5.0 cm high is placed 1.8 m in front of a convex lens. An image 6.8 cm high is produced on a screen. Find the focal length of the lens.

DEFECTS IN LENSES

19.6

Spherical aberration

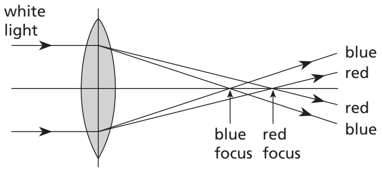
Spherical aberration is the inability of a convex lens to refract rays of light to a precise point. The word aberration comes from the Latin *aberrans*, meaning 'to wander'. This is caused by light rays that strike the outer edges of the lens being refracted more than those near the middle. This can be overcome by using a 'stop', which reduces the size of the aperture through which light can pass so that only the middle portion of the lens is used. It can also be overcome by using special parabolic lenses, or a combination of lenses as in camera lens systems.

Chromatic aberration

As discussed in Chapter 18, each colour of light is refracted by different amounts, therefore when white light passes through a prism violet light is refracted more then red light. In the case of convex lenses each colour of light is focused to a slightly different point (Figure 19.12). This produces a coloured haze around images and is called **chromatic aberration**, from the Greek word *chromo*, meaning 'colour'.

This can be overcome by using a concave–convex lens combination, as shown in Figure 19.13. Since the concave lens diverges light away from the principal axis it tends to cancel the effect of chromatic aberration produced by the convex lens. However, the two lenses need to be made of different materials, such as flint glass and crown glass, or this effect will still occur. The focal lengths of the two lenses are chosen to produce the desired focal length of the combination. The new lens is called an '**achromatic doublet**'. Today, especially in camera lens design, techniques such as multilayer coating onto the lens elements and low dispersion glasses in the lens elements all contribute to lens systems with very low aberration.

Figure 19.12
Chromatic aberration occurs when each colour of light is refracted by different amounts through a convex lens.

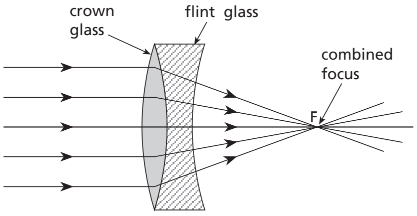


19.7 THE FOCAL LENGTH OF CONVEX LENSES

There are a number of practical methods used to find the focal length of convex lenses:

- You could locate the image of an object and then use the lens formula. The most accurate way is to move a small candle in until the image distance is the same as the object distance. The object and the image are then at 2*f*.
- A second method is to use a light box with slits producing a number of parallel rays. The point where these rays converge after passing through the lens is the focal point.
- The third method is similar to the second method but without using a light box. This involves finding the image of a distant object — a building, a tree, or the Sun. Since rays from these distant objects are just about parallel they will converge to the focus, producing an image at the focus.

Figure 19.13
Chromatic aberration can be overcome by using an achromatic lens.



19.8 USES OF LENSES

- Magnifying glasses consist of a single convex lens. To produce a large upright image the object needs to be placed inside the focal point.
- As many optical instruments use lenses, the use of these in such things as cameras, the eye, spectacles, telescopes, microscopes, etc. will be discussed in Chapter 20, Optical Instruments.

— Practice questions

The relative difficulty of these questions is indicated by the number of stars beside each question number: * = low; ** = medium; *** = high.

Review — applying principles and problem solving

*12 Find the power of the following lenses. **(a)** A concave lens of focal length 25 cm. **(b)** A concave lens of focal length 2.0 m. **(c)** A convex lens of focal length 20 cm. **(d)** A convex lens of focal length 100 cm.

13 What is the focal length and the type of lens that has an optical power of **(a) –5 D; **(b)** 10 D; **(c)** 25 D; **(d)** –50 D?

14 What type of lens **(a) converges light; **(b)** can form upright or inverted images; **(c)** always forms a virtual image; **(d)** always forms smaller images; **(e)** is used in a magnifying glass?

15 An object 15 mm high is placed in front of a convex lens of 12 cm focal length. The image is found 30 cm from the lens. **(a) Find the location of the object **(i)** using a ray diagram; **(ii)** using the lens formula. **(b)** Find the height of the image **(i)** using a ray diagram; **(ii)** using the magnification formula.

****16** A magnifying glass is used to read the fine print on a legal document.
(a) Where should the document be placed in respect to the magnifying glass?
(b) If the lens has a focal length of 20 cm, and the print is 1.0 mm high, find the size of the image when the lens is placed 15.0 cm above the document.

***17** A 4.0 cm high object is placed 52 cm in front of a convex lens of 25 cm focal length. Find **(a)** the position of the image; **(b)** the height of the image; **(c)** the nature of the image; **(d)** the position of the image if this object is moved to 10 cm from the lens.

***18** A 10 cm high bulb is placed 20 cm from a concave lens of 20 cm focal length. Find **(a)** the position of the image; **(b)** the magnification; **(c)** the height of the image.

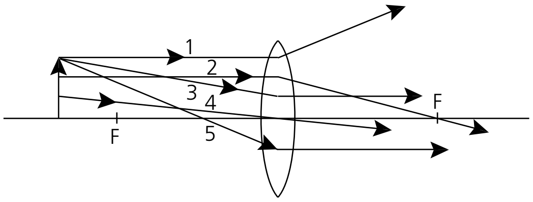
****19** A thin lens of optical power +2 D was used to read the small print (2.0 mm high) in a newspaper advertisement. If the paper was placed 20 cm from the lens **(a)** state what type of lens was being used; **(b)** describe the image produced; **(c)** find the position of the image; **(d)** find the height of the image.

****20** In order to take a sharp photograph of a distant object with a camera the position of the convex lens had to be adjusted until the lens was a distance of 50 mm from the film.
(a) What was the focal length of the camera lens?
(b) This camera was then used to take a photograph of a flower 50 cm from the camera; what was the distance between the lens and the film then?

***21** Describe experimentally how you would find the focal length of **(a)** a convex lens; **(b)** a concave lens.

***22** In Figure 19.14 which of the rays are drawn correctly?

Figure 19.14
For question 22.



***23** Explain how the focal length of a convex lens can be found by focusing the light from a distant building onto a screen.

****24** Students experimenting with convex lenses find the image distances corresponding to several object positions. These are shown in Table 19.2.

Table 19.2

Object distance (cm)	30	35	40	45	60	80	100	120
Image distance (cm)	150	88	67	56	43	36	33	32

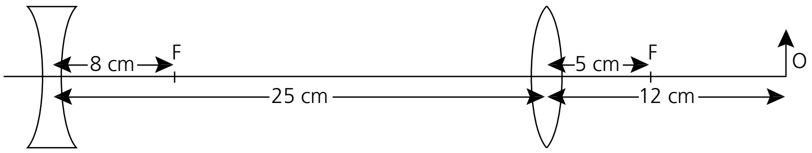
(a) Plot the graph of image distance against object distance.
(b) From the graph determine the focal length of the lens.
(c) Find the position of the image when the object is 70 cm from the lens.

***25** A philatelist (stamp collector) wishes to view stamps to identify detail in them.
(a) What type of lens would he require?
(b) Where would he need to place the stamps with respect to the lens?
(c) How would the optical power of the lens affect the image produced?

Extension — complex, challenging and novel

*****26** A convex lens of focal length 5.0 cm and a concave lens of focal length 8.0 cm are placed 25 cm apart. An object is placed 12 cm in front of the convex lens. Find the position of the image. (Refer to Figure 19.15.)

Figure 19.15
For question 26.



*****27** A plane mirror and a convex lens of 18 cm focal length are set up as shown in Figure 19.16. A small light source is placed 25 cm from the mirror.

(a) Describe what will be observed.

(b) Find the position of the image(s).

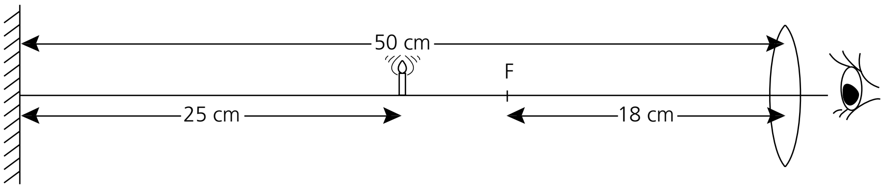


Figure 19.16
For question 27.

*****28** Students performing an experiment using a convex lens placed a small candle at various distances from the lens. The images were located and the image distance for each object distance was recorded. However, when the image distances were recorded they were recorded in the wrong places in the table (Table 19.3). Rewrite the table with the image distances in the correct columns and plot the graph of object distance against image distance. From the graph find the focal length of the lens.

Table 19.3

Object distance (cm)	25	30	45	50	60	80	100
Image distance (cm)	60	27	100	36	33	25	30

*****29** A slide projector is used to produce images on a screen 3.6 m from the slide. If the image is required to be 8 times as large as the object, what focal length lens is needed?

*****30** A candle and a screen are placed 4.0 m apart. What focal length lens could be used to produce an image of the candle on the screen 7 times as large as the candle itself?

*****31** A convex lens has a focal length of 20 cm. Where must an object be placed to produce a magnification of 5 if the image is **(a)** upright; **(b)** inverted?

*****32** Students wishing to find the focal length of a concave lens found that they had a dilemma. They could not focus parallel light from a distant object onto a screen. When they used a candle they could see the image but again could not obtain an image on a screen. However, they conducted an experiment where they moved a 10 cm high candle in from 100 cm to a point 5.0 cm from the lens. At certain object distances the height of the image was judged by a number of students. The results are given in Table 19.4. Use as many of these data as necessary, generate more data from this table, plot a graph, or do calculations, to find an accurate value of the focal length of this lens.

Table 19.4

Object distance (cm)	100	80	60	40	20	10	5
Estimated height of the image (cm):							
• Student 1	1.5	2	2.5	3	5	7	9
• Student 2	1.75	2	2.5	3.5	4.5	7	8
• Student 3	1.75	2	2.5	3.25	5.5	6	8

*****33** A lens forms an image on a screen with a magnification of 3.0. The screen is moved 20 cm closer and the object is then moved until the image is again in focus. The magnification is now found to be 2.5. Determine the focal length of the lens.