UNIT 10

Extension Topics

CHAPTER 30

Special and General Relativity

30.1 WHY 'SPECIAL', WHY 'GENERAL'?

Einstein's name is always attached to the **theory of relativity**, yet the work of many famous scientists before him underpins his theory. He questioned the accepted theories of time and motion of earlier nineteenth-century physics and came up with a special theory of his own. People today still ask some of the questions that bothered Einstein:

- Can you travel faster than light?
- Can you travel into the past or into the future?
- If I ran at the speed of light with a mirror in my hand, could I see my own reflection?
- When two rockets are moving relative to each other, can you tell which one is really moving?
- If a torch was moving, wouldn't its light travel faster than if the torch was at rest?
- In *Star Trek*, 'warp speeds' faster than light are equal to $2^n c$, where c is the speed of light and n is the 'warp number'. Can this be true?

In this chapter, we will investigate Einstein's Special Theory of Relativity and later the General Theory.

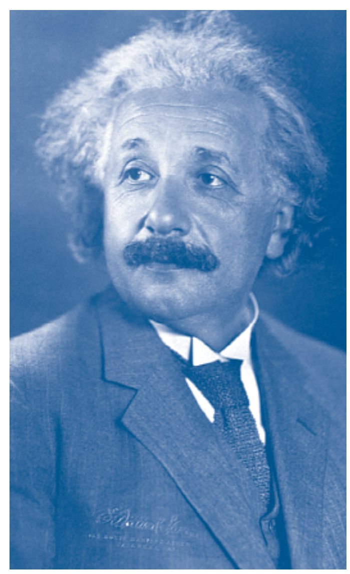
In the beginning

For more than two centuries after its inception, the Newtonian view of the world ruled supreme, to the point that scientists developed an almost blind faith in this theory. And for good reason: there were very few problems that could not be accounted for using this approach. Nevertheless, by the end of the nineteenth century new experimental data began to accumulate that were difficult to explain using Newtonian theory. New theories soon replaced the old ones. In 1884 Lord Kelvin said that there were 'nineteenth-century clouds' hanging over the physics of the time, referring to certain problems that had resisted explanation using the Newtonian approach. Among the problems of the time were the following:

- Light appeared to be a wave, but the medium for its propagation (the 'ether') was undetectable.
- The equations describing electricity and magnetism were inconsistent with Newton's description of space and time.
- The orbit of Mercury didn't quite match the Newtonian calculations.
- Materials at very low temperatures did not behave according to the predictions of Newtonian physics.
- Newtonian physics predicted that a hot object (a black-body radiator) at a stable constant temperature would emit an infinite range of energies — not so!

During the first quarter of the twentieth century, Albert Einstein created revolutionary theories that explained these phenomena. They also completely changed the way we understand nature. To deal with the first two problems he developed the **special theory of relativity** (in 1905). The third item required the introduction of his **general theory of relativity** (1915).

Photo 30.1
Albert Einstein.



The last two items can be understood only through the introduction of a completely new mechanics: quantum mechanics. This chapter deals with special and general relativity. The previous chapter introduced quantum mechanics.

Special Relativity is a deceptively simple theory and has only two assumptions or 'postulates'. They are presented here so you know what is coming, but without any explanation:

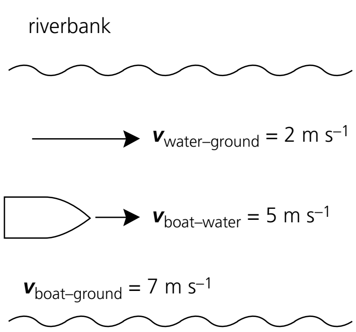
- The laws of physics are the same in all uniformly moving reference frames. No preferred frame exists.
- The speed of light in free space has the same value, c, in all uniformly moving reference frames.

Hmm! That doesn't seem too complicated. In fact, most physicists agree that the second postulate is redundant as it is a logical consequence of the first.

General Relativity does away with the restriction of 'uniform motion' which tends to make it more complicated — philosophically, physically and mathematically. In fact, it took Einstein 10 years, with many false starts and wrong turns, from introducing special relativity to the complication of his general theory in its final form. Along the way, the general theory became a whole new way of understanding gravity.

FRAMES OF REFERENCE 30.2

Figure 30.1
The speed of the boat is affected by the speed of the current.



In your earlier work on mechanics, you generally used the ground or Earth as your frame of reference. For example, when a car is going at 60 km h^{-1} along a road, this is with reference to the ground. But when a boat travels down a river, we can state its motion relative to the ground or relative to the water (Figure 30.1). The choice is arbitrary. This notion of reference frames had been discussed at length by Galileo and Newton and we need to begin there.

Inertial frames of reference

This chapter deals with **inertial reference frames** — that is, frames in which Newton's first law (the law of inertia) is valid. If an object experiences no net force due to other bodies, the object either remains at rest or remains in motion with constant velocity (in a straight line). Accelerating frames of reference, rotating or otherwise, are non-inertial frames, and we will not be concerned with them here. The Earth is not quite an inertial frame because it rotates. But it is close enough that for most purposes we can consider it an inertial frame. We could also carry out inertia experiments aboard a ship that is travelling at constant speed. It, too, is an inertial frame.

For Newton, there was a 'master' or absolute inertial frame: a frame stationary relative to absolute space. And any reference frame that is moving at a uniform velocity in a straight line relative to this master inertial frame, he said, will also be an inertial frame. Any reference frame that is accelerating with respect to absolute space, such as the car's frame when the light turns green and the driver accelerates, will not be inertial.

Now imagine that you are riding in the car at, say, 100 km/h down a straight highway and fluffy dice are hanging motionless from the rear view mirror. The principle of inertia is true for you. A second observer is standing beside the highway, watching the car go by. For her the dice are moving in uniform motion in a straight line. So the second observer is also in an inertial frame.

In this case, a good question is: 'Who is moving?' The answer is that you are moving relative to the observer beside the highway, but the observer beside the highway is moving relative to you. So you are both moving relative to each other. Both your inertial frame and her inertial frame are equally 'valid'. This realisation is often called 'Galilean relativity'.

An old favourite to illustrate this further is a cannonball dropped from the mast of a boat sailing along past an observer on the shore (Figure 30.2). For a sailor on the ship the cannonball appears to fall straight down (Figure 30.2**(a)**). From the point of view of an observer on shore, the ball falls with a uniform acceleration downwards while moving with constant speed in the horizontal direction — that is, it follows a parabolic path relative to the shore just like a rock thrown horizontally off a cliff (Figure 30.2**(b)**). However, for both observers the cannonball lands at the base of the mast, and the laws of inertia are the same in both reference frames although the paths are different. We can say:

> A reference frame that moves with constant velocity with respect to an inertial reference frame is itself also an inertial reference frame.

However, in frames moving relative to each other, the velocity of an object will appear different.

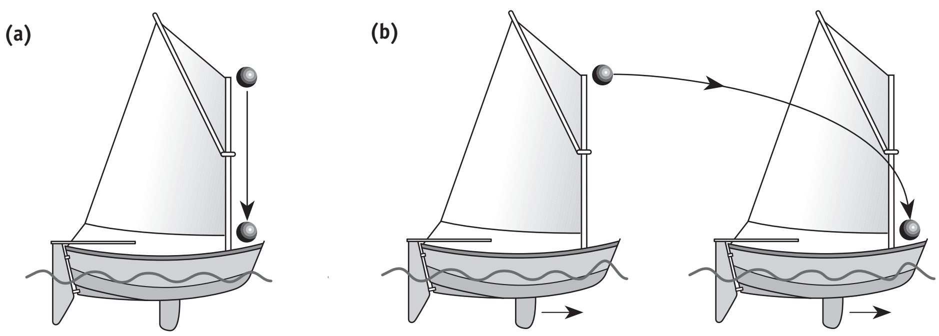


Figure 30.2
A falling cannonball travels different paths depending on your frame of reference: **(a)** from aboard the boat; **(b)** from the shore as the boat travels past you.

SR Activity 30.1 A 'GEDANKEN' (THOUGHT) EXPERIMENT

Before you read any further, you should sort out these questions (well, except for **(f)**):

(a) What would the path in Figure 30.2**(b)** look like if gravity was **(i)** less than that on Earth, **(ii)** more than that on Earth, **(iii)** zero?

(b) How would the path in Figure 30.2**(b)** differ if the cannonball was half the original mass?

(c) If the mast was 20 m high, and the boat sailed at 20 m s^{-1} relative to the shoreline, how many seconds would the cannonball take to hit the deck in Figure 30.2**(a)** and in Figure 30.2**(b)**?

(d) How far would the boat have travelled to the right in this time?

(e) Relative to the shore, what is the displacement and average velocity of the cannonball in its journey shown in Figure 30.2**(b)**?

(f) A very difficult one! How far would the cannonball have travelled in Figure 30.2**(b)** relative to the shore line? You will need to work out the 'arc length' of the parabola. How's your calculus?

Not all things change when viewed in different reference frames. For example, the number of atoms in an object doesn't change. If you time your pulse rate on Earth as 72 per minute, then you'll time it as 72 per minute aboard a moving bus. But if you are sitting down on the bus as it travels along a road at 5 m s^{-1}, you could say your speed is zero relative to the bus ($\boldsymbol{v}_{\text{person–bus}}$ = 0 m s^{-1}), and the speed of the bus relative to the Earth ($\boldsymbol{v}_{\text{bus–Earth}}$) = 5 m s^{-1}. Your speed relative to the Earth ($v_{person-Earth}$) would then also be 5 m s^{-1}. However, if you walk inside a bus towards the front with a speed of 1 m s^{-1}, and the bus moves at 5 m s^{-1} with respect to the Earth, then your speed is 6 m s^{-1} with respect to the Earth. (See Figure 30.3.)

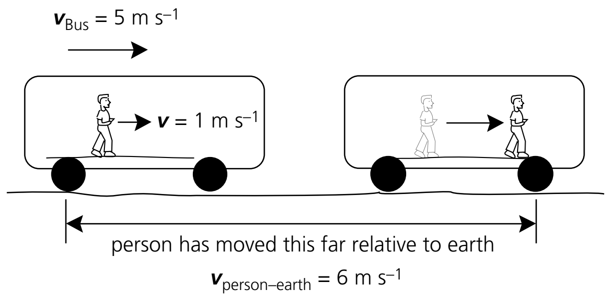


Figure 30.3
At ordinary speeds, the addition of relative velocities is quite straightforward.

Mathematically we can set this out using the Newtonian relativity equation as:

$$\begin{aligned} v_{person-Earth} &= v_{person-bus} + v_{bus-Earth} \\ 6 \text{ m s}^{-1} &= 1 \text{ m s}^{-1} + 5 \text{ m s}^{-1} \end{aligned}$$

You may be more familiar with the equation in the following form where the Earth is the assumed frame of reference and left out of the subscripts. The answer is the same.

$$\begin{aligned} v_{ab} &= v_a - v_b \\ v_{person-bus} &= v_{person} - v_{bus} \\ v_{person} &= v_{person-bus} + v_{bus} \\ &= 1 \text{ m s}^{-1} + 5 \text{ m s}^{-1} \\ &= 6 \text{ m s}^{-1} \end{aligned}$$

SR Activity 30.2 WHO'S REALLY MOVING?

Next time you are on a bus or car that is moving, try this 'thought' experiment. Imagine that you are stationary and it's the Earth that is moving. If so, why do your wheels turn? And why do the wheels of cars beside you turn? It might seem dumb but this is the very same question Einstein pondered 100 years ago. He called it a 'Gedanken experiment' (*Gedanken* is German for 'thought').

THE NATURE OF LIGHT 30.3

NOVEL CHALLENGE

Can a shadow travel faster than light? In the late afternoon, shadows are long and when you stand up your shadow shoots out along the ground much faster than you rose up — well over twice as fast. If you could fire a projectile upwards at 0.8c late in the afternoon then its shadow would scoot across the ground at 1.6c. *What is the problem here?*

As you may have seen in earlier chapters, changing magnetic fields produce electricity; conversely, changing electric fields produce magnetism. In the mid-nineteenth century, the great Scottish physicist James Clerk Maxwell deduced that as each field could create the other a 'wondrous new phenomena' would result. You get the idea: once a changing field of one type appears, self-perpetuating systems of electric and magnetic fields take on an independent existence, no longer associated with what started them, and would propagate through space as an electromagnetic wave. Maxwell explored the properties of these waves theoretically and calculated their speed as 3.00×10^8 m s^{-1}, and equal to the speed of light. The symbol 'c' was chosen to represent the speed of light. It was the initial letter in the Latin word *celeritas* meaning 'swiftness' (as in ac<u>celer</u>ate). That speed had been solidly established to an accuracy of a few per cent. So there was no doubt that the speed of Maxwell's electromagnetic waves was the speed of light and his brilliant conclusion was inescapable: light is an electromagnetic wave.

However, a question arose: in which frame of reference did light have this speed? It was thought that light would have a different speed in different frames of reference. For example, if a rocket ship was leaving Earth at half the speed of light ($0.5c$) and shone a beam of light forward at $1.0c$ relative to the spaceship, then the speed of light relative to observers on Earth would be $1.5c$:

$$\boldsymbol{v}_{\text{light–Earth}} = \boldsymbol{v}_{\text{light–rocket}} + \boldsymbol{v}_{\text{rocket–Earth}}$$
$$1.5c = 1.0c + 0.5c$$

Likewise, when you drive along a road with your headlights on, the light should travel more quickly than if your car was stationary. Something should start to sound a bit odd by now. Light just doesn't do these things!

Maxwell's equations made no provision for a frame of reference. They just said that the speed of light was c (3.00×10^8 m s^{-1}). So physicists thought there must be some special frame of reference where light had this value, and this frame would be the 'absolute' frame by which all other things could be measured. This was a problem for physicists at the beginning of the twentieth century.

Measuring the speed of light

Early attempts to measure the speed of light failed because light moves so quickly. Galileo attempted to measure the speed of light in the early part of the seventeenth century by measuring the time lag between one observer turning on a lamp and another observer noting this and turning on a second lamp on a distant hill several kilometres away. The method failed since the reaction time of the observers exceeded the time (10^{-5} s) that it took the light to travel the distance.

The Danish astronomer Olaus Römer made the first real measurement of the speed of light by using astronomical rather than terrestrial distances (Figure 30.4). By studying eclipses of the moons of Jupiter he was able to measure the speed of light as 2.26×10^8 m s^{-1} — about 75% of the value accepted today. Modern technology has enabled scientists to use Römer's method and obtain a value of 3.0×10^8 m s^{-1} for the speed of light. Albert Michelson, a Prussian-born American physicist, used rotating mirrors in 1926, and by reflecting light between two mountains 35.4 km apart, he was able to measure the speed of light as $2.997\,96 \times 10^8$ m s^{-1}.

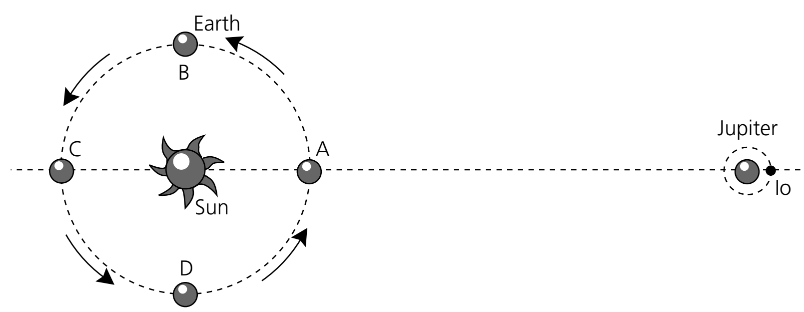


Figure 30.4
Römer's method of measuring the speed of light gave the first reliable value. The eclipse of Jupiter's moon Io occurs 16.6 minutes later than expected when seen from position C. This is the time it takes light to travel the diameter of the Earth's orbit A to C.

The accepted value for the speed of light is now $2.997\,925 \times 10^8$ m s^{-1} but for the purposes of simple calculations in this book we shall take the value to be 3.0×10^8 m s^{-1}. This is a distance of 300 000 km every second, about seven and a half times around the equator of the Earth. It takes about 5 microseconds for light to travel from Brisbane to Cairns.

NEI Activity 30.3 IS LIGHT SLOWING DOWN?

Use your library, CD-ROM or whatever means you can, to find out about and report on one of the following:

1. Some people believe that the speed of light is slowing down. Is there any evidence for this from published historical data or is it just that accuracy has been improving over the years? Use data to support your case.
2. Other people besides Römer and Michelson have tried to measure the speed of light. Report on one of these attempts, using drawings to illustrate how the process worked.

ABSOLUTE FRAMES OF REFERENCE 30.4

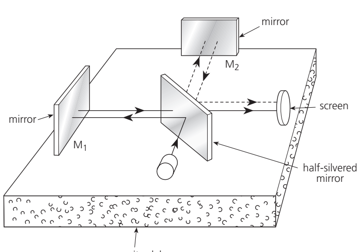
Nineteenth-century physicists were familiar with the properties of water waves, sound waves and waves on springs. These waves all needed some medium for their propagation, be it water, air or steel. There was no reason to think that Maxwell's electromagnetic waves should be any different. They called this transparent medium the *ether* and assumed it permeated all space. But don't think of this ether as the organic liquid used in chemistry. It came from the Greek *aithein* meaning 'to burn', referring to the invisible vapour given off by fires. By 'ether' the physicists referred to some mysterious fluid that filled the universe. The medium for light waves could not be air, since light travels from the Sun to Earth through nearly empty space.

It was therefore presumed that the velocity of light given by Maxwell's equations must be with respect to this 'ether'. Scientists soon set out to determine the speed of the Earth relative to this absolute frame, whatever it might be.

Michelson–Morley experiment

Photo 30.2
The American physicist A. A. Michelson refuted the 'ether wind' theory.

Figure 30.5
Light path in the Michelson–Morley experiment.



In 1887, two American scientists, A. A. Michelson (1852–1931) and E. W. Morley (1838–1923), were concerned that there was no experimental proof of the ether, which was supposed to be the absolute frame of reference for light. They argued that as the Earth moves around the Sun, it should be moving through an ether 'wind'. If there was an ether, and a beam of light was shone in the same direction as the Earth's movement through it, the velocity of light would be measured as greater compared to light travelling at right angles to the direction of motion of the Earth through the ether. Similarly, if light was shone in the opposite direction to the Earth's movement through the ether, the velocity should be measured as smaller. That seems logical!

However, light travels so rapidly that direct measurement of its speed was not possible at the time. But they were able to use an **interferometer** to measure the difference in the speed of light travelling with, against or across the ether. The path diagram for the interferometer is shown in Figure 30.5. It was a huge instrument consisting of a source of coherent (single-wavelength) light and some mirrors on a platform screwed to a massive stone block floating in a pool of mercury so that it could be rotated. This idea comes from lighthouses which also had their rotating lights floating in mercury. In fact, we have some mercury at school which came from the Cape Moreton lighthouse when the old light was removed.

Light was shone from the position marked 'source' where it struck a half-silvered mirror. (This is just a mirror with a half-thickness layer of silver over the whole of the glass so that half the light gets reflected and half passes through.) The reflected beam travelled to mirror M1, bounced off and travelled back to the half-silvered mirror where a lot of it passed through and onto a screen, shown at the right-hand side of Figure 30.5. The other half of the beam from the source initially passed straight through the first mirror where it struck mirror M_2. It also reflected back to the half-silvered mirror where a lot of it was reflected onto the screen.

The positions of mirrors M1 and M2 could be adjusted somewhat so that the path lengths for both beams from the source to the screen were equal (about 11 m). This meant constructive interference would occur and you would get a nice strong image on the screen. If the speed of light was different as it travelled the two paths, there would be a slight time delay in one of the beams and the constructive interference would be upset; the waves would be slightly out of phase and you would get a dimmer image (called a 'fringe shift'). They tried everything. They rotated it left and right; they did it morning and afternoon, in summer and in winter, yet there was no fringe shift.

Trying to explain this 'null' result was going to be difficult for physicists. Some suggested that the ether was really there but it was at rest with respect to the Earth because it was dragged along with the Earth; in this case the fringe shift would be zero. But this was quickly dismissed as fanciful nonsense after some experiments with high-flying balloons were undertaken.

In the 1890s, physicists G. F. Fitzgerald and H. A Lorentz argued that any length — including the stone slab on which the Michelson–Morley interferometer sat — would contract by a factor $\sqrt{1 - \boldsymbol{v}^2/c^2}$ in the direction of motion through the ether. This sounds like they just made it up to explain the null result, but Lorentz argued that the contraction — known as the Fitzgerald Contraction — could happen because the atoms moved closer together as a result of the ether upsetting the electrical interatomic forces. It was a good start, but more justification for his hypothesis was required.

In 1893 Michelson became head of the physics department of the University of Chicago. Later, in 1920, he did research work at the California Institute of Technology and the Mt Wilson Observatory and in 1907 he was awarded the Nobel prize for physics for his work on optical instruments and spectroscopic and meteorological investigations. He was the first American citizen to win this prize.

All of the theories attempting to explain the Michelson–Morley 'null' result were eventually replaced by the far more comprehensive theory proposed by Albert Einstein in 1905 — the special theory of relativity.

30.5 THE SPECIAL THEORY OF RELATIVITY

Einstein was eight years old when Michelson and Morley carried out their famous 1887 experiment. By 1905 he was a young father 26 years old, devoted to his family, to his work at the Swiss Patent Office, and to his physics. In this year he produced six scientific papers, all of which stood out as seminal works in the history of physics. The first one was on quantum mechanics, for which he received the 1921 Nobel prize. The second and third papers were on the size of molecules, for which he received his PhD at the Zurich Polytechnic. The fourth paper introduced the world to the famous formula $E = mc^2$, and the fifth paper dealt with relativity. It was stunning in its simplicity and ingenious, penetrating insight. It resolved completely the contradictions posed by Michelson and Morley.

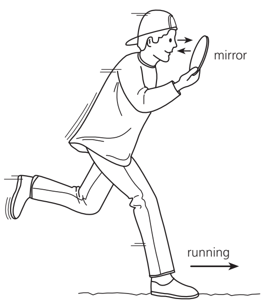
What motivated Einstein for this paper were what he called '*Gedanken*' (thinking) experiments like 'If I rode on a light beam, what would I see? Would I see light with a speed of zero?' He concluded that absolute space doesn't exist. Einstein's resolution was radical but profoundly simple. It can be stated in one brief sentence, called the Principle of Relativity:

> The laws of physics are the same in all uniformly moving reference frames.

That's it! One sentence implying all of Einstein's special theory of relativity. Historically, Einstein presented two postulates. The second one asserted that the speed of light is the same in uniformly moving reference frames. A more modern approach shows that the second postulate follows from the first and, in fact, by 1910 physicists had shown rigorously that the second postulate is superfluous.

Figure 30.6
The motion of light when you are at rest.

Figure 30.7
Face and mirror moving at speed *c*. Can you see yourself now?



In the following section we will examine some interesting consequences of Einstein's theory, particularly the 'invariance' (no variation) in the speed of light, even for observers who are moving relative to each other — and that's so troubling that it will lead to a radical revision of your commonsense notions of space and time. Students get particularly unsettled when commonsense appears to be thrown out the window. But remember, science is not about commonsense; most of the major developments in science appear to be intuitively wrong at first (for example, the Earth revolves around the Sun). As Nobel laureate Richard Feynman said, 'We never really understand physics, we just get used to it.' So, get used to the following!

SR Activity 30.4 ANOTHER 'GEDANKEN' EXPERIMENT

When Einstein was a boy he wondered about the following question: a runner holds a mirror at arm's length in front of his face. Can he see himself in the mirror if he runs at the speed of light?

When you look at yourself in a mirror, light travels from your face to the mirror and then is reflected back to your eyes (Figure 30.6). Einstein wondered how light could ever get from your face to the mirror if the mirror is travelling away from the light beam at the speed of light. (Figure 30.7.) The light would never catch up to the mirror! He soon realised the flaw in the logic. Can you? Propose your reasons.

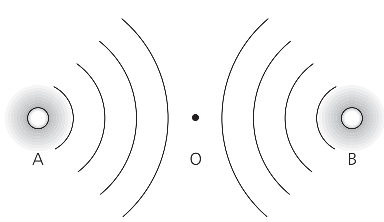
THE MEANING OF 'SIMULTANEOUS' 30.6

Imagine that at your school there are two bells, one at each end of the school. You hear both bells sound at the same time. But could there be a situation where an observer hears one bell before the other? In other words, can an event (sounding of the bells) be simultaneous to one observer but not to the other?

What does simultaneous mean? **Two events are simultaneous if they occur at the same time.** But how can you tell if two bells rang at the same time? If the bells were side by side there would be no problem; but when events are separated in space it gets difficult. If you were midway between the two bells and you heard them ring at the same time, you could say they were simultaneous. But what if you were closer to one bell than the other? If you still heard them at the same time, the more distant one must have sounded first because the sound had to travel further to your ears.

Does this apply to light as well? Say you were looking out your window at dusk and two street lights came on at the same time. You would say the events were simultaneous if you were midway between them (Figure 30.8). If you were not midway you would have to calculate the time it took to get from each event to your position so that you could work out when the events actually occurred. If both lights appeared to turn on at the same time but one was closer to you than the other, the closer one must have occurred after the more distant one. They were not simultaneous. Simultaneity can be defined thus: **Two events are simultaneous if light signals from the events reach an observer who is midway between them at the same time.** So the logic is the same for light as it was for sound.

Figure 30.8
A moment after street lights turn on at A and B, light waves travel outwards. If they arrive at observer O at the same time, she can say they are simultaneous because she is midway between the two lights.



The relativity of simultaneity

To show that two events that are simultaneous in a frame S are not simultaneous in another frame S′ moving relative to S, we will use an example introduced by Einstein.

Imagine a train moving past an observer sitting on an embankment at the side of the track. The train is the moving frame of reference and the embankment is the stationary frame.

Imagine that in the centre of the train carriage there is a person holding a device that can send out a beam of light in the forward direction and at the same time a beam of light in the backward direction (Figure 30.9). Also imagine that the front door and back door are opened automatically by these light beams.

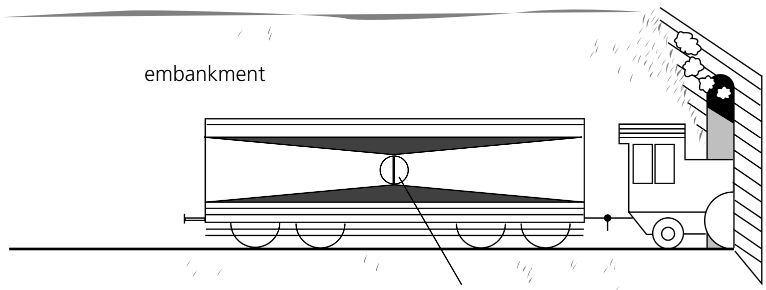


Figure 30.9
The train with the light pulse device.

To the person holding the device, the doors of the carriage will open simultaneously (Figure 30.10**(a)**). But to a person on the embankment, the back door will open before the front door (Figure 30.10**(b)**). This is because the stationary observer sees the back door move forward to meet the light pulse while the front door moves away from the light pulse. So the light gets to the back door before the other light can get to the front door.

Hence, the opening of the doors may be simultaneous to one observer (on the train) but not simultaneous to another (on the embankment). Students often say, 'Who is right? Do the doors really open together or not?' The answer is, 'Both are right.' It depends on your frame of reference. Remember, there is no best frame of reference; some are just more useful than others. You, as an observer, will decide which is the most useful frame.

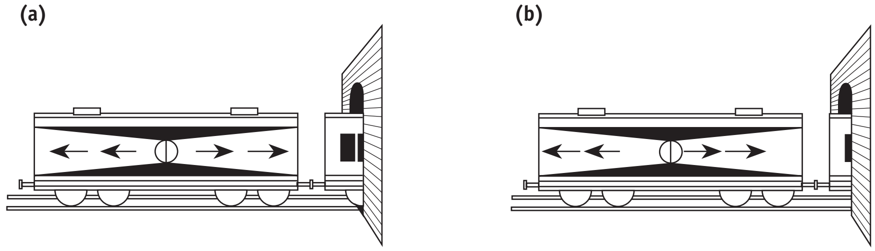


Figure 30.10
(a) Motion of the light pulse as seen by an observer inside the train;
(b) motion of the light pulse as seen by the observer on the embankment — it gets to the rear door first.

30.7 THE VARIABILITY OF TIME

We usually think of time marching on, oblivious to anything we may be doing. Although you may think time drags when you are doing something boring and goes fast when you're having fun, this is not time in a technical sense, only psychological time.

We're now going to convince you that the time interval between two events cannot be the same for two observers in motion with respect to each other. Imagine a bus that has a light source on the floor and a mirror directly above on the ceiling. A brief flash leaves the source and travels upwards to hit the mirror, reflect and return to the source (Figure 30.11**(a)**).

Consider how this looks to an observer sitting at a bus stop at the side of the road. The flash occurs when the bus is located to the left, strikes the ceiling mirror when it is in the centremost position, and returns to the source when the bus is towards the right (Figure 30.11**(b)**).

The labelling of the diagram is as follows. The distance from the source to the ceiling mirror is D. To the roadside observer the bus is travelling to the right at velocity $\boldsymbol{v}$, and moves a distance L in the time between the flash and the light striking the ceiling. The bus moves another distance L by the time the light goes from the mirror back to the source. This makes the total distance moved by the bus equal to $2L$. Light, of course, travels at a speed c for both observers (Einstein's second postulate).

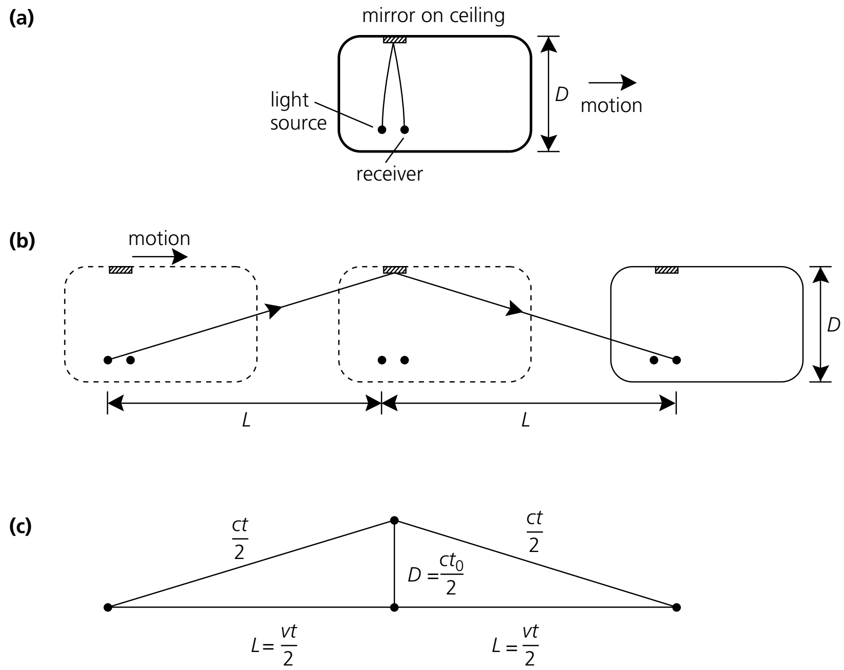


Figure 30.11
(a) Path of light as viewed by the observer aboard the bus; **(b)** path of light as viewed by the observer at the bus-stop — the path looks much longer; **(c)** derivation of Einstein's formula.

To the observer aboard the bus, the light travels a distance of $2D$ (up plus down) in a time t_0. This time is called **proper time** because the start and finish **occur in the same place in space.** Using our velocity formula $\boldsymbol{v} = s/t$, $t = s/\boldsymbol{v}$, hence $t_0 = 2D/c$. When rearranged, the distance $D = ct_0/2$.

To the observer at the bus stop, the light has travelled a triangular path in time t (t with no subscript, as distinct from proper time, t_0). As the speed of light is the same for both observers, the light actually travelled a longer path from the observer at the bus stop's viewpoint, so it must have taken a longer time. This is called 'relativistic' time or 'dilated' time (Latin *dilatare* = 'to spread out'), that is, to bring them away from each other or make them bigger. Dilated time is the time between two events that occur in two different places in space; in this case the two events (the flash leaving the source and the flash arriving back at the source) are separated by a distance of $2L$.

Looking at Figure 30.11**(c)**, the total distance travelled by the light is calculated by using the formula $\boldsymbol{v} = s/t$, or $s = \boldsymbol{v}t$. In this case, the distance travelled by the light will be ct. In the diagram the length of the hypotenuse will equal $ct/2$. The time taken (t) for the bus to go from start to finish will be given by $t = 2L/\boldsymbol{v}$, hence $L = \boldsymbol{v}t/2$ (the base of each triangle).

Using Pythagoras's theorem on one of the right-angled triangles:

$$\left(\frac{ct}{2}\right)^2 = \left(\frac{ct_0}{2}\right)^2 + \left(\frac{vt}{2}\right)^2$$

$$(ct)^2 = (ct_0)^2 + (vt)^2$$

$$t^2 = t_0^{\,2} + \frac{v^2}{c^2}t^2$$

$$t_0^{\,2} = t^2 - \frac{v^2}{c^2}t^2$$

$$= t^2\left(1 - \frac{v^2}{c^2}\right)$$

$$t^2 = \frac{t_0^{\,2}}{1 - \frac{v^2}{c^2}}$$

Take square roots of both sides:

$$t = \frac{t_0}{\sqrt{1 - v^2/c^2}}$$

Because v is always less than c, the value of $\sqrt{1 - v^2/c^2}$ must always be less than 1, so we see that $t > t_0$. That is, the time between the two events (the sending of the light, and its arrival at the receiver) is greater for the Earth observer than for the travelling observer. This is a general result of the theory of relativity, and is known as time dilation. Sometimes this is stated simply as **moving clocks are measured to run slow**, but most physicists hate this catchphrase. They say it is utterly meaningless, or worse, since it applies an absoluteness to motion. They prefer the better, but wordier, description of time dilation:

The time between two events is shortest when measured in a reference frame where the two events occur in the same place.

If you want to learn this as 'moving clocks run slow', just remember that you are referring to clocks moving with respect to you. The clocks will be stationary to someone 'moving' along with the clock — that is, to someone in the same reference frame as the clock. Moving is relative! Time really does appear to pass more slowly in a frame of reference moving with respect to you. For example, you could measure the time between two events in a frame moving relative to you as taking 3 seconds. To someone stationary with respect to the clocks, the time between the events may be 2 seconds. They say 2, you say 3, and that seems longer to you. This sounds amazing and is the inevitable outcome of the two postulates of the theory of relativity. Students often ask, 'But is it really true? or does it just appear to be true'. All we can say is that it does not violate the laws of physics and that it has been confirmed by many experiments, so it can be called a 'fact'. However, like all 'facts' in science, they can be replaced if better theories and experiments come along. For now, special relativity works beautifully well, but one day, when you're older ...

We know that the concept of time dilation is hard to accept, for it violates commonsense. We can see from the equation that the time dilation effect is negligible unless v is reasonably close to c. Table 30.1 shows the ratio of v/c (called the speed factor β) for different speeds, and the ratio of t/t_0 (called the **Lorentz factor** γ, after the Dutch physicist H. A. Lorentz who developed the formula before Einstein but didn't realise its significance).

Table 30.1 RELATIVISTIC EFFECTS

SPEED	$\beta = v/c$	$\gamma = t/t_0$
Car at 60 km h^{-1}	0.0000	1.000 000
Jet at 8000 km h^{-1}	0.000 006 8	1.000 000
1 GeV electron	0.999 999 88	2000
20 GeV electron	0.999 999 999 67	40 000
Light pulse	1.000	infinite

Table 30.1 shows that at ordinary speeds (e.g. the car), relativistic effects are negligible, but at speeds approaching the speed of light, the effects are dramatic. An experimenter working with 1 MeV electrons travelling at 0.94 times the speed of light ($0.94c$) would have to realise that 1 second of time in the electron's frame of reference is the same as 2.9 seconds of time in the laboratory frame of reference. Imagine that the electron spun once on its own axis every second when viewed from a frame of reference stationary to the electron (that is, if you rode along with the electron). To an observer in the laboratory, the electron would take 2.9 seconds to spin once. The electron's 'clock' appears to run slow.

Thousands of experiments have confirmed the theory of relativity. For example, in 1971 extremely accurate clocks were flown around the world on commercial aircraft, and when they were compared to the clocks left back in the laboratory a time dilation effect was confirmed. One of the most famous 'natural' confirmations of relativity is in the 1960s measurement of the lifetime of the unstable elementary particle known as a muon. Muons' 'rest' lifetime (that is, their lifetime as measured by someone at rest to them) is 2.2 microseconds. When they are created in the upper atmosphere by cosmic rays from the Sun (at an altitude of about 5 km) they travel towards Earth at close to the speed of light. Calculations using classical mechanics show that they would be expected to travel $2.2 \times 10^{-6} \times 3 \times 10^{8} = 660$ m before decaying. However, we observe them on the Earth's surface, so their lifetime is longer than expected and in full agreement with the relativity formulas.

Which clock is moving?

The thing that confuses students most of all in this work is defining which is the moving clock and which is the stationary clock. Einstein said that motion is relative so you could say either is stationary. So which is 't' and which is 't_0' in the formula?

It all depends on what event you are timing. Imagine that you are timing a rocket flight to the moon. There is a clock aboard the rocket for the space travellers and there are synchronised clocks on the Earth and on the Moon. The two events — take-off and landing — are measured by the space travellers with a single clock aboard the spacecraft but the observers on Earth need two clocks for their timing — one on Earth to register the time of take-off and one on the Moon to register the time of landing. We say that the space travellers have measured **proper time**, t_0, because the two events were measured in the one place by one clock at rest with respect to both events. The people on Earth measured **dilated time**, t, or **relativistic time**. Dilated time is longer than proper time.

Example

What will be the mean lifetime of a pion (an elementary particle) as measured in the laboratory if it is travelling $0.669c$ with respect to the laboratory? Its mean lifetime at rest is 3.5×10^{-8} s.

Solution

If an observer were to move along with the pion (the pion would be at rest to this observer), the pion would have a mean life of 3.5×10^{-8} s. This is proper time, t_0, and for elementary particles is sometimes called the **rest life**. To an observer in the laboratory, the pion lives longer because of time dilation:

$$t = \frac{t_0}{\sqrt{1 - v^2/c^2}}$$

$$= \frac{3.5 \times 10^{-8}\ \text{s}}{\sqrt{1 - (0.669)^2}} = \frac{3.5 \times 10^{-8}}{0.743} = 4.71 \times 10^{-8}\ \text{s}$$

Space and time units

A **light-year** (ly) is the distance travelled by light in one year. Numerically it is equal to $3 \times 10^{8} \times 60 \times 60 \times 24 \times 365$ or 9.5×10^{15} m. The distance to the red star Betelgeuse in the constellation Orion is 650 ly while our Sun is only 500 light-seconds away from us.

Space travel

If time dilation means that time slows down, it could be possible to live long enough to travel to distant parts of the universe. For example, say you wanted to travel to the star Rigel which is 800 light-years away. If you could travel at close to the speed of light as measured by someone on Earth, it would take about 800 years as measured by Earth observers for you to return. The original observers would all be dead by then. Let's say your speed was 0.999*c*. Then the time (t_0) for such a trip, as measured by the astronauts, could be calculated:

$$t = \frac{t_0}{\sqrt{1 - v^2/c^2}}$$

$$t_0 = t\sqrt{1 - v^2/c^2} = 800 \times \sqrt{1 - 0.999^2} = 35.8 \text{ years}$$

Thus you could make the trip and get back to Earth within your lifetime. Students often ask: 'Is it just the clocks aboard the spacecraft that slow down?' The answer is that everything slows down — all life processes slow down — and the astronauts would experience 35.8 years of normal sleeping, eating, working and so on.

NOVEL CHALLENGE

You want to get to a star 100 light-years away and have calculated you could do it in 4.5 years if you travelled at 0.999*c*. The problem is that the human body can't stand accelerations greater than 5 'g'. How much time, as measured by an Earth observer, would it take you to reach 0.999*c* from rest at an acceleration of 5 'g'? This next question may be too hard: estimate how much time it would take in the spacecraft's frame of reference.

Questions

1 How fast must a pion be travelling if its rest life is 2.6×10^{-8} s but to a laboratory observer it appears to live for 3.1×10^{-8} s?

2 A beam of a certain type of elementary particle travels at a speed of 2.85×10^{8} m s^{-1}. At this speed, the average lifetime is measured to be 2.50×10^{-8} s. What is the particle's lifetime at rest?

30.8 CONTRACTION OF LENGTH

There was a young man named Fisk,
whose fencing was exceedingly brisk.
So fast was his action, the Fitzgerald Contraction,
reduced his rapier to a disk.

In a previous example, you saw that a spaceship can travel a distance of 800 light-years in 35.8 years. You may wonder how it can do this in such a short time if nothing can go faster than light, and even it takes 800 years to get there. The answer is that, although time gets stretched in different reference frames, length gets squeezed. This is called c**ontraction of length**. An example involving a rocket departing Earth for the star Rigel may help.

The Earth–Rigel frame of reference

Imagine that an observer on Earth watches a rocket take off for the star Rigel at a speed of v. Both the astronauts and the Earth observer will agree on the speed of the rocket. The Earth and Rigel are at rest to one another so they form a single frame of reference. The distance between Earth and Rigel is 800 light-years and has the symbol L_0. This is called the **rest length** or **proper length** (hence the subscript 0) because it is the length measured by an observer at rest to both the Earth and Rigel.

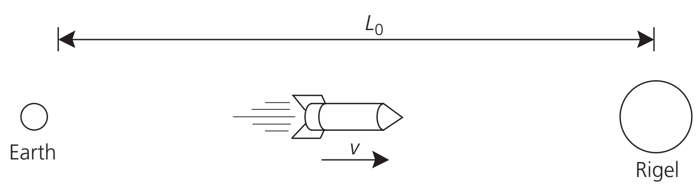


Figure 30.12
As viewed from Earth, the rocket is the moving frame.

The time taken for the journey according to Earth or Rigel observers is **dilated time** (t), because the departure and arrival events are separated in space and require two separate clocks for its measurement.

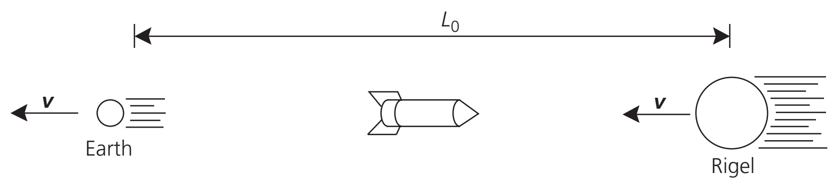
The spaceship frame of reference

Figure 30.13 shows the journey from Earth to Rigel from the astronauts' perspective. They can picture themselves as being stationary and assume the Earth's is rushing away from them and Rigel approaching them. This frame is moving with respect to the astronauts, so they measure the distance between the Earth and Rigel as the **contracted** or **relativistic length** L.

The time taken for the journey as measured by the astronauts is t_0 (proper time), because they are measuring the departure and arrival events in the same place in space (inside their rocket ship) and can use one clock to do this. The space travellers measure t_0, the **proper time**, because the take-off and landing occur at the same place in space for them. Only one clock is needed.

The space travellers and the Earth observers do, however, agree on the relative velocity ($\mathbf{v}$) between the two frames of reference.

Figure 30.13
As viewed from the rocket, Earth–Rigel is the moving frame. Either way, the distance is L_0.



Relationships between the frames

The time for the journey is t for the Earth observers and t_0 for the astronauts. The distance is L_0 for the Earth observers and L for the astronauts. They both agree on the velocity of the spaceship as $\mathbf{v}$. As we agree that the relationship between t and t_0 is given by: $t_0 = t\sqrt{1 - \mathbf{v}^2/c^2}$, the time measured by the astronauts (t_0) is less than that measured by Earth observers (t), hence $t_0 < t$. But as they agree on the velocity of the spaceship ($\mathbf{v}$), the distance travelled by the astronauts must also be less than that measured by the Earth observers. In other words, $L < L_0$. We now have two relationships: $\mathbf{v} = L_0/t = L/t_0$. If we rearrange the second part we get $L = \mathbf{v}t_0$ and if we replace the t_0 with the earlier equation we get this:

$$L = \mathbf{v} \times t_0 = \mathbf{v} \times t\ \sqrt{1 - \mathbf{v}^2/c^2} = L_0\ \sqrt{1 - \mathbf{v}^2/c^2}$$

That is:

$$L = L_0\ \sqrt{1 - \mathbf{v}^2/c^2}$$

Table 30.2 RELATIONSHIP BETWEEN THE FRAMES

FRAME OF REFERENCE	EARTH–RIGEL	SPACECRAFT
Time for journey	t	t_0
Distance travelled	L_0	L
Velocity	$\mathbf{v}$	$\mathbf{v}$

Summary of relationships for space travel (Table 30.2): $\mathbf{v} = \dfrac{L}{t_0} = \dfrac{L_0}{t}$.

This length contraction applies not only to distances between heavenly bodies but also between atoms — so objects shrink as they speed up. But this contraction occurs only along the direction of motion. For example, if a car travelled forwards at high speed, it would appear to shrink in length (from say 4 m to 2 m) but its height would remain the same at 1.5 m and its width the same at 2 m. If you could run as fast, your height would remain the same but you'd get thinner — but stay just as broad.

Example 1

A spaceship passes you at a speed of 0.80c. You measure its length to be 90 m. What length would it be to observers aboard the spaceship?

Solution

- $\mathbf{v} = 0.80c$.
- relativistic length L = 90 m.
- proper length, L_0 = ?

$$L = L_0\sqrt{(1 - \mathbf{v}^2/c^2)}$$

$$L_0 = \frac{L}{\sqrt{(1 - \mathbf{v}^2/c^2)}} = \frac{90}{\sqrt{1 - 0.8^2)}} = \frac{90}{0.6} = 150 \text{ m}$$

Example 2

A certain star is 36 light-years away. How many years would it take a spacecraft travelling at 0.98c to reach that star from Earth as measured by observers **(a)** on Earth; **(b)** on the spacecraft? **(c)** What is the distance travelled according to observers on the spacecraft? **(d)** What will the spacecraft occupants compute their speed to be from the results of **(b)** and **(c)**?

Solution

- $\mathbf{v} = 0.98c$; distance between stars and planets is proper length, L_0 = 36 ly.

(a) Observers on Earth measure dilated time, t:

$$\mathbf{v} = \frac{L_0}{t} \text{ or } t = \frac{L_0}{\mathbf{v}} = \frac{36 \text{ ly}}{0.98c} = 36.73 \text{ y}$$

Note: when a *distance* in light-years (ly) is divided by a *speed* in units of 'c', the answer is *time* in years (y).

(b) Space travellers measure proper time, t_0:

$$t_0 = t\sqrt{1 - \mathbf{v}^2/c^2} = 36.73 \times \sqrt{1 - 0.98^2} = 7.31 \text{ y}$$

(c) Space travellers measure relativistic length, $L = \mathbf{v} \times t_0 = 0.98c \times 7.31$ y = 7.16 ly.
Alternatively: $L = L_0\sqrt{(1 - \mathbf{v}^2/c^2)} = 36 \text{ ly}\sqrt{(1 - 0.98^2)} = 7.16$ ly.

(d) $\mathbf{v} = \dfrac{L}{t_0} = \dfrac{7.16 \text{ ly}}{7.31 \text{ y}} = 0.98c$ (same as for Earth observer).

Questions

3 Convert **(a)** 1.8×10^7 m s^{-1} to units of c; **(b)** 0.95c to m s^{-1}; **(c)** 30 ly to km; **(d)** 3×10^{15} km to ly.

4 An aeroplane whose rest length is 40.0 m is moving at a uniform velocity with respect to Earth at a speed of 630 m s^{-1}. Calculate the length of the aircraft as measured from the Earth.

5 Suppose you decide to travel to a star 85 light-years away. How fast would you have to travel so the distance would only be 20 light-years to you?

6 After the Sun, the nearest star visible to the naked eye is Rigel Centaurus, which is 4.35 light-years away. If a spacecraft was sent there from Earth at a speed of 0.80c, how many years would it take to reach that star from Earth as measured by observers **(a)** on Earth; **(b)** on the spacecraft? **(c)** What is the distance travelled according to observers on the spacecraft?

NOVEL CHALLENGE

Imagine you have slid into a parallel universe in which the year is 1840. A US Marshall is travelling by in a train at 0.75c and approaches two gunslingers about to have a duel. The gunslingers are both the same distance from the railway line. Gunslinger A is closer to the Marshall.

If the Marshall sees both men draw their guns at the same time, who actually drew first in ***(a)*** *the Marshall's frame of reference;* ***(b)*** *the gunslingers' frame of reference?*

NOVEL CHALLENGE

A physicist driving a very fast sports car is booked by police for travelling through a red traffic light. The physicist argues that because he was travelling fast wih respect to the light, the colour of the light had its wavelength altered and appeared green to him. The judge said that he would let him off the charge of running a red light but would fine him 1 cent for every m/s he was travelling over 100 km/h.

How much was he fined?

Note: the frequency of red light is 4.5×10^{14} Hz and green light, 6.0×10^{14} Hz. The transverse frequency shift formula

$$f_0 = f\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}},$$

is where f_0 = the frequency of the light with respect to an observer in the frame of reference of the source (i.e. the police).

THE TWIN PARADOX 30.9

Not long after Einstein proposed the special theory of relativity, an apparent paradox was pointed out. According to this *twin paradox*, suppose one of a pair of 25-year-old twins takes off in a spaceship travelling at very high speed to a distant star and back again, while the other twin remains on Earth. When the travelling twin returns they have aged differently according to the concept of time dilation (Figure 30.14). The question is: who has aged more? In the examples below we'll assume the travelling twin had an average speed of 0.80*c* for the journey and had aged 30 years aboard the spacecraft before returning.

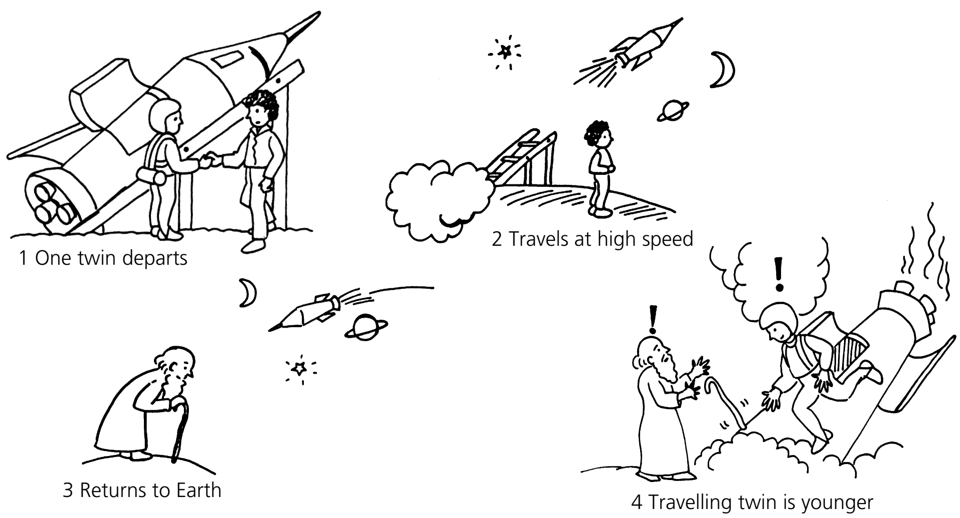


Figure 30.14 The twin paradox — how we can age at different rates.

Scenario 1: Earth's reference frame

According to the Earth twin, the travelling twin will age less.

- Proper time (aboard spacecraft): t_0 = 30 years.
- Velocity: $\boldsymbol{v} = 0.8c$.
- Dilated time:

$$t = \frac{t_0}{\sqrt{1 - \boldsymbol{v}^2/c^2}} = \frac{30}{\sqrt{1 - 0.8^2}} = 50 \text{ years}$$

Hence, the time elapsed on Earth is 50 years; this is how much the Earth twin will have aged. The travelling twin will be 55 years old (25 + 30) whereas the Earth twin will be 75 years old (25 + 50).

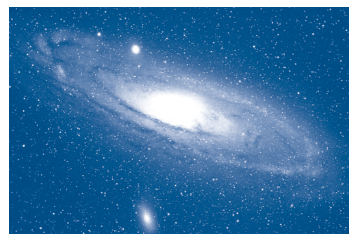
Scenario 2: spaceship's reference frame

Since 'everything is relative', all inertial reference frames are equally good. The travelling twin could make all the claims the Earth twin does, only in reverse. The travelling twin could claim that since the Earth is moving away at high speed, time passes more slowly on Earth and the twin on Earth will age less. In this case, proper time will be that as measured aboard the 'moving' Earth (30 years) and dilated time will be measured by the 'stationary' spacecraft (50 years). This is the opposite of what the Earth twin says. They both can't be right, can they? When the spacecraft returns to Earth they can stand beside one another and compare ages and clocks. Only one of the above scenarios will be correct — but which one?

Resolution

The problem can be resolved by deciding who is travelling with uniform motion. The travelling twin must change velocity at the beginning and end of the trip and also when turning around out in space, so he must be really moving, even if these acceleration periods occupy only a tiny portion of the total time. So the Earthbound twin measures proper length and the travelling twin measures the contracted (shortened) length. But as both twins agree on the relative velocity, the travelling twin must measure a shorter time (to cover the shorter length) and thus returns to Earth having aged less than the Earthbound twin. Even when the acceleration periods are considered, Einstein's general theory of relativity, which deals with accelerating reference frames, confirms this result. The ultimate judge, of course, is experiment, and the 1971 experiment of precise clocks sent around the world in jet planes confirms that less time passes for the traveller.

Photo 30.3
The Andromeda galaxy is 100 light-years away but astronauts would age only 20 years (in their time) if they travelled at 0.98c to get there. Andromeda is the nearest major galaxy to our own Milky Way galaxy, and our galaxy is thought to look much like Andromeda. Together these two galaxies dominate the local group of galaxies. The diffuse light from Andromeda is caused by the hundreds of billions of stars that compose it. The several distinct stars that surround Andromeda's image are actually stars in our galaxy, and are well in front of the background ob



Time travel

There was a young man named White,
who wasn't exceedingly bright.
He went out one day in a relative way,
and came back the previous night.

The prospect of travelling into the past or into the future has always excited people. However, it is always discounted as a non-scientific idea. But it's not! It all depends on how you look at events.

In the twin paradox, the space traveller arrived back on Earth as a 50-year-old to meet his Earth twin who was 75 years old. The space traveller had travelled into his twin's future. You can travel into the future — but once you are there, you can't go back.

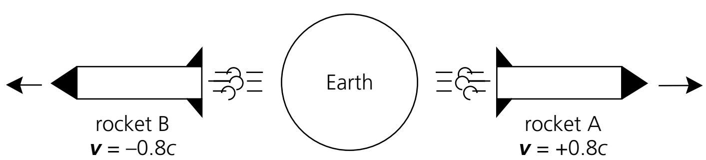
Likewise, the Earth twin went into his travelling twin's past to see the travelling twin at age 50. You can travel into the past — but only someone else's past.

30.10 RELATIVISTIC ADDITION OF VELOCITIES

In the classical view of motion, velocities were vector quantities that could be added or subtracted according to certain simple rules. In modern physics, this notion no longer holds. Consider the following two cases.

Case 1: Anti-parallel tracks

Consider two rocket ships, both leaving Earth in opposite (anti-parallel) directions and both travelling at 0.8c relative to the Earth (Figure 30.15).



Anti-parallel tracks

Figure 30.15
Anti-parallel tracks means moving in opposite directions.

The velocity of A relative to B can be calculated in the classical Newtonian method by:

$$\boldsymbol{v}_{AB} = \boldsymbol{v}_{AE} + \boldsymbol{v}_{EB}$$

Or, if rearranged, the relationship becomes:

$$\boldsymbol{v}_{AB} = \boldsymbol{v}_{AE} - \boldsymbol{v}_{BE}$$

It can be simplified by assuming the velocities are with respect to the same stationary frame of reference, in this case the Earth, and omitting the subscript E:

$$\boldsymbol{v}_{AB} = \boldsymbol{v}_A - \boldsymbol{v}_B$$

(this is the basic formula for relative motion from Chapter 2.)
If we substitute the data from the current scenario:

$$\boldsymbol{v}_{AB} = 0.8c - -0.8c = 1.6c$$

This is clearly wrong because no object can travel faster than the speed of light in any reference frame.

Einstein's modification

Einstein showed that since length and time are different in different reference frames, the old addition of velocities formula is no longer valid. Instead, the correct formula, he said, is:

$$\boldsymbol{v}_{AB} = \frac{\boldsymbol{v}_A - \boldsymbol{v}_B}{1 - \boldsymbol{v}_A\boldsymbol{v}_B/c^2}$$

When applied to the current situation:

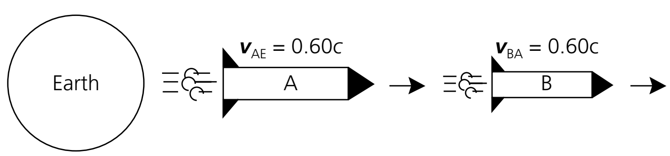
$$\boldsymbol{v}_{AB} = \frac{0.8c - -0.8c}{1 - \dfrac{(0.8c)(-0.8c)}{c^2}} = 0.98c$$

which is less than the speed of light.

Case 2: Parallel tracks

Consider rocket ship A, which travels away from the Earth with velocity $\boldsymbol{v}_{AE} = 0.60c$, and assume that this rocket has fired off a second rocket, B, on a parallel track, which travels at velocity $\boldsymbol{v}_{BA} = 0.60c$ with respect to the first (Figure 30.16).

Figure 30.14
Parallel tracks means moving in the same direction.



Parallel tracks

We might expect that the velocity $\boldsymbol{v}_{BE}$ (or $\boldsymbol{v}_B$) of rocket B with respect to Earth can be determined by:

$$\boldsymbol{v}_{BA} = +0.60c;\ \text{hence}\ \boldsymbol{v}_{AB} = -0.60c$$

Substituting into $\boldsymbol{v}_{AB} = \boldsymbol{v}_A - \boldsymbol{v}_B$:

$$-0.60c = 0.60c - \boldsymbol{v}_B$$
$$\boldsymbol{v}_B = 1.20c$$

This is again clearly wrong because no object can travel faster than the speed of light in any reference frame.

Using Einstein's relativistic addition of velocities formula:

$$\boldsymbol{v}_{AB} = \frac{\boldsymbol{v}_A - \boldsymbol{v}_B}{1 - \boldsymbol{v}_A\boldsymbol{v}_B/c^2}$$

$$-0.60c = \frac{0.60c - \boldsymbol{v}_B}{1 - \dfrac{(0.60c)(\boldsymbol{v}_B)}{c^2}}$$

$$\boldsymbol{v}_B = +0.88c$$

Questions

7 Spaceships A and B are approaching Earth at velocities of 0.75c and 0.50c respectively. Calculate the velocity of A relative to B if the ships are on **(a)** parallel tracks; **(b)** anti-parallel tracks.

8 The meteorite watch officer on a spaceship reports that two fast micrometeorites are approaching the ship on parallel tracks (i.e. from the same direction), one at a velocity of 0.90c and the other at 0.70c. What is the velocity of either of them with respect to the other?

9 A radioactive lithium nucleus is travelling at 0.70c in an accelerator when it emits a beta particle directly forward at a velocity of 0.80c relative to the nucleus. What is the velocity of the beta particle relative to the laboratory frame of reference?

30.11 MASS AND FORCE

In terms of Newtonian mechanics, if a rocket engine was used to propel a spacecraft through frictionless space, then its speed would continue to increase forever. Newton's second law of motion implies that as long as there is an unbalanced force, acceleration will continue ($\boldsymbol{F} = m\boldsymbol{a}$). For example, if a 10 000 N force was applied to a 100 kg satellite, then its acceleration would be 100 m s^{-2}. To go from rest to the speed of light could be calculated thus:

$$\boldsymbol{a} = \frac{\boldsymbol{v} - u}{t} \text{ or } t = \frac{\boldsymbol{v} - u}{\boldsymbol{a}} = \frac{3 \times 10^8 - 0}{100} = 3 \times 10^6 \text{ s (about 1 month)}$$

If the force was continued for another month then the speed would be twice that of light, and so on. Clearly, this is wrong.

Mass increase

So far, we've seen that time and length are relative, but another relative quantity is mass. In 1909 physicist Hans Bucherer was investigating beta rays (electrons) being emitted by radium. He found that they were being emitted at different velocities but the greater the velocity, the greater the mass. He applied Einstein's relativistic mass formula and found good agreement with the observations. The formula is:

$$m = \frac{m_0}{\sqrt{1 - \boldsymbol{v}^2/c^2}}$$

In applying this formula, Bucherer used the rest mass of the electron for m_0. This is the mass of the electron as measured by an observer at rest to the electron; that is, travelling along with it. The symbol m is used for the relativistic mass; that is, in the frame in which the electron is moving at a velocity $\boldsymbol{v}$. In Bucherer's experiment this was the laboratory.

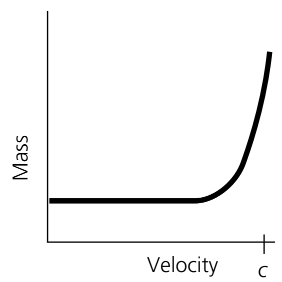


Figure 30.17
At a speed of c, mass would be infinite.

From the formula it should be obvious that relativistic mass is always greater than rest mass ($m > m_0$). But what may not be as obvious is how mass increases with velocity. A plot of the data produces a graph as shown in Figure 30.17. As velocity increases so does mass, but it increases exponentially; hence, you need more and more force for the same increase in velocity. As velocity approaches the speed of light, mass approaches infinity (as $\boldsymbol{v} \to c$, $m \to \infty$).

Example

The rest mass of a proton is 1.67×10^{-27} kg. What is the mass of a proton travelling at $\boldsymbol{v} = 0.865c$?

Solution

$$m = \frac{m_0}{\sqrt{1 - \boldsymbol{v}^2/c^2}} = \frac{1.67 \times 10^{-27}\text{ kg}}{\sqrt{1 - 0.865^2}} = \frac{1.67 \times 10^{-27}\text{ kg}}{0.50} = 3.34 \times 10^{-27}\text{ kg}$$

(At a velocity of $0.865c$, the mass of an object is double its rest mass.)

Questions

10 What is the mass of a neutral pi-meson $\pi°$ ($m_0 = 2.4 \times 10^{-28}$ kg) travelling at a velocity of $0.87c$?

11 Escape velocity from the Earth is 40 000 km h^{-1}. What would be the increase in mass of a 3.8×10^5 kg spacecraft travelling at that velocity?

The ultimate speed

Perhaps the most astonishing prediction to come out of the special theory of relativity is that there is a certain velocity beyond which nothing can go. The contraction of length formula suggests that, as the velocity of an object approaches c, its length approaches zero. This means that at a speed equal to c the object would disappear. You cannot go faster than light. You just can't! You can't even equal it. This is a fundamental result of the special theory of relativity. The speed of light is a natural speed limit in the universe. As an object is accelerated to greater and greater speeds, you are doing more work on it and giving it more kinetic energy. This extra energy is converted to mass ($E = mc^2$) so its mass becomes larger and larger and, at a speed of c, mass would be infinite and this is impossible. However, Einstein's equations do not rule out the possibility that objects exist whose speed is always greater than c. If such particles exist (the name 'tachyon' — Greek *tachy* = 'fast' — was proposed), the rest mass m_0 would have to be imaginary; in this way the mass m would be the ratio of two imaginary numbers for $v > c$, which is real. Did you do that in maths? For such hypothetical particles, c would be a lower limit of their speed. In spite of extensive searches for tachyons, none have been found. **It seems that the speed of light is the ultimate speed in the universe.**

MASS AND ENERGY 30.12

As you know from Newton's laws of motion, when a net force is applied to a body it accelerates ($\boldsymbol{F} = m\boldsymbol{a}$) and so it gets faster. Since the force is acting over a distance, work is done on the object ($W = \boldsymbol{F}\boldsymbol{s}$) and its kinetic energy increases as well. In classical mechanics, the work done on an object leads to an increase in speed and kinetic energy, but in relativistic mechanics, the work done on an object also increases its mass.

Hence, the relativistic kinetic energy formula incorporates the notion of relativistic mass. Einstein proposed the following formula:

$$E_k = mc^2 - m_0c^2$$

where m is the relativistic mass of an object travelling at speed v, and m_0 is the rest mass of the same object. Einstein called the first term (mc^2) **total relativistic energy** (or E_{tot}) and the second term (m_0c^2) **rest energy** (or E_0). Hence, kinetic energy is the difference between total energy and rest energy:

$$E_k = E_{tot} - E_0 \text{ or } E_{tot} = E_k + E_0$$

This is a mathematical statement of the principle of conservation of mass–energy: **The total energy (rest energy plus all other forms of energy) in a closed physical system is a constant** (and is equal to mc^2). You can change rest mass into kinetic energy and vice versa. In nuclear reactions or radioactive decay, kinetic energy is produced as mass is lost. In other words, rest mass is converted to kinetic energy as the particles fly away from each other. Einstein's formula $E = mc^2$ is one of the most famous in science. If you know the change in mass of an object, you can calculate how much energy this is equivalent to.

Example 1

(a) How much energy would be released if a pi-meson ($\pi°$) of rest mass of 2.4×10^{-28} kg decayed and its entire rest mass was transformed completely into electromagnetic radiation? (*Hint:* assume it is not moving and has no kinetic energy to contribute.)
(b) How much energy would be released if it was travelling at 0.8c when it decayed? (*Hint:* consider the kinetic energy as well.)

Solution

(a) $E = mc^2 = 2.4 \times 10^{-28} \times (3 \times 10^8)^2 = 2.2 \times 10^{-11}$ J.
(b) The total energy is equal to its relativistic mass (m) times c^2.

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} = \frac{2.4 \times 10^{-28}}{\sqrt{1 - 0.80^2}} = 4.0 \times 10^{-28} \text{ kg}$$

$$E = mc^2 = 4.0 \times 10^{-28} \text{ kg} \times (3.0 \times 10^8)^2 = 3.6 \times 10^{-11} \text{ J}$$

Note: you could also calculate the rest energy ($m_0c^2 = 2.2 \times 10^{-11}$ J) and add it to the E_k, which equals $(m - m_0)c^2$ or 1.4×10^{-11} J, giving a total the same as above (3.6×10^{-11} J).

Example 2

An electron (rest mass = 9.109×10^{-31} kg) moves with a speed of 0.8c. **(a)** Calculate its kinetic energy. **(b)** Compare this with the Newtonian kinetic energy.

Solution

(a) The mass of the electron at 0.8c is:

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}} = \frac{9.109 \times 10^{-31} \text{ kg}}{\sqrt{1 - 0.8^2}} = 1.5 \times 10^{-30} \text{ kg}$$

Thus its kinetic energy is:

$$\begin{aligned} E_k &= mc^2 - m_0c^2 \\ &= 1.5 \times 10^{-30} \times (3 \times 10^8)^2 - 9.109 \times 10^{-31} \times (3 \times 10^8)^2 \\ &= 5.30 \times 10^{-14} \text{ J} \end{aligned}$$

(b) The Newtonian calculation would give:

$$E_k = \tfrac{1}{2}mv^2 = \tfrac{1}{2} \times 9.109 \times 10^{-31} \times (0.8 \times 3 \times 10^8)^2$$
$$= 2.62 \times 10^{-14}\ \text{J}$$

but this is not the correct formula.

Questions

12 Calculate the kinetic energy of a proton travelling 9.2×10^5 m s^{-1}. The rest mass of a proton is 1.673×10^{-27} kg.

13 What is the kinetic energy of an electron whose mass is 5.0 times its rest mass?

HOW REAL IS RELATIVITY? 30.13

Because of the complexity of modern scientific theories, an adage has developed among physicists: 'You never really understand new theories — you just get used to them'. This is true of special relativity. Try explaining relativity to someone who knows nothing about it and you'll soon find out that people are quite sceptical. But relativity is included in all sorts of applications these days. When NASA began the Apollo program in the 1960s, they had to figure-in relativity to get their spacecraft trajectories right. Relativity theory works — it is used. It is essential in modern technology.

Photo 30.4
Global positioning system: a hand-held unit worth about $300. The one shown here is the Magellan GPS 310, which can get a precision fix on a location by tracking up to twelve GPS satellites simultaneously, and to an accuracy of 15 m or better.

NEI

Activity 30.5 NAVIGATION SATELLITES

In modern long-range navigation, the precise location and speed of a moving craft are continuously monitored and updated. With a modern system of navigation satellites called NAVSTAR, the location and speed anywhere on Earth can be determined to within about 15 m and 2 cm s^{-1}. However, if relativity effects were not taken into account, the accuracy would be unacceptable to modern navigation systems.

1. Suggest how you would find out if relativity effects are taken into account with the more common global positioning system (GPS) satellite used by small boats and outback adventurers. What would you ask?
2. If you have time, carry out your suggestion, and report back to your class. *Hint:* write, phone, look up a GPS manual or use one of the Internet newsgroups.

GENERAL RELATIVITY 30.14

Since special relativity requires all objects and particles to be limited by the speed of light, all forces and interactions must also travel at or below the speed of light. Newton's gravitational theory is in contradiction with this principle because it states that the gravitational force acts instantaneously. Einstein spent many years attempting to create a gravitational theory that would not require forces to act faster than light. Unlike special relativity, his theory of general relativity — developed in 1915 — allows for a simple treatment of objects moving with non-uniform (accelerated) motion. But like special relativity it begins with another of his simple postulates: 'Inertial mass and gravitational mass are the same.' This is called the **principle of equivalence**. It may not seem to be very profound, as we have never questioned the difference. Let us investigate. You may like to review pages 78 and 79 first.

In Newton's theory, two kinds of mass appear:

(a) **inertial mass** from his second law of motion: $F_{net} = m \times a$ or $a = \dfrac{F_{net}}{m_{inertial}}$

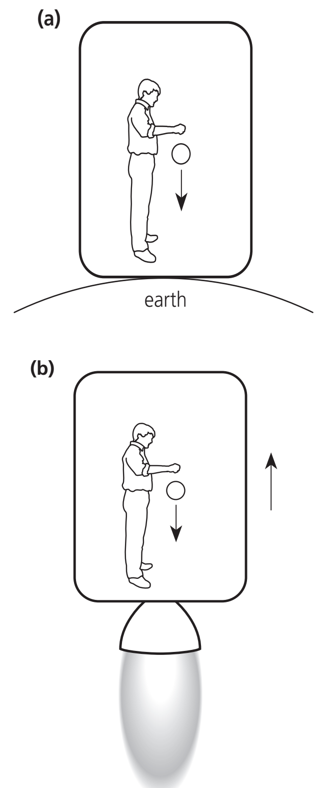
(b) **gravitational mass** from his law of gravity where the force between objects of masses M and m separated by a distance r is $F = GMm/r^2$, where G = the universal gravitational constant. Taking m to be an object near the Earth's surface, with M and r being the Earth's mass and radius, the force on m is:

$$F_{gravitational} = \frac{GM_{Earth}}{r_{Earth}^{\ 2}} \times m = 9.8 \text{ m s}^{-2} \times m_{gravitational}$$

In Newton's theory there is no physical reason why these masses should be related to each other. Why should the pull of gravity on an object be related to the reluctance of an object to accelerate when a net force is applied? Consider what happens if you drop an object from shoulder height. The only force acting on the object is supplied by gravity and hence the net force F_{net} is $F_{gravitational}$. The acceleration is given by $a = F_{net}/m_i$ and when the equation for F_{net} is substituted into it, the acceleration is given by $a = 9.8\ m_g/m_i$. If m_g and m_i were different, the acceleration of an object under the influence of gravity would depend on the inertial mass and thus fall differently. We know this is not true. In fact, the inertial and gravitational masses are equal in value to a very high precision (1 part in 10^{12}), and this is an astonishing mystery in Newton's theory that begs to be explained.

In characteristic fashion, Einstein hypothesised that these two kinds of mass are, in fact, one and the same and he sought to deduce the remarkable consequences of this hypothesis. Einstein was very good at using simple, but profound, physical reasoning to get to the heart of things. His discussion of free-falling elevators provided two good examples.

Figure 30.18
'Einstein's elevator' thought experiment. A stationary observer near Earth experiences similar forces to a person accelerating in a rocket in free space, where gravitational influences are almost zero.



Case 1: On Earth versus an accelerated reference frame in space

Imagine you are in a small closed room on Earth (such as an elevator). You are holding an apple, which you drop, and it accelerates towards the Earth (at 10 m s^{-2}) because of the constant force of gravity acting on it. Now let's attach a rocket motor to the room and travel to a distant region of intergalactic space, so far from any planets or stars that gravity appears to have no effect. The rocket engine propels the room with an acceleration of 10 m s^{-2}. If you hold the apple in your hand it shares your motion and you have the same motion as the room. So the room, you and the apple are all accelerating at the same rate. Now repeat the apple-dropping experiment. As soon as you let go of the apple there is no longer any net force acting on it. So what does it do? It obeys the law of inertia and tries to retain its current velocity. But the room is accelerating and its velocity is increasing relative to the apple so the floor catches up to the apple. To you, it appears that the apple hits the floor. Einstein asked, 'How does this differ from apple-dropping on Earth?' The answer is that it is no different. An unaccelerated frame in Earth's gravity is equivalent to an accelerated frame in the absence of gravity.

Case 2: Free fall on Earth versus an unaccelerated reference frame in space

Imagine you are in an elevator that is at rest relative to the Earth (Figure 30.19**(a)**). The gravitational force on your body, called your **weight** ($\boldsymbol{F}_W$), pulls you down onto the floor of the elevator. However, because you are neither going through the floor nor being thrown into the air it follows that the floor must be pushing up on you with exactly the same force. This is the normal reaction force ($\boldsymbol{F}_N$). You experience this reaction force as your weight. Suddenly disaster strikes; the elevator cables snap and you become weightless as you are in free fall (Figure 30.19**(b)**). The floor of the elevator is accelerating towards Earth as fast as you are (10 m s^{-2}). This has two consequences: the elevator cannot exert any force on your body and you remain at rest relative to the elevator. You take an apple from your pocket and let go of it. The apple appears to remain suspended in thin air. Again, this is because the apple is in free fall and is accelerating towards the Earth at the same rate as the elevator. Einstein asked, 'How different is this from being inside an elevator drifting around in outer space?' (Figure 30.19**(c)**). Einstein postulated that all physical phenomena occur exactly the same way in a frame (i.e. the elevator) accelerating in gravitational free fall as they do in a frame without gravity. Within the confines of the room, no experiment could help you decide which situation you were in. Nor could any other experiment involving the laws of motion.

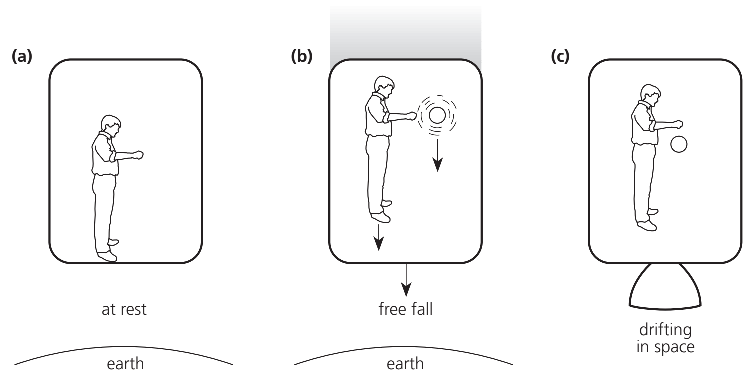


Figure 30.19
'Einstein's elevator' thought experiment. Free fall in a gravitational field is like weightlessness in the absence of a field.

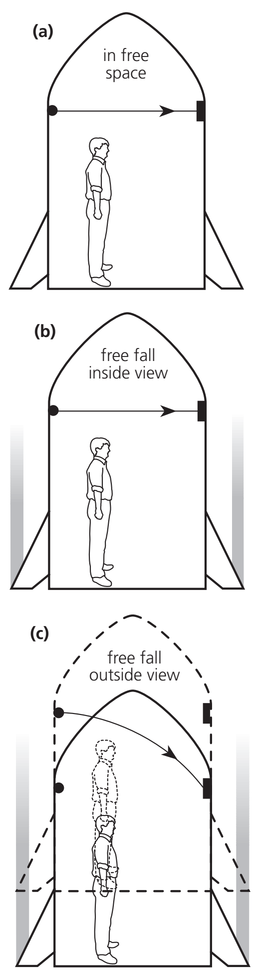


Figure 30.20
A laser beam has different paths to observers inside and outside a falling spaceship. Both are correct.

With his 'Gedanken' experiments, Einstein showed that gravity can be made to vanish, merely by going to a frame of reference that is in free fall. If gravity can be so easily banished, he reasoned that what we call the *force* of gravity may be imaginary. Perhaps gravity is not a force at all, but is somehow related to free motion in '**space-time**'. Einstein concluded that, since gravity has been transformed away within the elevator, all experiments conducted inside it should give the same results as experiments carried out in empty space, where there are no net gravitational influences. In 1915 Einstein translated his thoughts about nature into a rigorous mathematical theory, the **general theory of relativity**. Einstein summarised the results of his reasoning in his **principle of equivalence**, which can be restated as:

> All experiments will give the same results in a local frame of reference in free fall and in a local frame of reference far removed from gravitational influences.

That is, there is no experiment we can perform that will tell us whether we are in a free-falling reference frame (like the elevator above) or in a reference frame far away in space.

Consequences and tests of general relativity

The consequences of Einstein's profound hypothesis are quite remarkable. In the following section you will see some of the logical deductions of the hypothesis and how they have been verified.

Consider Figure 30.20. In a rocket in free space (somewhere far away from stars and planets) a laser beam is emitted from one side of the rocket towards a light detector on the opposite side (diagram **(a)**). The astronaut sees the laser beam travel in a straight line from one side to the other. How would this appear if the same rocket were in free fall near Earth (diagram **(b)**)? From inside, the astronaut will still see the laser beam travel in a straight line across the room. So far, nothing strange! But now consider the same experiment viewed from the reference frame of someone on Earth (diagram **(c)**). The stationary Earth observer also sees the laser beam hit the light detector, but by the time the light beam has crossed the cabin, the rocket and the light detector will have fallen a small distance. So the observer who is stationary with respect to Earth will see the laser beam follow a *curved path*.

Since the stationary observer believes himself to be in a gravitational field (because he feels his weight) he will conclude that gravity bends light. Einstein assumed that light, nonetheless, travels in as straight a line as possible. The fact that light's natural motion is curved could be understood if the space-time through which it travelled were itself curved.

The principle of equivalence is only the *basis* of general relativity. Just as the contraction of length and time dilation equations follow from the postulates of special relativity, a mathematical framework follows from this 'equivalence' postulate. Unfortunately, general relativity theory is too complicated to discuss quantitatively here (it involves the mathematics of **tensors** and **differential geometry**). However, some of its astonishing predictions make interesting reading. Four have become very popular with physicists: (1) the deflection of light by the Sun, (2) gravitational lensing, (3) the precession of the perihelion of Mercury, and (4) gravitational red shift.

Tests of general relativity

Deflection of light

Previously we imagined observing a beam of light in an accelerated elevator and saw that the light path was curved. By the equivalence principle the same must be true for light whenever gravitational forces are present. This was tested by carefully recording the position of stars near the rim of the Sun during an eclipse (see Figure 30.21), and then observing the same stars a year later when the Sun was not in a line between us and the stars.

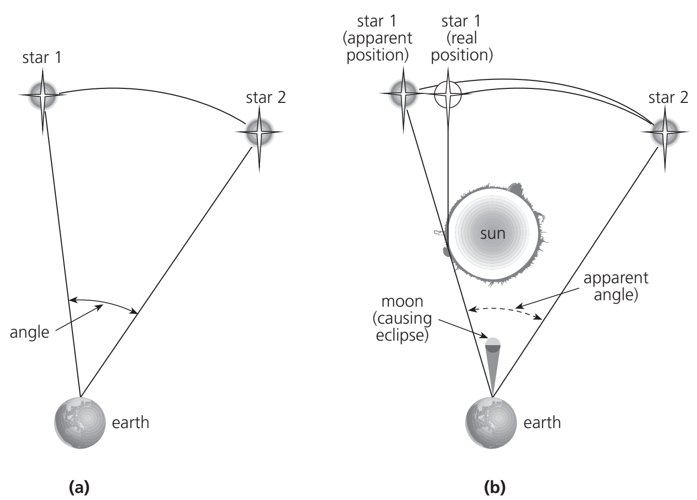


Figure 30.21
Distant stars appear to have a fixed angular distance between them, but if the rays of light from one should pass near the Sun, they are deflected by gravity.

During the eclipse the observed starlight reaches us only after passing through a region where gravitational effects from the Sun are very strong (that is why only stars near the rim are used), but the observations a year later are done at a time where the gravitational effects of the Sun on starlight are negligible.

It is found that the positions of the stars are displaced when photographs of both situations are compared. The deviations are the same as the ones predicted by general relativity. Eddington first observed this effect in 1919 during a solar eclipse.

Gravitational lensing

In the late 1970s, a double quasar was discovered. These quasars are powerful astronomical sources of radiation in the 'radio' band of the spectrum (10 cm to 10 m wavelength). The fact that it was a double quasar was unusual, but both had identical radio 'signatures' so everything about the two quasars seemed to be exactly the same, except that one was fainter than the other. It was suggested that perhaps there was only one quasar and something was producing multiple images. This 'something' was likely to be a massive but optically faint object which had bent its radio waves. A faint galaxy was subsequently discovered between the two quasar images, which confirmed this hypothesis. Other examples have since been discovered (see Figure 30.22).

Figure 30.22
A massive but optically faint galaxy can bend light from a distant quasar, producing multiple images. This is called 'gravitational lensing'.

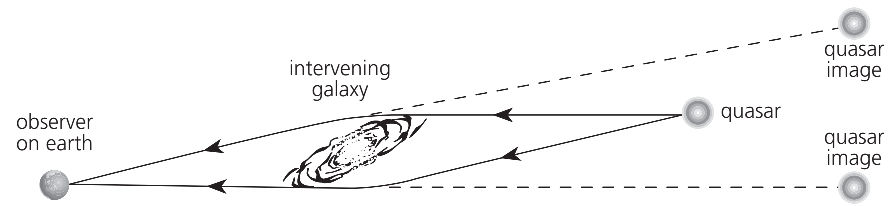
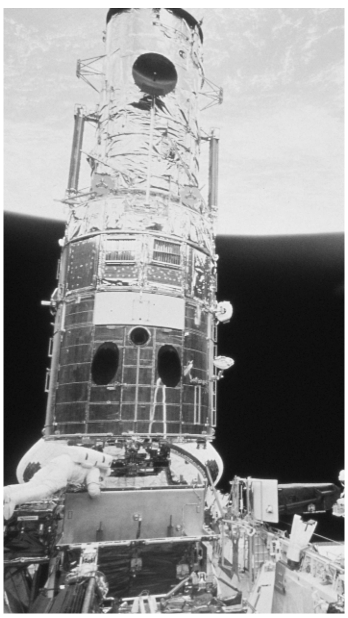


Photo 30.5
Hubble Space Telescope. Now over 10 years old (launched in 1990), the telescope is basically a new machine. Upgrades and maintenance keep Hubble operating in top condition to give us the best scientific data possible.



Gravitational lenses can be considered to be a test of general relativity. This concept has also given general relativity a new role in astronomy. By examining multiple images of a distant galaxy or quasar, their relative intensities and so on, astronomers can gain information about an intervening galaxy whose gravitational field causes the bending of light. The light may take several months or even years longer on one of the light paths than on the other to traverse the ten or more billion light-years to the quasar.

Such an effect has been observed by the Hubble Space Telescope (Photo 30.5). On a photo of a remote galaxy, two mirror images of the structure were observed on opposite sides of the picture (see Photo 30.6: Einstein's Cross). These were believed to be caused by the gravitational lensing of an intervening cluster of galaxies containing much **dark matter** that does not emit electromagnetic radiation and so cannot be detected by regular observations.

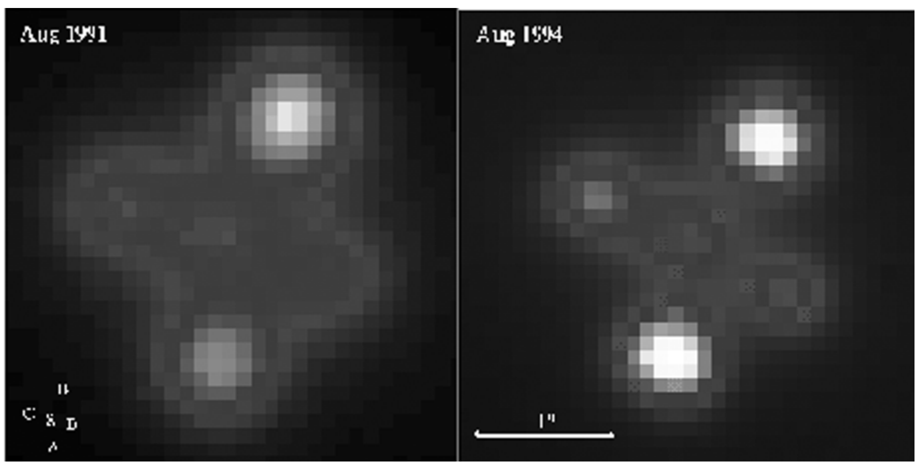


Photo: Geraint Lewis and Michael Irwin, William Hershel Telescope

Photo 30.6
Einstein's Cross is a multiple image of a quasar approximately 8 billion light-years away, formed by gravitational lensing of light as it passes a galaxy twenty times closer (400 million light-years). This is a famous example of an object that is seen four times. Here a very distant quasar happened to be positioned right behind a massive galaxy. The gravitational effect of the galaxy on the distant object was similar to the lens effect of a drinking glass on a distant street light — it created multiple images. But stars in the foreground galaxy have been found to act as gravitational lenses here too! These stars make the images change in brightness relative to each other. The brightness changes are visible on these two photographs of Einstein's Cross, taken about three years apart.
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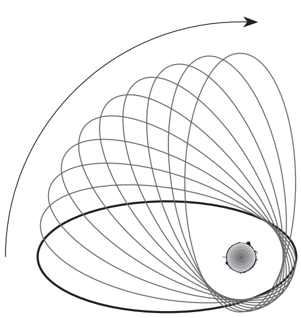
Figure 30.23
The slightly elliptical orbit of Mercury precesses due to the perturbing effects of the other planets.



Perihelion of Mercury

A long-standing problem in the study of the Solar System was that the orbit of Mercury did not behave as required by Newton's equations. As Mercury orbits the Sun, it follows an ellipse — but only approximately. It is found that the point of closest approach of Mercury to the Sun does not always occur at the same place, but slowly moves around the Sun (see Figure 30.23). This rotation of the orbit is called a *precession*.

The precession of the orbit is not peculiar to Mercury; all the planetary orbits precess. In fact, Newton's theory predicts these effects, as being produced by the pull of the planets on one another. The question is whether Newton's predictions agree with the *amount* an orbit precesses. It is not enough to understand qualitatively what is the origin of an effect; such arguments must be backed by hard numbers to give them credence. The precession of the orbits of all planets except for Mercury's can in fact be understood using Newton's equations. But Mercury seemed to be an exception.

The problem was that Newton's law of gravitation is correct only for a weak gravitational field. A strong field might well cause observation to deviate from the 'classical' expectation. If any planet shows deviation, it should be Mercury, for it orbits where the Sun's field is strongest. Using general relativity, a correction to the classically expected precession rate of Mercury may be calculated. The result of 43 seconds of arc per century is in good agreement with observation.

Gravitational red shift

One prediction of general relativity is that gravitation affects time by causing it to slow down. The greater the gravitational field, the greater is the slowing of time. In a region that has a strong gravitational field, such as the Sun, the slowing of time should be noticeable. The consequence is that the electronic vibrations of the Sun's atoms should also be slower. From your studies of wave motion and visible light you know that the rate of vibration (the frequency) is related to the colour of the light: the lower the frequency, the more the colour is shifted towards the red end of the spectrum. This is different from the Doppler red shift, which is caused by the relative movement of the source (stars) and observer. In 1960, two American physicists, R. V. Pound and G. A. Rebka, Jr, detected the red shift resulting from the Earth's gravitational field in agreement with Einstein's predictions.

The future

The special theory of relativity showed that relative motion can dilate time intervals and contract length. With general relativity we have seen that a gravitational field can also change (warp) time intervals, even when there is no relative motion. So we can ask, 'Can gravity warp space intervals as well?' The answer is 'Yes'. General relativity predicts that a massive heavenly body warps space-time nearby.

Representing warped space-time in three dimensions is difficult. It is easier in two dimensions, in which space is *area*. Figure 30.24 shows a massive heavenly body disturbing the uniformity of a two-dimensional space. All 'cells' in this 2D space are of equal area, but you should be able to see that to 'outside observers' like us, the cells near the heavenly body are really larger — but only from our 'extra-dimensional' viewpoint. You wouldn't notice the warping of real three-dimensional space if you were located within it — it is only apparent to someone outside it. We ourselves live within our space of three dimensions, and are not able to stand back and view our universe on four-dimensional axes. However, 'reduced-dimensional' views such as Figure 30.24 help to provide a qualitative understanding of some features of general relativity. These are the challenges facing physicists today.

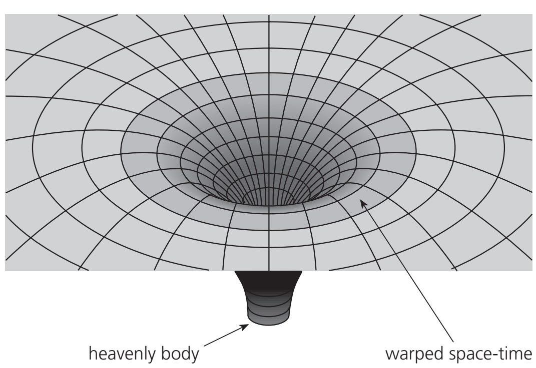


Figure 30.24
A massive heavenly body disturbs the regularity of a two-dimensional space.

Another challenge relates to new evidence for and against relativity. After a century, Einstein's theories have held up remarkably well. But as scientists probe the edges of the current knowledge of physics with new tests, they may find effects that require modifications of the venerable theory.

Several current theories, designed to encompass the behaviour of black holes, the Big Bang and the fabric of the universe itself, could lead to violations of special relativity. So far, recent, updated versions of century-old experiments show no signs that Einstein's vision is reaching its limits. Various tests are ongoing, however, and a new generation of ultra-precise, space-based experiments is set to launch in the next few years, offering some chance — however slim — of observing signs of the laws that will eventually supersede relativity.

In an updated version of the Michelson–Morley test, researchers have already sent laser light into two optical cavities set at right angles to each other. The light forms a standing wave in each cavity, with a frequency that depends on the cavity length and the speed of light in that direction. If light can go faster in one direction of space than another, rotating the apparatus should reveal this effect as a change in the relative frequencies between cavities. The team's preliminary results showed no deviation from special relativity.

In 2003 physicists also found that Einstein's theory passed the most accurate version yet of the Kennedy–Thorndike test – perhaps the most critical test of relativity first done in the 1930s. They compared the resonance of a standing light wave with an atomic clock over a period of 190 days, during which time Earth's orbital speed changes by 60 km s^{-1}. The result was ten times as accurate as previous Kennedy–Thorndike measurements, and no deviation from relativity theory was found.

Others physicists have monitored highly stable atomic clocks. These are collections of atoms that radiate at a certain frequency. Deviations from special relativity would show up as changes in their frequencies, depending on which way Earth is pointing. Earth-bound atomic clocks start to become unstable after just a few hours. Gravity, daily temperature changes and mechanical degradation are all sources of error. So the next generation of atomic clock measurements (the Primary Atomic Reference Clock in Space, PARCS) will compare the frequency of an ultra-cold caesium atomic clock against a hydrogen microwave laser (or maser). These measurements are scheduled to run on satellites or the International Space Station, where microgravity and a shorter rotation time should allow higher accuracy.

Finally, two important tests of relativity will come late this decade: the Superconducting Microwave Oscillator (SUMO) will keep time with a microwave-filled superconductor cavity. It will make measurements on its own and in conjunction with the Rubidium Atomic Clock Experiment (RACE) and the Space Time Mission (STM) will slingshot a satellite containing three atomic clocks around Jupiter for a close view of the Sun. The high speeds involved will offer more sensitive tests of relativity.

Practice questions

The relative difficulty of these questions is indicated by the number of stars beside each question number: * = low; ** = medium; *** = high.

Review — applying principles and problem solving

***14** If you were standing on top of a moving train and threw a rock straight up (as it appeared to you):
(a) how would the motion of the rock appear to your mother, who is standing on the platform; **(b)** would it land behind the carriage or on top of it;
(c) would you be in trouble when she got hold of you?

***15** If you were on a rocketship travelling at 0.6*c* away from the Sun, at what speed would the sunlight pass you?

****16** How fast must a pion be travelling if its rest life is 2.6×10^{-8} s but to a laboratory observer it appears to live for 5.2×10^{-8} s?

****17** A certain type of elementary particle travels at a speed of 2.6×10^{8} m s^{-1}. At this speed, the average lifetime is measured to be 2.20×10^{-7} s.
(a) Express its speed in units of '*c*'.
(b) What is the particle's lifetime at rest?

***18** Do mass increase, time dilation, and length contraction occur at ordinary speeds, say, 100 km h^{-1}?

****19** A spaceship passes you at a speed of 0.75*c*. You measure its length to be 120 m. How long would it be when at rest?

****20** Suppose you decide to travel to a star 80 light-years away. How fast would you have to travel so the distance would be only 40 light-years?

****21** If you were to travel to a star 24 light-years from Earth at a speed of 2.4×10^8 m s^{-1}, what would you measure this distance to be?

****22** Spaceships Alpha and Beta are approaching Earth at speeds of 0.85*c* and 0.65*c* respectively. Calculate the speed of Alpha relative to Beta if the ships are on **(a)** parallel tracks; **(b)** anti-parallel tracks.

****23** In the medical procedure called positron emission tomography (PET), a radioactive nucleus emits a positron that is captured by an electron. Both particles immediately self-destruct, forming two gamma photons that travel out in opposite directions. Knowing that photons travel at '*c*', calculate the speed of one relative to the other.

****24** At what speed will an object's mass be twice its rest mass?

****25** **(a)** What is the speed of an electron whose mass is 800 times its rest mass?
(b) If the electrons travel in a lab through a linear accelerator 1.5 km long, how long will this tube be in the electrons' reference frame?

****26** **(a)** How much energy can be obtained from conversion of 1.0 μg of mass?
(b) How much mass could this energy raise to a height of 10 m?

***27** If you were travelling away from Earth at a speed of 0.5*c*, would your mass, height, or waistline change? What would observers on Earth using telescopes say about these things?

***28** Consider the piece of paper on which one page of this book is printed. Which of the following properties of the piece of paper are absolute, that is, which are independent of whether the paper is at rest or in motion relative to you?
(a) The thickness of the paper; **(b)** the mass of the paper; **(c)** the volume of the paper; **(d)** the number of atoms in the paper; **(e)** the chemical composition of the paper; **(f)** the speed of the light reflected by the paper; **(g)** the colour of the coloured print on the paper.

Extension — complex, challenging and novel

****29** Because of the rotational motion of the Earth about its axis, a point on the Equator moves with a speed 460 km h^{-1} relative to a point on the North Pole. Does this mean that a clock placed on the Equator runs more slowly than a similar clock placed on the Pole?

****30** A 100 MeV electron, travelling at 0.999 987*c*, moves along the axis of an evacuated tube that has a length of 3.00 m as measured by a laboratory observer S with respect to whom the tube is at rest. An observer S′ moving with the electron, however, would see this tube moving past her. What length would the tube appear to the observer S′?

****31** A friend of yours travels by you in her fast sports car at a speed of 0.760*c*. It appears to be 5.80 m long and 1.45 m high.
(a) What will be its length and height at rest?
(b) How many seconds did you see elapse on your friend's watch when 20.0 s passed in the Earth's frame of reference?
(c) How fast did you appear to be travelling to your friend?

*****32** The star Alpha Centauri is 4.0 light-years away. At what constant velocity must a spacecraft travel from Earth if it is to reach the star in 3.0 years, as measured by travellers on the spacecraft?

*****33** Suppose that a special breed of cat (*Felix schrödingerus*) lives for exactly 7.00 years according to its own body clock. When such a cat is born, it is put aboard a spaceship and sent off at a speed of 0.80c toward the star Alpha Centauri.

(a) How far from the Earth (reckoned in the reference frame of the Earth) will the cat be when it dies?

(b) As soon as the cat dies, a radio signal announcing the death of the cat will be sent from the spaceship. How many years after departure will the signal reach Earth (radio signals travel at the speed of light)?

****34** Suppose a spacecraft of rest mass 20 000 kg is accelerated to 0.25c.

(a) How much kinetic energy would it have?

(b) If you used the classical formula for kinetic energy, by what percentage would you be in error?

*****35** An object with a rest mass of 1.000 kg is accelerated to high speed. Calculate the dilated mass (to 4 significant figures) of the object for speeds increasing from rest in increments of 0.1c up to 0.9c and also at 0.95c, 0.99c, 0.999c and 0.9999c. Plot a graph with speed on the x-axis. Use a computer spreadsheet if you like.

*****36** You are sitting in your Holden when a very fast sports car passes you at a speed of 0.18c. A person in the car says his car is 6.00 m long and yours is 6.15 m long. What do you measure for these two lengths?

*****37** Apart from the Sun, our nearest star is Proxima Centauri which is 4.225 light-years away. How many years would it take a spacecraft travelling 0.80c to reach that star from Earth as measured by observers **(a)** on Earth **(b)** on the spacecraft? **(c)** What is the distance travelled according to observers on the spacecraft?

*****38** Prove that the kinetic energy of an electron of rest mass 9.11×10^{-31} kg, which has a relativistic mass of 2.0×10^{-30} kg, is 0.62 MeV (1 J = 6.24×10^{18} eV) and that its speed is 2.7×10^{8} m s^{-1}.

****39** A proton has a total energy three times its rest energy.

(a) Calculate its rest energy in MeV.

(b) Calculate its KE in MeV.

(c) How fast is it travelling?

*****40** In relativity, momentum is conserved just as it is in classical physics. The formula for relativistic momentum is the same except that the relativistic mass (m) must be used instead of rest mass (m_0).

(a) Derive a formula for relativistic momentum (mv = m_0 etc.).

(b) Square both sides and rearrange to show that $m^2 = m_0{}^2 + (mv)^2/c^2$.

(c) Multiply both sides by c^4 and show that $m^2c^4 = m_0{}^2c^4 + c^2p^2$.

(d) For a photon (rest mass = zero), show that this equation becomes E_{total} (= mc^2) = cp.

(e) Using the equation in part **(c)**, show that the relativistic momentum of an electron described in Question 38 above is 5.4×10^{-22} kg m s^{-1}.

*****41** Imagine a rocketship takes off for a distant planet and can travel at many times the speed of light. (We know that this is impossible but let's just say you can for this question.) Observers on the planet are viewing the incoming spaceship through a powerful telescope. Describe what they will see from the moment the rocketship leaves Earth until it lands on the observers' planet.