Measurement of the speed of sound in a metal rod

Se-yuen Mak, Yee-kong Ng and Kam-wah Wu
Department of Curriculum and Instruction, Faculty of Education, The Chinese University of Hong Kong, Shatin, NT, Hong Kong

Two improved methods are suggested to measure the speed of sound in a metal rod. One employs a fast timer to measure the time required for a compression pulse to travel along the rod from end to end. In the other, a microphone is used to measure the frequency of the fundamental mode of a freely suspending singing rod, and the speed is calculated using $v = f\lambda$.

The measurement of the speed of sound in a solid rod is one of the most difficult demonstrations done in the A-level physics course. Traditionally, two methods are commonly used. In the Nuffield approach (Harris 1992, Duncan 1995), a signal generator is connected to a CRO via a 'giant make-and-break switch' comprising a hanging metal rod and a hammer (figure 1). When the rod is tapped with the hammer at one end, contact is made for a short period of time, during which the compression pulse travels along the rod to the far end and is reflected back as a rarefaction pulse to the near end. The time of contact is read from the time-base setting or by counting the number of complete waves displayed on the CRO with the sweep mode set to NORMAL†. The sound path is twice the length of the rod and the speed is equal to distance over time. As the time of contact is of the order of only 0.1 ms, counting is done by memory from the image sensation, which persists for about 1/20 s. Under most circumstances, only an order-of-magnitude estimation of the speed can be made.

† Under the auto-trigger-sweep mode as recommended by Nuffield, the sweeping of the electron beam is 'automatic' when no external trigger source is present. If contact is started while the sweeping voltage is increasing, only a portion of the trace can be seen. However, with the sweep mode set to 'normal' and the trigger level set above the noise level, there will be no trace on the screen unless the sweeping unit is triggered by the selected input. Thus, the waveform will always start from the left end of the trace. To our surprise, few textbooks adopting the Nuffield approach consider this caveat.

Another perpetual problem associated with this demonstration is that most students have only a vague perception about the phase change of the compression pulse at the far end and the breaking of the contact at the near end of the rod by the reflected rarefaction pulse. To illustrate the propagation of the disturbance in the rod, a concrete 'slow motion model' making use of a chain of spring-coupled trolleys, which is equally difficult to carry out, is often used to enhance understanding.

The second method can be found in some demonstration resource books, such as *The Dick and Rae Demo Notebook* (Carpenter and Minnix 1993). A metal rod of length L is held at the midpoint and excited to vibrate along its length by rubbing it with the thumb and forefinger. The fundamental frequency is determined either by an attached Kundt's tube (figure 2) (e.g. Berry 1986), using $f = v_{\mathrm{air}}/\lambda_{\mathrm{air}} = v_{\mathrm{rod}}/\lambda_{\mathrm{rod}}$, or by beats sounding from the singing rod and a source of known frequency. The speed is calculated from $v = f\lambda = 2fl$. The difficulty associated with this method is that the stick-slip action of the fingers requires some practice and luck. Also, the related frequency measurements suggested in the literature, to the best of our knowledge, were tedious and inaccurate.

In this note, we introduce two simplified methods, modified from each of the methods mentioned, to measure the speed of sound in a solid rod. In the first method, a fast timer is used instead of a CRO to measure the time required for a compression pulse to travel in a metal rod from end to end. In the second method, the frequency

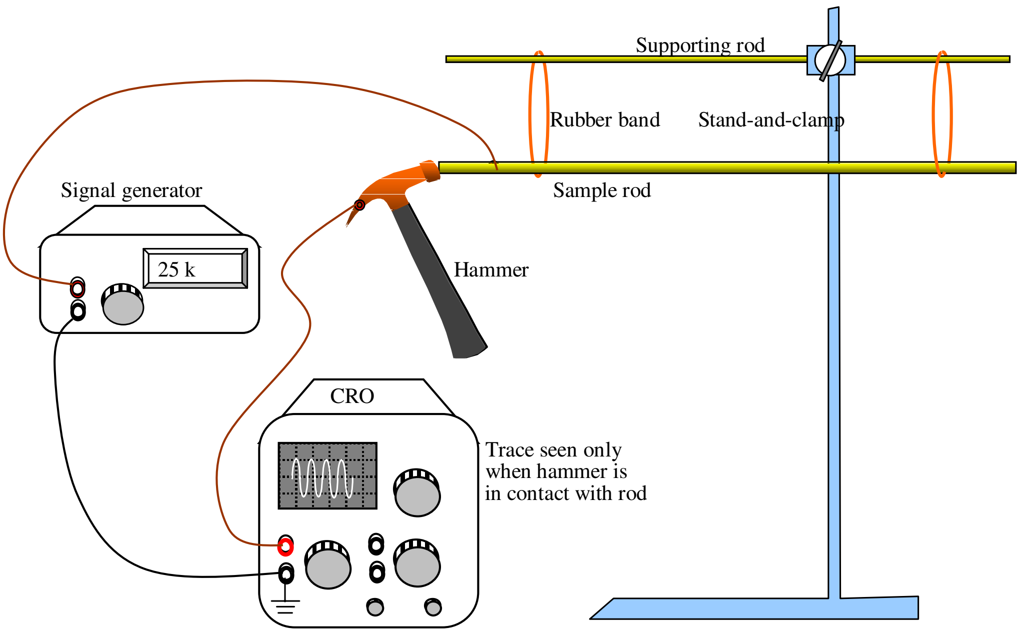

Figure 1. Timing the speed of sound in a metal rod with a signal generator and a CRO.

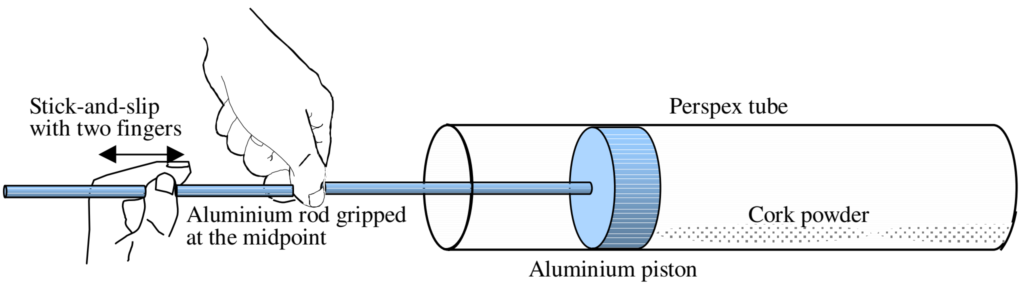

Figure 2. Exciting a metal rod and an adjacent air column to vibrate in the longitudinal direction by rubbing it with two fingers.

of the fundamental is measured using a horned microphone connected to a CRO.

Specimen rods

Looking back at the history of scientific endeavours, the speed of sound in a solid was first measured directly by Biot in 1808 (Beyer 1999). He used an iron water pipe about 1000 m long. A bell was sounded at one end and the time of arrival of sound at the other end through the rod and through air was measured. Since the speed of sound in air was known, by comparing the two times of arrival, the speed in the iron rod could be determined. Timers used in a school laboratory are now over 1000 times more accurate than those used in Biot's time, so we shall follow his general line of thought but use rods 1000 times shorter.

Materials commonly available in rod form include aluminium, copper, brass and iron or steel. Only rods satisfying certain length and mass requirements can be considered. As the final selection criteria are more or less the same in the two methods, only those related to method 1 will be elaborated here. In order to keep the percentage error below a certain level, the rod must be long enough to ensure that the time of travel is much longer than the random error and end corrections, if any, in the timing circuit. It should be massive enough so that a longitudinal pulse

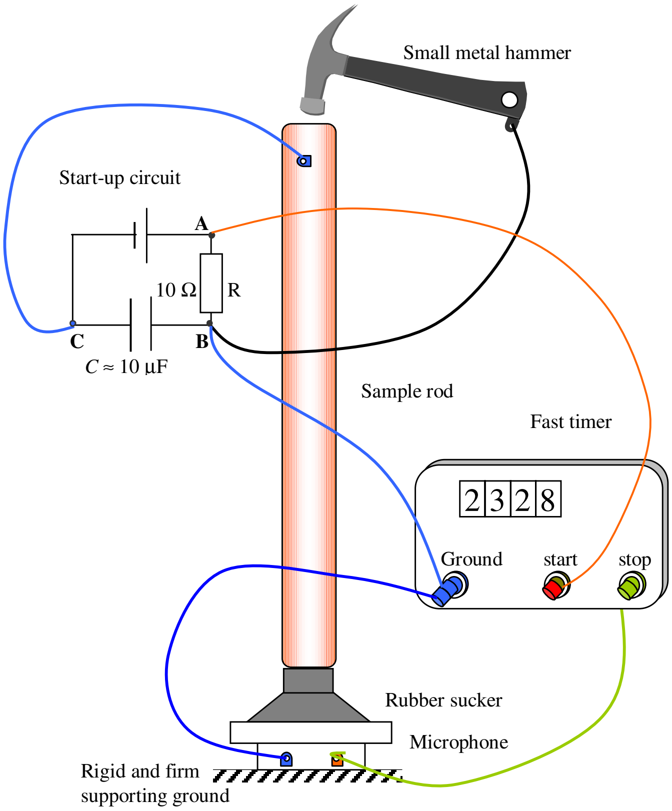

Figure 3. Set-up of the rod and the timing circuit for speed measurement.

can travel from end to end with little attenuation. Also, when one end of the rod is tapped, a large inertia is essential to prevent the rod as a whole from making a significant displacement during the timing period. A general criterion for selection is that when the rod is struck with a small hammer, a lingering humming sound can be heard for several seconds. Unlike the historical case, pipes are not recommended because hitting the edge may create unwanted transverse vibrations, but filling the bore at the ends may introduce a large damping, and skidding of the filling material may cause a time delay.

Method 1. Direct timing using a fast timer

Set-up

Normally, rods about 1 m long but below 5 kg can be held in a vertical position by hand or using a stand-and-ring. Timing is done using a fast timer with a 1–999 μs range and the timing circuit is connected as shown in figure 3. If very long and heavy rods (which yield better results) are used, for safety reasons and to avoid damaging the microphone, each should be hung horizontally and pressed against a vertical wall (figure 4).

The start-up circuit is simply a 1.5 V dry cell,

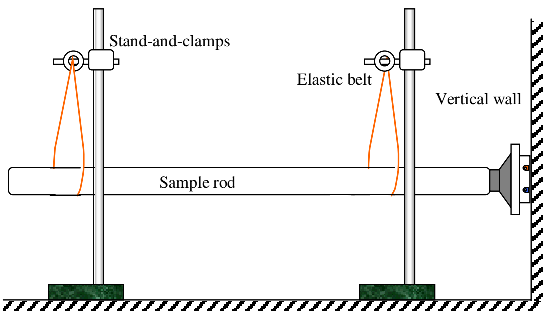

Figure 4. Hanging a long rod in a horizontal position against a vertical wall. The timing circuit is the same as that shown in figure 3.

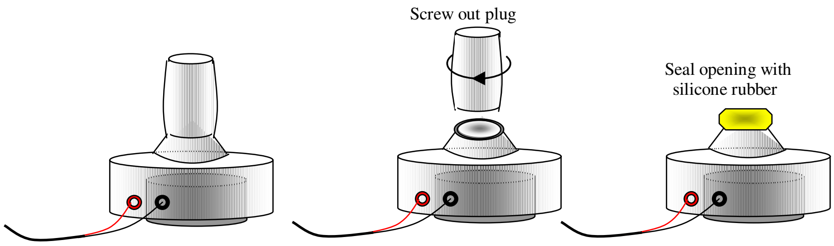

Figure 5. Converting a crystal earphone into a pulse collector–detector.

Table 1. Time required for sound to travel 1 m in a rod determined by a fast timer.

Sample material	T_1 (μs)	T_2 (μs)	Time required to travel 1 m (μs)	Percentage error
Aluminium	290 ± 10	85 ± 20	205 ± 30	14.6%
Brass	408 ± 10	108 ± 15	300 ± 25	8.3%
Cast iron	476 ± 10	250 ± 10	226 ± 20	8.8%
Copper	382 ± 10	105 ± 20	277 ± 30	10.8%

a capacitor C and a resistor R connected in series. The values of R and C can be fairly arbitrary. In our case, $R = 10\ \Omega$ and $C = 10\ \mu$F. The positive end (A) of the resistor is connected to the start-gate of the timer and its negative end (B) is grounded. The positive end (B) of the capacitor is connected to the hammer and its negative end (C) is connected to the rod.

A microphone with an attached sucker is placed under the rod over a hard surface. Its terminals are connected to the stop-gate of the timer. Alternatively, an earphone with the plug replaced by a patch of silicone rubber can be used in the stopping circuit (figure 5).

How does it work?

When the upper end of the rod is tapped by a small hammer, the capacitor is short-circuited; a pulse is sent out from the resistor to start the timer. As the compression pulse reaches the far end of the rod, it is converted into a pressure pulse in air by a rubber sucker and transmitted to the diaphragm of the microphone. With a suitable tapping force, the microphone will generate an electric pulse large enough (say, $V > 50$ mV) to stop the timer.

End correction

As the sucker is also used as a shock absorber to protect the microphone, the impact time is prolonged. Moreover, the inductive reactance in the receiver circuit may also produce a lag in timing. To minimize the inductive damping, a crystal microphone is preferred over moving-coil instruments because the time delay of the latter could be of the same order of magnitude as the target result. Whatever the cause may be, the total end correction, say ΔT, can be eliminated if timing is done using a pair of rods of different lengths for each sample material tested. In our experiment, all rods except the iron rod are about 2 cm in diameter and the lengths are 1.25 m and 0.25 m. The pair of iron rods used are 1.0 and 2.0 m long. Iron is exceptional because it is cheap and easily available. A length of 2 m is more or less the maximum length that can be handled safely in the laboratory and transported conveniently.

Let v be the speed of sound in the rod. If we denote the lengths of the long rod and the short rod by L_1 and L_2, and the corresponding timer readings by T_1 and T_2 respectively, then

$$T_1 = \text{time of travel} + \text{end correction} = \frac{L_1}{v} + \Delta T \tag{1}$$

$$T_2 = \frac{L_2}{v} + \Delta T. \tag{2}$$

Eliminating ΔT, we find

$$v = \frac{L_1 - L_2}{T_1 - T_2}. \tag{3}$$

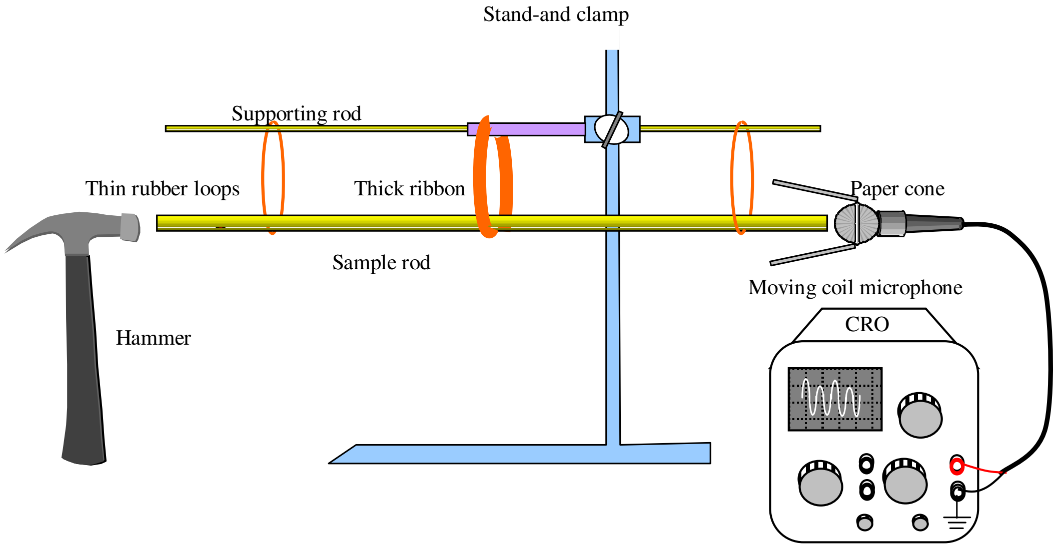

Figure 6. Determination of the frequency of the fundamental mode using a CRO and a horned microphone.

Results obtained from four sample materials are summarized in table 1. Each entry represents an average of five trials.

Method 2. Measuring the frequency of the fundamental mode of vibration

Set-up and procedure

The experimental set-up for method 2 is shown in figure 6. The sample rod, with a length ranging from 1.0 to 2.0 metres and 2 cm in diameter, is suspended at its midpoint by a thick ribbon. The rod is also supported loosely by two thin rubber bands, one near each end to keep it horizontal and prevent it from swinging about the pivot.

The rod is excited to vibrate by striking one end with a hammer. Once struck, a longitudinal standing wave generated by multiple reflection of the sound pulse is set up in the rod and a humming sound will stay ‘in the air’ for seconds. A moving-coil microphone connected to a CRO is held near the far end of the rod. A horn cut from a paper cup is used to feed more sound into the microphone. This arrangement enlarges the amplitude and prolongs the duration of the CRO trace.

Result

Because of our crude method of excitation, the sound produced by the singing rod is rich in harmonics, and so is the trace initially displayed on the CRO screen. However, since the rod is supported at its midpoint, overtones with an antinode at the midpoint, like f_2, f_4, etc (figure 7), will damp out more rapidly. Moreover, the inductance in the moving-coil microphone will filter out higher frequency harmonics. So the overall effect is that most overtones appearing in the trace will subside before the humming sound dies away completely. The CRO trace in the final stage is primarily that produced by the fundamental mode. The period of the fundamental can be measured easily because its zero-crossings stay in the same position on the screen for several seconds irrespective of the presence of other harmonics. The results obtained for rods of four different materials are summarized in table 2.

The percentage error depends on the amplitude and the duration of the trace. A very large error is obtained for aluminium because its density is much lower than that of other materials and the humming sound dies out in a much shorter time. In contrast, the best result is obtained in brass because the material is more elastic and the corresponding rod sings for a much longer time.

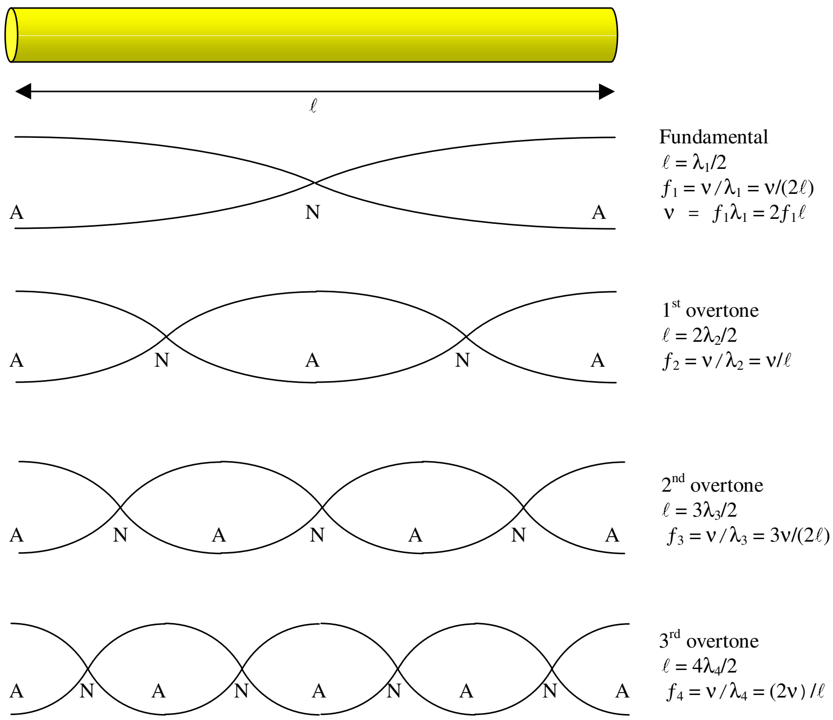

Figure 7. The first few longitudinal vibrational modes of a singing rod in transverse representation. A = displacement antinode; N = displacement node.

Table 2. Frequency of the fundamental mode of a longitudinal standing wave in a metal rod.

Sample material	Rod length (m)	Wavelength, λ_1 (m)	Period (T_1) measured in multiples of 0.1 ms	Frequency of fundamental, $f_1 = 1/T_1$ (kHz)	Percentage error
Aluminium	1.25	2.5	5.0 ± 1.0	2.0 ± 0.4	20%
Brass	1.0	2.0	6.0 ± 0.2	1.67 ± 0.06	3.5%
Cast iron	2.0	4.0	8.9 ± 0.5	1.12 ± 0.06	5.6%
Copper	1.25	2.5	7.0 ± 0.5	1.40 ± 0.10	7.1%

Final result and conclusion

An overall summary of our results obtained from four different materials using the above two methods together with data from a handbook for physicists are given in table 3. In method 1, an error of about 10% is obtained for all samples. This error enters our results because a much shorter rod, which satisfies our selection criterion only marginally, is used to eliminate the end correction. In method 2, with the exception of aluminium, the percentage error of speed measurement is about 5%. This error corresponds to a ± 0.2 mm uncertainty in determining the period of the fundamental using the CRO trace.

Besides the basic experimental skills in handling timing devices and error treatment, students also benefit from the above methods in a number of ways. First, instead of following a recipe, pupils are encouraged to make small changes to improve the set-up or procedure when doing experiments in a way similar to what we have done here. For example, a sucker can be used to transmit the compression pulse from a solid rod to the microphone; and a paper cup can be used to collect and amplify the sound signal. Second, these investigations enable pupils to see the connection between different physical quantities, such as the speed of sound and the Young modulus, which appear to be independent of one

Table 3. Comparison of experimental results and standard values for the speed of sound in a rod, $v = \sqrt{E/\rho}$ where E is the Young modulus and ρ is the density of the material.

Material	Accepted value of v from handbook† at 20 °C (km s^{-1})	Measured value of v and percentage error from method 1 using equation (3) (km s^{-1})		Measured value of v and percentage error from method 2 using equation $v = 2f_1 l$ (km s^{-1})	
Aluminium	5.00	4.9 ± 0.7	14.6%	5.0 ± 1.0	20%
Brass	dependent on composition	3.3 ± 0.3	8.3%	3.34 ± 0.11	3.5%
Cast iron	4.40	4.4 ± 0.4	8.8%	4.48 ± 0.25	5.6%
Copper	3.75	3.8 ± 0.4	11.2%	3.50 ± 0.26	7.1%

† In some handbooks, the speed of sound in a solid bulk is given. The latter is calculated from $(B/\rho)^{1/2}$ where B is the bulk modulus of the material. This value may be a few per cent different from the linear speed in a rod.

another. Third, these experiments suggest that some physical constants, like the Young modulus, can be measured using an indirect method. Fourth, pupils will recognize, through concrete experiences, that the mechanical properties of certain materials such as brass render them more suitable for making (wind and percussion) musical instruments. Also, they will discover in depth that there are some subtle differences in the response characteristics of various types of microphones and will appreciate why different instruments have to be used in different situations.

Note added in proof. It was discovered by Mr Lee Chung Kay of the Physics Department (CUHK) that a ceramic piezoelectric buzzer plate could be used to collect the sound pulse in method 1. To do this, the specimen rod is hung or placed horizontally on two pegs. The buzzer plate is attached to the far end of the rod by double-sided adhesive tape and its leads are connected directly to the stop-gate of the faster timer. When the buzzer plate is used, the end correction ΔT in equation (2) is reduced from about 35 μs to less than 10 μs.

Received 3 July 2000
PII: S0031-9120(00)15149-4

References

Berry D A 1986 *A Potpourri of Physics Teaching Ideas* (College Park, MD: American Association of Physics Teachers) p 246

Beyer R T 1999 *Sounds of Our Times* (New York: Springer) p 36

Carpenter D R and Minnix R B 1993 *The Dick and Rae Demo Notebook* (Lexington: Dick and Rae) section on wave: W-365

Duncan T 1995 *Advanced Physics for Hong Kong—Volume 2: Waves, Atoms and Materials* 2nd edn (London: John Murrray) pp 296–7

Harris J (ed) 1992 *Revised Nuffield Advanced Physics* 4th impression (Essex: Longman) pp 261–5