Air-Water Rockets

Air-water rockets are good fun to make and you can learn a lot from studying how they work. The following is a brief explanation of how they work by Peter Nielsen, Associate Professor in Civil Engineering, The University of Queensland.

What does the thrust force F_{thrust} depend on?

The thrust force F_{thrust} from any rocket exhaust is the mass flow $\dot{m}$ times the velocity u_{ex} of the exhaust relative to the rocket

$$F_{thrust} = \dot{m}\, u_{ex} = \rho A\, u_{ex}^2 \qquad (1)$$

where ρ is the density of the exhaust fluid, and A is the outlet cross section.

For air-water rockets, *Bernoulli's equation* gives $u_{ex} = \sqrt{2P^+ / \rho_w}$ where P^+ is the internal over-pressure and ρ_w the density of water. The thrust force is then, remarkably

$$F_{thrust} = 2P^+ A \qquad (2)$$

which is <u>twice</u> the force required to hold the water back before start.

Finding the enclosed air volume *V*

The pressure inside the rocket decreases as the water escapes and the air volume V increases. The expansion process is complicated by the fact that some water vapour condenses and some heat is received from the surroundings. However, a reasonably simple model is obtained by assuming the process to be like the adiabatic expansion of an ideal gas. In that case:

$$P = P_o\, (V_o/V)^{1.4} \qquad (3)$$

if the volume and absolute pressure at take-off are V_o and P_o.. The rate of increase of the enclosed air volume due to the exhaust flow is then

$$u_{ex} A = \sqrt{\frac{2P^+}{\rho_w}} A = \sqrt{\frac{2[P_o (V_o / V)^{1.4} - P_a]}{\rho_w}}\, A \,, \quad V<V_{tot} \qquad (4)$$

P_a is the atmospheric pressure and $P^+=P-P_a$. The air volume at a slightly later time $t+\delta_t$ can then be calculated as

$$V(t+\delta_t) \approx V(t) + \delta_t u_{ex} A = V(t) + \delta_t \sqrt{\frac{2[P_o (V_o / V)^{1.4} - P_a]}{\rho_w}}\, A \,, \quad \text{for } V<V_{tot} \qquad (5)$$

These formulae hold untill the water is used up and $V=V_{tot}$.

Finding the mass *m* of the rocket

If the solid parts of the rocket have the mass M_s, and the total volume of enclosed air and water is V_{tot}, the mass of the rocket as function of time is

$$m(t) = M_s + M_a + \rho_w (V_{tot} - V) \quad \text{for } V < V_{tot} \qquad (6)$$

M_a is the mass of enclosed air which can be estimated as

$$M_a = \frac{P_o V_o}{288 T_o} \qquad (7)$$

where T_o is the starting temperature in degrees Kelvin.

The temperature of the enclosed air

As the enclosed air expands, it cools down considerably. Under adiabatic conditions with no heat received from the surroundings or from condensing water vapour, it would vary as

$$T = T_o \left(\frac{P}{P_o} \right)^{0.286} = T_o \left(\frac{V}{V_o} \right)^{-0.4} \qquad (8)$$

The drag force from the surrounding air

As the rocket moves through the air it is slowed down by air drag. The magnitude of the drag force F_D depends on the density of the surrounding air ρ_{air} (≈1.2kg/m^3), on the speed u of the rocket, its cross sectional area A_R and on a drag coefficient C_D. The drag coefficient is of the order 1.0 for non-streamlined objects but may be as low as 0.1 for very streamlined objects. The formula for the drag force is

$$F_D = -½ \rho_{air} C_D A_R |u| u \qquad (9)$$

Finding the acceleration *a* of the rocket

If the rocket is moving vertically upwards against gravity, Newton's Second Law gives its acceleration a by

$$ma = F_{thrust} + F_D - mg = 2P^+A - \frac{1}{2}\rho_{air} C_D A_R |u| u - mg \qquad (10)$$

where g is the acceleration due to gravity, 9.8m/s^2 at the surface of planet Earth.

Using the expression (6) for the instantaneous mass and rearranging, this means

$$a = \frac{2P^+A - \frac{1}{2}\rho_{air} C_D A_R |u| u}{M_s + M_a + \rho_w (V_{tot} - V)} - g, \quad \text{for } V<V_{tot} \qquad (11)$$

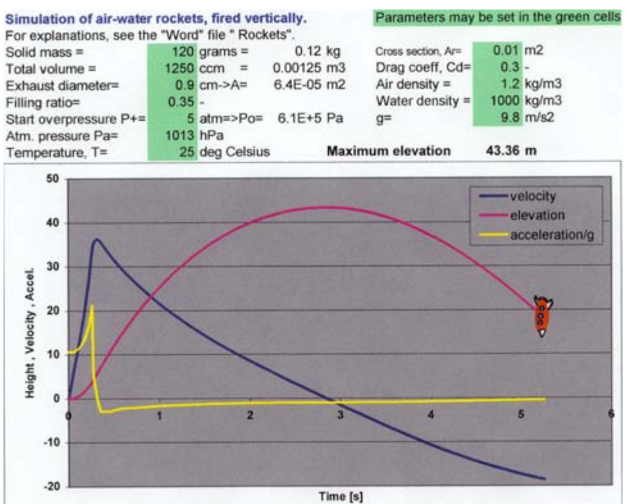
After the water is used up ($V \to V_{tot}$) , air is expelled at great speed. Initially (for $P^+ > 1.89P_a$), this gives $F_{thrust} = 0.89P^+A$, and the air is escaping at the speed of sound.

Finding the speed *u* and height *h* of the rocket

The formulae above can be used to calculate speed u and height h of the rocket in a spreadsheet. You start with u=0 and h=0 at time zero and then update u and h after each time increment δ_t using

$$u(t+\delta_t) = u(t) + \delta_t \frac{[a(t) + a(t+\delta_t)]}{2}$$
$$h(t+\delta_t) = h(t) + \delta_t \frac{[u(t) + u(t+\delta_t)]}{2} \qquad (12)$$

In the spreadsheet you can try different parameters and optimize by trial and error. For example, how much water to put in to get the best result.



time count	time [s]	V/Vtot [-]	P/Pa [-]	mass [kg]	Thrust [N]	accele-ration [m/s2]	speed [m/s]	Plot Speed	height [m]	Drag [N]
0	0	0.65	6.00	0.563	64.4	104.6	0.00	0	0.00	0
1	0.010	0.666	5.80	0.543	61.8	104.1	1.04	1.04	0.01	0.0
2	0.020	0.683	5.60	0.523	59.3	103.7	2.11	2.11	0.02	0.0
3	0.035	0.706	5.35	0.493	56.0	103.7	3.67	3.67	0.06	0.0

It will also help you find the best bottle dry mass (M_s). You will find that the lightest bottles are not the best!

You can also experiment with jet propulsion of aircraft.

For this type of aircraft you will find that the trust force has to be reduced be reducing A. Otherwise, the wings get ripped off the plane at take-off.