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Coupled pendulums: a physical system for laboratory investigations at upper secondary school

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Abstract
The topic of coupled oscillations is rich in physical content which is both interesting and complex. The study of the time evolution of coupled oscillator systems involves a mathematical formalization beyond the level of the upper secondary school student's competence. Here, we present an original approach, suitable even for secondary students, to investigate a coupled pendulum system through a series of carefully designed hands-on and minds-on modelling activities. We give a detailed description of these activities and of the strategy developed to promote both the understanding of this complex system and a sound epistemological framework. Students are actively engaged (1) in system exploration; (2) in simple model building and its implementation with an Excel spreadsheet; and (3) in comparing the measurements of the system behaviour with predictions from the model.

Introduction

Physics laboratory activities for upper secondary school students (16–17 years old) are usually limited to investigating the behaviour of simple systems that can be easily described by means of well-known laws. In laboratory courses, students are often required to do no more than verify what they expect on the basis of *a priori* theoretical predictions. Such an approach optimizes the time schedule, but often fails to stimulate the student's interest and, perhaps, also fails to provide a correct epistemological framework. In fact, much research [1–6] indicates that a high proportion of students:

- consider most experiments performed in the physics laboratory unchallenging, often cumbersome and with predictable outputs; and
- have serious difficulties in appreciating the basic epistemological status of physics.

Specific activities which adopt a modelling approach aim to improve understanding and attitudes toward physics, stimulating effective learning and sound epistemological appreciation of the discipline [7–10].

In the last few years we have developed laboratory activities based on a system of coupled

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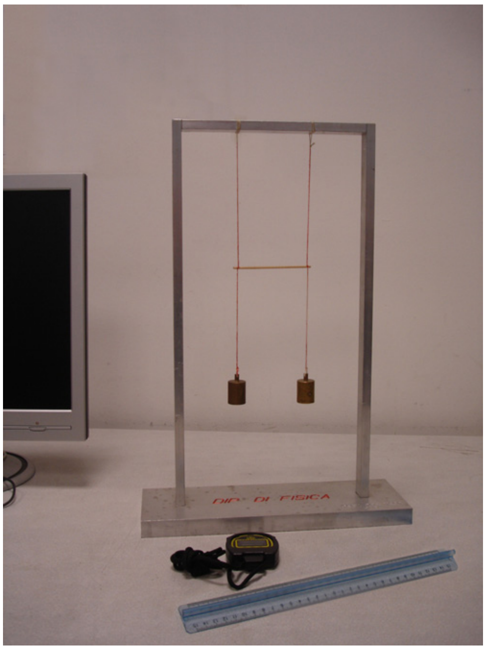

Figure 1. A picture of the coupled pendulum system.

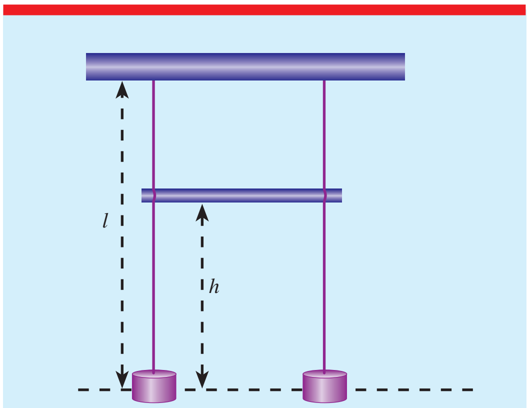

Figure 2. A sketch of two pendulums of same length l coupled by a thin horizontal rod at distance h from the bottom.

pendulums and trialled them with upper secondary school students, freshmen and senior university students, and with in-/pre-service teachers. This topic presents a rich physics content and can be analysed in different ways, taking into account both the laboratory equipment available and the students' mathematical backgrounds [11–15].

The topic is not usually included among standard laboratory activities for secondary school students and university entrants, since its mathematical formalization is beyond them.

In this article:

- we discuss an original approach to investigating a system of two coupled pendulums, suitable even for young students;
- we give a detailed description of the activities proposed and the strategy developed to promote both understanding of this complex system and a sound epistemological framework; and
- we discuss its impact on students and teachers.

The coupled pendulum system and the activities proposed

The coupled pendulum system under analysis is shown in figure 1. It consists of two cylindrical bobs suspended from a fixed support by two thin strings of the same length l. Each bob can oscillate independently and represents, as is well known to students, a simple pendulum. These two independent pendulums can be coupled by a thin rod, around which each string is wrapped. The coupling rod is positioned horizontally at height h with respect to the centre of mass of the two bobs, as shown in figure 2.

Such a system can be made to oscillate by displacing one of the bobs from its equilibrium position, in a vertical plane containing the rod, and then releasing it. The motion of this bob transfers to the other one. Over time, the amplitude of the first bob's oscillation decreases to a minimum amplitude as the other bob oscillates with increasing amplitude. The process is then reversed: the amplitude of the second bob decreases until it reaches a minimum amplitude, as the first bob increases its amplitude to a maximum. This interchange of motions continues until the two pendulums come practically to rest, because of unavoidable dissipative forces. The motion of the two bobs is coupled: energy transfer from one pendulum to the other is clearly due to the coupling rod. Moreover, the time interval between two consecutive minimum amplitudes of a given bob, or in other words, the time needed to transfer the energy of the motion, depends on the rod position, i.e. on the height h: the time increases as h approaches the pendulums' length l.

In the following subsections, we describe the three phases of the activities in which students can be engaged: system exploration, model development, and model validation.

System exploration

The aim of this phase is to motivate students to represent what they observed about the behaviour of the oscillatory coupled pendulums.

First of all, we encourage students to *play* with the system in order to:

- investigate its phenomenological characteristics, e.g. how the time T_t of energy/motion transfer between the two bobs varies as a function of the length l of the string and of the height h of the coupling rod;
- discover, by careful observation of the system, that the resulting motion of each bob is a superposition of two oscillations, one with the oscillation centre at the fixed support and the other one with the oscillation centre at the coupling rod; and
- identify possible *normal modes* corresponding to an oscillation with respect only to one of the two centres of oscillations. If the students are not able to identify these conditions by themselves, the teacher can help them only to verify the existence of normal modes. In one configuration, both pendulums are initially displaced from the equilibrium position in the same direction by the same angle and are then released. The behaviour of the system oscillating in this way is one of the normal modes, known as *in-phase oscillation*: both pendulums oscillate with respect to the centre of oscillation, whose position is in the supporting rod. In the other configuration, the two pendulums are initially displaced from the equilibrium position in the opposite directions by the same angle, and when the two bobs are released they oscillate with respect to the centre of oscillation, whose position is in the coupling rod. This last normal mode is known as the *opposite-phase oscillation*: displacements of the two bobs are always equal but opposite. In both configurations, the students should observe that, when the pendulums oscillate in these two normal modes, there is no energy transfer from one pendulum to the other.

At the end of this phase, the students should have acquired sufficient confidence with the system and its phenomenological behaviour.

Model development

After the exploration phase, students are required to build a simple mathematical model describing and interpreting the behaviour of the system.

Students need to know:

- that a simple pendulum of length l oscillates for small angles with a period T given by

$$T = 2\pi\sqrt{\frac{l}{g}} \tag{1}$$

  with g = acceleration of gravity;
- that the time dependence of an oscillating pendulum displacement can be described, for small angles, by a harmonic function of the type

$$x = A\cos(\omega t + \phi) \tag{2}$$

  where A is the amplitude of motion, ϕ is its initial phase of motion and ω is the angular frequency, given by $\omega = \frac{2\pi}{T}$;
- how to use, at an elementary level, an electronic spreadsheet such as Excel, to quickly estimate and graph the displacement x as a function of t.

On the basis of this knowledge, students are expected to simulate, with a mathematical model, what they hypothesized during the exploration phase, i.e. that the motion of each bob results from two harmonic oscillations with oscillation centres respectively at the fixed support and at the coupling rod.

The students themselves soon conclude that the system can be modelled using a simple electronic spreadsheet such as Excel. We then suggest that they tabulate and graph the two harmonic oscillations $x_1(t) = A_1\cos(\omega_1 t)$ and $x_2(t) = A_2\cos(\omega_2 t + \phi)$, with $\omega_1 = \sqrt{\frac{g}{l}}$ and $\omega_2 = \sqrt{\frac{g}{h}}$ respectively, and their sum $x(t) = x_1(t) + x_2(t)$ for fixed l and h. In our experimental conditions, i.e. in the approximation of small angle oscillations, the two harmonic oscillations were assumed to be of nearly equal amplitude [11]: this assumption is not essential, but if the amplitudes of harmonic oscillations are different, then it follows that the energy transfer is not complete.

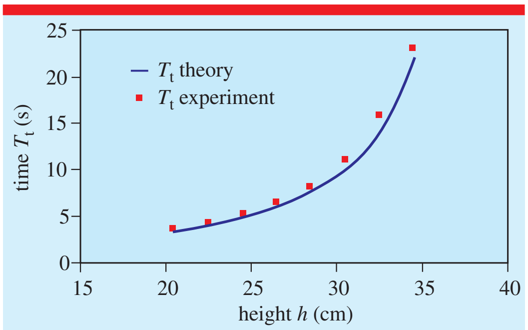

Figure 4. The mean time T_t of the energy transfer versus h. Dots represent the experimental results. The continuous curve interpolates the T_t values as estimated from the simulated graphs of the resulting motion of the coupled pendulums with the same experimental values of the lengths l and h.

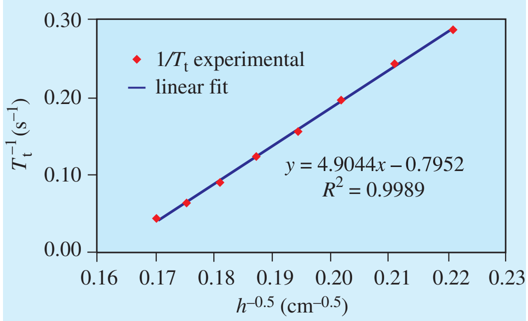

Figure 5. Linearization of functional dependence of time T_t of the motion transfer between the two pendulums on the parameter h. Comparison of the experimental results with the simulated data.

the bob initially displaced from its equilibrium position to the other that was at rest. This time interval is measured by starting the stopwatch when the bob initially displaced reaches the minimum amplitude condition (i.e. practically at rest). To reduce measurement error it was suggested they stop the stopwatch after the first bob reaches its minimum amplitude five or more times. The measured values are then divided by five in order to have a mean time T_t. Measurements were made with a fixed length l of the coupled pendulum, for at least eight h values, corresponding to $\omega_1 = 5$ rad s^{-1} and 5.3 rad s$^{-1} \leqslant \omega_2 \leqslant 6.9$ rad s^{-1}. The values obtained by measurements of the mean time T_t versus h are then compared with those extracted from the spreadsheet graphs for the same experimental values of the lengths l and h. An example of the results is shown in figure 4. It becomes evident that the spreadsheet model adequately describes what is found experimentally.

Again, if the students have good mathematical backgrounds and, in particular, know the sum–product identities for the trigonometric functions sine and cosine, they can perform another experimental test of the model predictions. In fact, starting from two simple harmonics motions, described by $x_1(t) = A_1 \cos(\omega_1 t)$ and $x_2(t) = A_2 \cos(\omega_2 t + \phi)$, and considering the case of equal amplitudes to simplify the calculus, the teacher can easily prove that the resultant oscillatory motion $x(t) = x_1(t) + x_2(t)$ has an amplitude modulated by a frequency equal to the difference of the frequencies of the two component motions:

$$\nu = \nu_2 - \nu_1 = \frac{1}{2\pi}\sqrt{\frac{g}{h}} - \frac{1}{2\pi}\sqrt{\frac{g}{l}} \tag{3}$$

where

$$\begin{aligned} \nu_1 &= \frac{\omega_1}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \quad \text{and} \\ \nu_2 &= \frac{\omega_2}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{h}}. \end{aligned} \tag{4}$$

That is, for fixed length l, time T_t depends on the variable h in this way:

$$\frac{1}{T_t} = \nu = m\frac{1}{\sqrt{h}} + b \tag{5}$$

with

$$m = \frac{1}{2\pi}\sqrt{g} \qquad \text{and} \qquad b = -\frac{1}{2\pi}\sqrt{\frac{g}{l}}.$$

The linear dependence between transformed variables is evident. Predictions can be compared with experimental data. This is done in figure 5, where the expected dependence of time of motion transfer between the pendulums on the parameter h is evident.

The result obtained by this approach, despite the equal amplitude hypothesis, is valid only in the approximation of small angle oscillations, as is shown in Priest and Poth [11]; they are, in general, in quite good agreement (better than 5%) with the expected value of the acceleration due to gravity g.

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