Investigating Flight with a Toy Helicopter

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Flight fascinates people of all ages. Recent advances in battery technology have extended the capabilities of model airplanes and toy helicopters. For those who have never outgrown a childhood enthusiasm for the wonders of flight, it is possible to buy inexpensive, remotely controlled planes and helicopters. A toy helicopter offers an opportunity to investigate and study some basics of flight.[1]

An airplane is able to fly because of its wings.[2-4] A helicopter flies by using a spinning rotor blade.[5] Those familiar with the law of conservation of angular momentum immediately recognize one of the problems a successful helicopter has to overcome. Since the rotor is spinning in one direction, once the helicopter lifts off, the body of the helicopter will spin in the opposite direction so that the total angular momentum remains zero. This is not an ideal situation for either the pilot or the passengers in a helicopter. A small rotor mounted on the tail of the helicopter with rotor plane perpendicular to the ground provides a torque to counter the tendency of the helicopter body to rotate. Large helicopters with two rotors overcome the difficulty by the simple expedient of having the rotors spin in opposite directions.

The rotor blade of a helicopter forces air downward. Newton's third law requires that the air in turn exert an equal force upward on the rotor. For a helicopter to hover, the force exerted by the rotor blade on the air must be equal to the weight of the helicopter. With a few simple assumptions and basic laws of physics, we can find a reasonable expression for the rotational frequency of the rotor blade given the weight of the helicopter.[6] The appendix shows a derivation of the relation between the rotational frequency of the rotor blade and the helicopter weight:

$$f^2 = mg/(8\pi^3\rho\lambda^2 R^4). \tag{1}$$

In the equation, m is the mass of the helicopter, g is the free-fall acceleration, ρ is the air density, λ is a parameter called the *rotor inflow ratio*, and R is the radius of the rotor blade.

It is possible to make an experimental test of Eq. (1). Adding mass to a toy helicopter without disturbing its center of mass is not simple. It is also problematic to measure the rotational frequency of the rotor blade while the helicopter is in flight. It is easier to keep the helicopter grounded. A small platform was constructed with a few pieces from an erector set. The helicopter was fixed to the platform. The mass of the helicopter mounted on the platform is more than the helicopter is capable of lifting. The platform and helicopter are then placed upon the pan of an electronic balance. As power is applied to the helicopter via its infrared-linked controller, the helicopter attempts to lift off. As the rotor spins, the helicopter mounted on the platform tends to vibrate and slide off the pan of the balance. The simple expedient of taping the platform to the pan of the balance eliminates the problem. The lift produced by the helicopter reduces the mass recorded by the electronic balance. Increasing amounts of power produce increasing lift. The mass lifted by the helicopter is the difference between the mass reading of helicopter plus platform with no power applied and the mass reading when the rotor is spinning. The frequency f of the rotor blade can be determined with a stroboscope. A number of measurements of the mass lifted were made at different frequencies. Table I shows the results and Fig. 1 represents a graph of the rotational frequency squared versus the mass lifted by the helicopter. There is a linear relationship between the mass lifted and the square of the rotational frequency of the rotor.

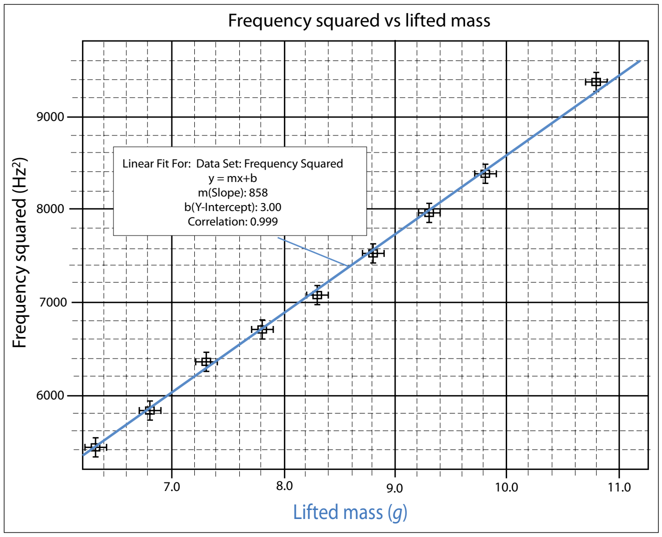

Fig. 1. Graph of frequency squared vs lifted mass.

Table I. Apparent mass loss for different rotational frequencies of the rotor blade of the helicopter.

Lifted mass (g)	Frequency (Hz)
10.8	97
9.8	91
9.3	89
8.8	87
8.3	84
7.8	82
7.3	80
6.8	76
6.3	74

The toy helicopter used in the experiment has two rotor blades, a smaller stabilizer mounted above a larger rotor. Any contribution of the stabilizer blade to the lift was ignored. The

 DOI: 10.1119/1.3488189

toy helicopter used for this experiment has a mass of 11 g. The radius of the larger rotor blade is 6.5 cm. Take the density of air to be 1.3 kg/m^3 and the free-fall acceleration g to be 9.8 m/s^2. The graph in Fig. 1 of the frequency squared versus the mass lifted yields a slope that, when combined with the other known values, enables one to experimentally determine the value of λ. The best fit to the data results in a value of λ = 0.044. Since λ is the ratio of inflow air speed to rotor tip speed, it is possible to arrive at a speed for the air passing through the plane of the rotor. At a frequency of 80 Hz, ωR is 32.7 m/s. That gives an air speed through the rotor plane of 1.4 m/s, which seems a reasonable result.

It is possible with practice to fly the helicopter so that it hovers with little motion. A stroboscopic measurement of the rotational frequency of the flying helicopter while hovering resulted in a frequency of 84 Hz. This measurement indicates that the helicopter produces more lift when it is not tied to a platform. That is interesting because it is well known that a hovering helicopter produces better lift near the ground. Some factor in being tethered to the platform makes the toy helicopter seem less capable of lifting than it actually is. With the aid of Eq. (1), a mass of 11 g for the freely hovering helicopter and a frequency of 84 Hz for the rotor blade yields a value of $\lambda = 0.052$.

In spite of ignoring many complicated effects in the flight of a helicopter, it is interesting that a simple physics model can predict the relationship between the rotor frequency and the lifting capability of a toy helicopter.

Appendix

The rotor blades of a helicopter that is hovering with the rotor plane horizontal will push air through the circular plane swept out by the blades (see Fig. 2). Assume a smooth, well-defined airstream flowing through the rotor disk area. The mass flux through the disk is given by the expression:

$$dm/dt = \rho A v. \tag{2}$$

In the equation dm/dt is the rate at which a mass of air passes through the rotor disk area, ρ is the density of the air, A is the area swept out by the rotor blade, and v is the speed of the air moving down through the rotor disk area. The force exerted by the rotor is equal to the time rate of change of momentum:

$$F = dp/dt. \tag{3}$$

Far above the helicopter, the air is at rest. Below the helicopter, assign the speed of the downstream airflow to be w. Assume the air stream is incompressible and inviscid. Then the force exerted by the rotor is:

$$F = (dm/dt)w. \tag{4}$$

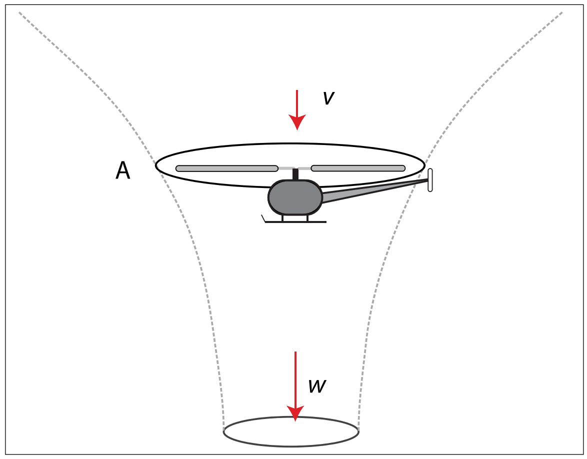

Fig. 2. Air stream for hovering helicopter.

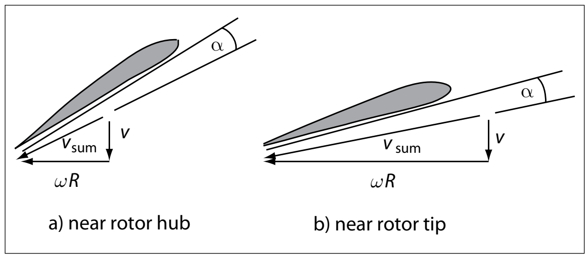

Fig. 3. Angle of attack α as measured from the vector sum of v and ωR.

The conservation of energy requires that the rate of work done by the rotor Fv be equal to the rate of change of kinetic energy in the fluid, $1/2(dm/dt)w^2$:

$$Fv = 1/2\,(dm/dt)w^2. \tag{5}$$

Dividing Eq. (5) by Eq. (4) leads to the result that $w = 2v$. Combining the information, the force that the helicopter rotor exerts must be:

$$F = 2(dm/dt)v = 2\rho A v^2. \tag{6}$$

The speed of the tip of a rotor blade is ωR, where ω is the angular frequency of the rotor as it turns and R is the radius of the rotor. In general, the relationship between the speed of the air v through the plane of the rotor blade and the speed of the tip of the rotor ωR is complicated. The rotor of a helicopter has a blade twist. The blade is twisted in order to keep its angle of attack as constant as possible as one moves out along the rotor blade. The angle of attack α is measured away from the vector sum of v and ωR (see Fig. 3). The speed of the air v drawn down past the rotor blade is relatively constant. But the tangential velocity of the rotor blade ωR increases with radius. Near the rotor hub, the airfoil of the rotor needs to be at a large angle to the horizontal plane defined by the path of the rotor because of the relative magnitudes of v and ωR. Far from the hub near the rotor tip, the airfoil of the rotor needs to be at a smaller angle to the horizontal plane because the direction of the vector sum of v and ωR has changed.[7] If we limit ourselves to the situation in which a helicopter is hovering, one can now define a parameter λ called the *rotor inflow ratio.* By definition λ is the ratio of the speed of the air v through the rotor plane to the speed of the rotor tip ωR:

$$\lambda = v/\omega R. \tag{7}$$

Using this equation to replace v, we arrive at the expression:

$$F = 2\rho A\lambda^2\omega^2 R^2. \tag{8}$$

Since ω is the angular frequency, $\omega = 2\pi f$, and since $A = \pi R^2$, we can relate the force required to make a helicopter hover to the frequency of the rotor blade:

$$mg = 8\pi^3\rho\lambda^2 R^4 f^2. \tag{9}$$

Note that the expression indicates that the weight lifted by the helicopter is proportional to the square of the rotor blade frequency.

References

1. The toy helicopter is an Air Hogs RC Apache Havoc. It is powered by a 3.7-V lithium poly battery. The charge capacity of the battery is 50 mAh. The large rotor and the tail rotor are controlled by an infrared-linked transmitter. This inexpensive helicopter is available in large discount stores.
2. Chris Waltham, "Scaling in model aircraft," *Am. J. Phys.* **65**, 1082–1086 (Nov. 1997).
3. Chris Waltham, "The flight of a balsa glider," *Am. J. Phys.* **67**, 620–623 (July 1999).
4. Holger Babinksy, "How do wings work?" *Phys. Educ.* **38**(6), 497–503 (Nov. 2003).
5. Yee-kong Ng, Se-yuen Mak, and Choi-man Chung, "Demonstration of Newton's third law using a balloon helicopter," *Phys. Teach.* **40**, 181 (March 2002).
6. The development of the equations in the appendix follows the outline of texts on helicopter aerodynamics. See for example Wayne Johnson, *Helicopter Theory* (Princeton University Press, Princeton, NJ, 1980); J. Seddon and Simon Newman, *Basic Helicopter Aerodynamics*, 2nd ed. (Blackwell Science Inc, 2001); or J. Gordon Leishman, *Principles of Helicopter Aerodynamics,* 2nd ed. (Cambridge University Press, New York, 2005).
7. Thanks to the reviewer whose expertise provided many helpful suggestions throughout the article.

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