# Wine Glasses, Bell Modes, and Lord Rayleigh

*By Thomas D. Rossing*

Many physics teachers, in a lecture demonstration or at the dinner table, have rubbed the edge of a wine glass with a moist finger to make it sing. But how many of us could correctly describe the vibrational motion of the glass?

Test time! Before you read any further, try to answer the following questions:

1. When you rub the glass, are you exciting standing waves, traveling waves, or both?
2. Are sound waves excited in the glass?
3. Does the glass move in the direction of a diameter, in the direction of the circumference, or both?
4. Does the frequency go up or down when you add water (or wine) to the glass?
5. If you rub two glasses that have identical diameters but different thicknesses, which glass will have the higher frequency?
6. If you rub two glasses of the same thickness but with different diameters, which glass will have the higher frequency?
7. If you rub two glasses of the same thickness and diameter but having different heights, which glass will have the higher frequency?

If you have difficulty answering any of these questions, don't feel bad. I have found incorrect answers to nearly all of them in physics textbooks. Fortunately, Rayleigh[1] gave us most of the correct answers a hundred years ago, and French[2] has more recently discussed the theory in considerable detail.

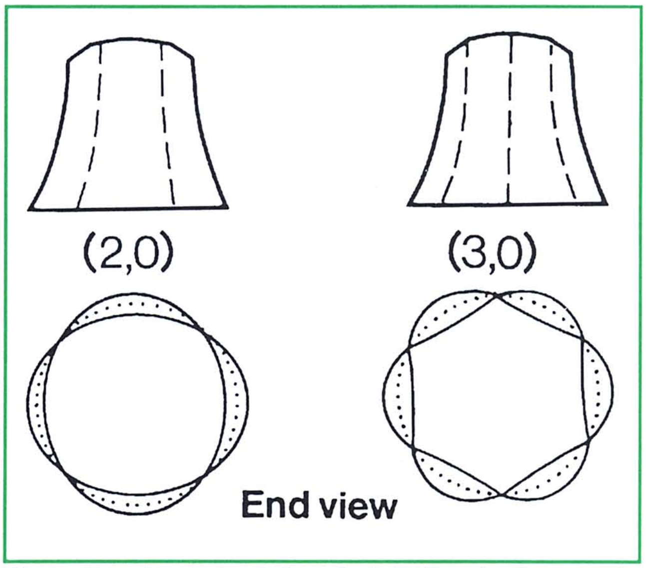
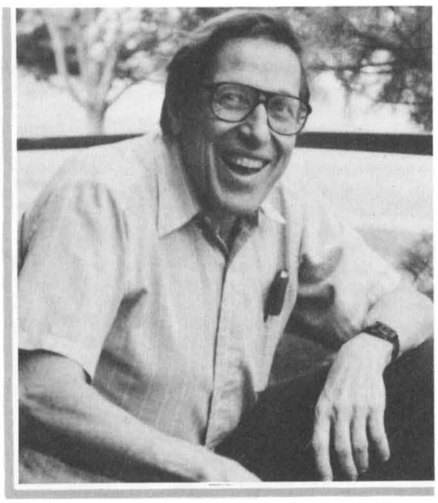



**Fig. 1. Side view and end view of a bell vibrating in two of its lowest modes, having two and three nodal meridians.**

To understand the vibrational motion of the glass, we should first note that tapping the glass with a spoon excites it to vibrate at essentially the same frequency as does rubbing its edge. This suggests that the motion is both radial and tangential and provides answers to Questions 2 and 3. In fact, tapping the glass excites a number of "bell modes," whereas rubbing it mainly excites the lowest of these, the (2,0) mode, with two nodal meridians (the same one that radiates the hum note of a bell; see Fig. 1).[3,4]



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An interesting feature of bell modes is their similarity to the vibrational modes of a flat plate. Nodal meridians replace the modal diameters of flat plates; we attach a label $(m,n)$ to the mode that has $m$ complete (over-the-top) meridians and $n$ nodal circles. (By slightly modifying Chladni's law,[5] it is possible to apply it to a wide variety of nonflat circular plates, including tuned church bells and carillon bells.[6,7]) In a flat circular plate, a carillon bell, or a wine glass, the fundamental mode of vibration is the (2,0) mode, having two complete nodal meridians (or diameters) and no nodal circles. The frequency of this mode is proportional to thickness in all three cases, because plate stiffness increases with $t^2$ but mass is only proportional to $t$. So if you answered Question 5 by saying that the thicker glass has the higher frequency, you are quite correct. Of course adding water only adds mass, so the frequency goes down (Question 6).

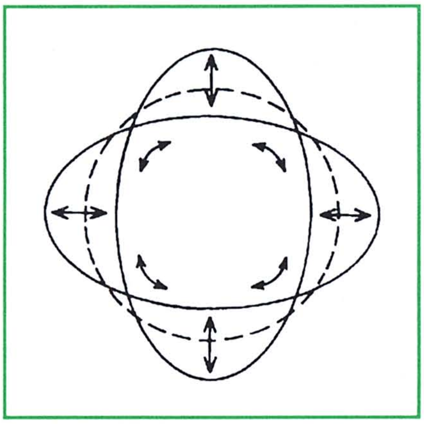

**Fig. 2. Top view (exaggerated) of a wine-glass vibrating in its fundamental mode (similar to the 2,0 mode in Fig. 1). For $m = 2$, the tangential amplitude is half the normal amplitude.**

Let's take another look at Question 3. To a first approximation at least, the upper rim of the glass changes from circular to elliptical twice per cycle, as shown in Fig. 2. If we assume that the motion is almost entirely flexural (as opposed to extensional), we see that there must be both radial and tangential motion. The radial and tangential components of the motion are proportional to $m \sin m\theta$ and $\cos m\theta$, respectively;[6] for the (2,0) mode the maximum tangential motion is half the maximum normal motion. In general the maximum tangential amplitude is $1/m$ times the maximum normal amplitude, and each occurs at a minimum of the other.[1] We can excite the glass to vibrate either by striking it (in the normal direction) with a spoon or by rubbing it (in the tangential direction) with our finger. In his lecture demonstrations, Rayleigh apparently used to excite large bells by bowing them (in the normal direction) with a violin bow.[1]

Rayleigh describes an interesting experiment that I have not yet tried to duplicate (but perhaps some reader has). A glass bell jar ("air-pump receiver," he calls it) is set into vibration by rubbing the edge with a moistened finger. "A small chip in the rim, reflecting the light of a candle, gave a bright spot whose motion could be observed with a Coddington lens suitably fixed. As the finger was carried round, the line of vibration was seen to revolve with an angular velocity double that of the finger; and the amount of excursion (indicated by the length of the line of light), though variable, was finite in every position." What I think Rayleigh is describing is a rather complicated stick-slip (tangential) motion, somewhat like the so-called Helmholtz motion of a bowed violin string.[8] No doubt someone has investigated wine-glass motion using modern stroboscopic and photographic equipment, but I am not aware of any report in the published literature.

How do we answer Question 1? I would think that we are observing standing waves resulting from flexural waves racing around the glass in both directions (at a speed much greater than our finger can move). While our finger is in contact with the glass, we force one of the nodes to occur close to (but not precisely at) the point of contact. When we remove our finger, the nodes may be free to rotate (if the glass has nearly perfect symmetry) or they may be pinned at one location by small imperfections in the glass. We have already answered Question 2—they are not sound waves but flexural waves.

With respect to Question 6, Rayleigh cites the work of Fenkner (1879), who found the vibrational frequencies of a thin-walled cylinder to be inversely proportional to the square of the radius and very nearly independent of the height. Why the square of the radius? Because the speed of flexural waves is proportional to $\sqrt{f}$. Thus, if the time for a wave to travel the circumference is taken to be its period, we can say that

$$1/f = \frac{2\pi r}{k\sqrt{f}} \tag{1}$$

from which $f \propto 1/r^2$.

What about higher modes of vibration in the glass? The next mode will nearly always be the (3,0) mode for which $2\pi r = 3\lambda$. Since the wave velocity is proportional to $\sqrt{f}$, the frequencies of the $(m,0)$ modes are nearly proportional to $m^2$, so the (3,0) mode frequency is approximately 9⁄4 times that of the (2,0) mode. Many modes can be identified in large bells. My colleagues in Loughborough, England have identified 134 modes of vibration in an English church bell.[9] Higher modes of vibration can be excited in a wine glass by striking it or by applying a sinusoidal force by means of a small magnet and coil or by placing it in front of a loudspeaker. I have not been able to excite higher modes by rubbing the edge with my finger; has anyone done this?

Standing waves in a wine glass have been likened to waves in the Bohr model of the atom.[10] It is true that $2\pi r = m\lambda$ in both cases (not $\lambda$ as stated in Ref. 10), but here the analogy ends. In the Bohr atom, the orbital radius increases with $m^2$, but in the wine glass it remains constant. Thus the dispersion relationships are quite different. The atomic analog of the wine glass might rather be an electron whirling about the nucleus on the end of a string.

Now that we understand something about the physics of wine glasses, perhaps we should consider making some serious music. If we carefully select a set of glasses, we should be able to play tunes on them. Right? Indeed we